CAD-Based Object Reconstruction Using Line Photogrammetry for Direct Interaction between GEMS and a Vision System

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Abstract
The paper presents an algorithm for reconstructing Geometric Element Modeling System (GEMS)-based industrial objects using line photogrammetry (LP) in order that there can be direct interaction between GEMS and a vision system. In the GEMS system, each object is constructed by Boolean set operators of constant solid geometry (CSG) primitives, each face of which is described by a Boundary representation (B-pre); and straight lines, curves, planes, and curved surfaces are described by parametric equations. Based on this type of data structure and representation, the geometric elements which describe industrial objects in GEMS, are taken as unknowns, and are directly solved for by matching images (2D) and objects (3D). Compared to traditional photogrammetric DEM products, the advantage of this algorithm is that the product of reconstruction is parameters describing the industrial object, which can be directly fed back to the GEMS system. It is convenient to communicate between GEMS and the vision system without any transformation. On the other hand, the developed mathematical model can be used not only for line-to-line correspondences but also for line-to-surface or surface-to-surface correspondences. A number of simulations and practical experiments demonstrated that this algorithm is feasible and reliable in industrial computer vision.

Introduction
Our vision research group at the Department of Computer Science and Technology of Tsinghua University was designing a vision system for the Chinese Aerospace Industrial Department. The objective of the system was to provide sufficient and reliable vision information for an Experimental Testbed of Space Intelligent Robotic Manipulators. We call this system the Space Intelligent Vision Equipment (SIVE) (Yu, 1995).

One of tasks in SIVE is to reconstruct Geometric Element Modeling System (GEMS)-based in industrial objects in order that the operator of SIVE can manipulate and locate a robot arm by comparing the reconstructed object and one represented in GEMS (GEMS, a type of CAD system, is described in detail in the next section). However, the traditional product of reconstruction is a DEM or DSM (digital surface model); and this form of product cannot be fed back into the GEMS system directly. On the other hand, when reconstructing these industrial objects, the following problems are encountered: (1) except for geometric features (e.g., lines and edges), distinct features can seldom be found on a surface; and (2) it is difficult to find 2D-to-2D corresponding points due to the vacuum of space, non-uniform lighting, object drift or instability, and the dynamic SIVE environment in space. Obviously, the traditional approach of point-to-point correspondence is not suitable for this real situation.

Thus, we had to develop an algorithm which could reconstruct 3D industrial objects based on the prior knowledge of GEMS when considering the above real situations.

For this reason, this paper presents an algorithm for reconstructing GEMS-based industrial objects using line photogrammetry (LP). This is because

- 3D spatial lines (straight or curved) and surfaces (planar or curved faces) are valuable and available because their 2D projections in images are more easily extracted than are point features in industrial images;
- line features (the term line features refers to one-dimensional straight lines and curves, or to the projection of spatial lines or spatial surfaces) are more robust, are much more geometric, and contain more semantic information than do point features;
- an object is constructed by combining Construct Solid Geometry (CSG) and B-representation (B-pre) in GEMS. Each primitive is described by parameters; and
- the data structure and topologic information for industrial objects in GEMS are available.

Based on the above characteristics, we first developed a mathematical model for reconstructing a single primitive, and then went on to develop a mathematical model for reconstructing complex objects while considering the data structure and topologic information contained in GEMS. In the model for reconstructing a single primitive, geometric elements, which represent industrial objects in GEMS, are taken as unknowns, and are directly solved for by matching images (2D) and objects (3D). The reconstruction of complex objects will make full use of prior knowledge of the data structure and representation in GEMS as well as the mathematical models for reconstructing an individual primitives.

Some references in the photogrammetric community to previous research and practical work based on line-to-line correspondences in computer vision and photogrammetry include Doehler (1975), who used horizontal and vertical lines as control features for close-range photogrammetric

In the computer community, Yen and Huang (1983) and Liu and Huang (1988a; 1988b) solved a set of non-linear equations for the motion parameters using straight line correspondences. Mitche et al. (1986) interpreted structure and motion using the property of angular invariance between line correspondences. Faugeras et al. (1987) approximated the non-linear equations by linear equations and used an iterative extended Kalman filter to estimate the motion parameters using point-to-line correspondences. Spetsakis and Aloimions (1990) and Liu et al. (1990) developed linear algorithms for estimating motion and structure parameters from line correspondences. Chen (1991) discussed the existence condition and closed-form solutions from line-to-plane correspondences. 2D-to-3D matching between lines and surfaces was published by Faugeras and Hebert (1986), Kim and Aggarwal (1987), Chen and Huang (1990), and Zhang and Faugeras (1991). Weng et al. (1992) discussed the closed-form solution, uniqueness, and optimization when estimating motion and structure parameters using line correspondences.

Representation and Modeling of Industrial Objects in GEMS

The mathematical model of reconstruction to be described is based on a single primitive. The reconstruction of complex objects is based on single primitive reconstruction, data structure, and the modeling method in the GEMS system. Thus, we first briefly overview the data structure and modeling method in GEMS. The GEMS is a CAD system developed by the CAD research group in the Department of Computer Science and Technology at Tsinghua University (Sun, 1989; Sun, 1990; Ren, 1991). In GEMS, each object is described by combining CSG (construct solid geometry) and Boundary representation (B-pre).

CSG represents a complex object using geometric transformations and Boolean set operators of primitives. These usual primitives are cubes, cylinders, spheres, and so on. The transformations indicate geometric transformations (shift, rotation, and scale) and Boolean set operators, including mono-tuple operators (intersection, merger, and difference) and mono-tuple operators (rotation, transfer, and scale transformation). This kind of representation can be illustrated by a binary tree, whose leaves represent various primitives and whose nodes represent the Boolean operator (Figure 1). In CSG, the important representation is the description of the primitives. The so-called primitive is an enclosed polyhedral solid, whose surfaces are composed of three elements (vertex, edge, and face). Each primitive is defined by a group of parameters, which include location information (element) and size information (element). For example, a cylinder has the central coordinates of its bottom, the direction of its axis (location elements), and its radius (size element). Some common primitives and their representations are illustrated in Figure 2.

This representation for primitives is concise, integrated, and distinct. Its topological relations, however, are not distinct. B-pre can give a complete and explicit description. In B-pre, each object is composed of finite surfaces, and each surface is enclosed by finite boundaries. Thus, this representation is divided into volume, faces, loops, edges, and vertices. Moreover, B-pre can still provide detailed geometric and topologic information (see Figure 3).

Geometric information indicates how to describe the geometric characteristics of an object. For example, a point is described by \( X, Y, Z \) in a Cartesian coordinate system; a straight line is represented by \( P = C + d \cdot t \) (\( P \) denotes an arbitrary point on line, \( d \) denotes a direction vector, and \( C \) denotes a
fixed point on a line); and a surface, such as sphere, is represented by \( x = R \cos(\theta) \sin(\phi), y = R \sin(\theta) \sin(\phi), \) and \( z = R \sin(\theta) \) (where \( R \) denotes the radius, and \( \theta \) and \( \phi \) are angular parameters).

Topological information is used to describe quantity, type, and relation of vertices, edges, and faces in order to guarantee that the yielded object is unique and legal.

Because CSG and B-pre each have their own advantages and disadvantages, an object is constructed by combining CSG and B-pre in the GEMS system. The process of an industrial object construction in GEMS (CSG and B-pre) is illustrated in Figure 4. Our mathematical model for reconstructing 3D objects using line photogrammetry is based on the GEMS data structure. In this way, the geometric elements can directly be solved for by matching (un-correspondence point) image to object.

Mathematical Model of Reconstruction

In the following analysis it is assumed that extraction of linear features and their locations in image space have already been accomplished and that the correspondences between these features has been established.

Reconstruction Principle of Linear Features

Each object is constructed using three coordinate systems (see Figure 5) in GEMS: the camera coordinate system \( C - x_c y_c z_c \), the object (model) coordinate system \( O - x y z \), and the primitive coordinate system \( P - x_P y_P z_P \). The following mathematical models are based on the primitive coordinate system. The mathematical model based on the model coordinate system is for complex objects, which will be handled in the next subsection.

Assume that \( T \) is a spatial linear feature in the scene (see Figure 6), and that its geometric information can be expressed by

\[
\begin{align*}
X &= X(s) \\
Y &= Y(s) \\
Z &= Z(s)
\end{align*}
\]
where \( Z \) denotes parameters describing the primitive, including position parameters and geometric parameters. Assuming that the linear features \( t \) and \( t' \) in the left and right image planes, respectively, are perspective projections of the spatial linear feature \( T \), and that an arbitrary point \( p \) in linear feature \( t \) is observed. The collinearity equations can be formulated for point \( P \) by ray line \( pP \):

\[
x_p - x_o = -\frac{a_1(x_p - x_o) + b_1(y_p - y_o) + c_1(z_p - z_o)}{a_1(x_o - x_o) + b_1(y_o - y_o) + c_1(z_o - z_o)}
\]

\[
y_p - y_o = -\frac{a_2(x_p - x_o) + b_2(y_p - y_o) + c_2(z_p - z_o)}{a_2(x_o - x_o) + b_2(y_o - y_o) + c_2(z_o - z_o)}
\]

where \( a_i, b_i, c_i \) are components of the rotation matrix for the left image; \( x_o, y_o, z_o \) are coordinates of the perspective center of the image sensor; \( f \) denotes a constant of the image sensor; \( x_p, y_p \) denote the coordinates of the point \( p \) in the image plane; and \( x_o, y_o, z_o \) denote the coordinates of the scene point \( P \).

In addition, the point \( P \) in the object space still satisfies the equation describing the object:

\[
X_p = X(s)
\]

\[
Y_p = Y(s)
\]

\[
Z_p = Z(s)
\]

The mathematical model of reconstruction using line photogrammetry is formed by substituting the parametric Equation 3 into the collinearity Equation 2. We then have:

\[
x_p - x_o = -\frac{a_1(x_p - x_o) + b_1(y_p - y_o) + c_1(z_p - z_o)}{a_1(x_o - x_o) + b_1(y_o - y_o) + c_1(z_o - z_o)}
\]

\[
y_p - y_o = -\frac{a_2(x_p - x_o) + b_2(y_p - y_o) + c_2(z_p - z_o)}{a_2(x_o - x_o) + b_2(y_o - y_o) + c_2(z_o - z_o)}
\]

If the other arbitrary point \( P' \) (non-correspondence with point \( P \)) in the feature \( t' \) is observed, the scene point \( P' \) corresponds to the point \( P' \). Similar equations hold. For any point, six fixed unknowns \( X_s, Y_s, Z_s, \alpha, \beta, \gamma \) and an additional unknown \( t \) or \( t' \) are required in order to solve Equations 7 and 8. A unique solution to this problem is that the unequal condition \( 2(N + N') > 6 \) is meant for two images. However, the fixed point \( C \) can be selected arbitrarily, and the module of directional vector \( d = (\alpha, \beta, \gamma) \) varies with components \( \alpha, \beta, \gamma \). That means that \( t \) is correlated with point \( C \). In order to uniquely determine the straight lines, Mulawa and Mikhail (1988) separately solve for the fixed point \( C \) in the directional vector \( d = (\alpha, \beta, \gamma) \) and parameters \( t \) with constraint \( \|d\|^2 = 1 \) and \( (C - U) \cdot d = 0 \) (the vector from fixed point to exposure center is orthogonal to straight line). The extended model is:

\[
F_1 = \hat{P} \cdot (\hat{C} - \hat{U}) = 0
\]

\[
F_2 = \|\hat{d}\|^2 = 1
\]

\[
F_3 = (\hat{C} - \hat{U}) \cdot \hat{d} = 0
\]

where \( U \) is the exposure center and \( P \) is the coordinates of any point.

- **Conic Curve**

Conic curves include circles, hyperbolas, and ellipses, as well as the perspective projections of regular surfaces such as spheres, spheroids, and so on. Let us take sphere as an example to derive the mathematical model.

A spherical surface can be expressed by:

\[
x = R \sin(\varphi) \cos(\theta)
\]

\[
y = R \sin(\varphi) \sin(\theta)
\]

\[
z = R \cos(\varphi)
\]
The primitive coordinate system describing a sphere is illustrated in Figure 8. An arbitrary measured point \( m \) in the right image is a perspective projection of the point \( M \) on the spherical surface. The collinearity equations can be formed by ray line \( mM \). Moreover, the point \( M \) will still satisfy Equation 10. Thus, we have (for the right image):

\[
x_m - x_M = -f \left( \frac{R \sin(\varphi) \cos(\theta) - X_M}{R \sin(\varphi) \cos(\theta)} \right) + b \left( \frac{R \sin(\varphi) \sin(\theta) - Y_M}{R \sin(\varphi) \sin(\theta)} \right) + c \left( \frac{R \cos(\varphi) - Z_M}{R \cos(\varphi)} \right) \tag{11a}
\]

\[
y_m - y_M = -f \left( \frac{R \sin(\varphi) \cos(\theta) - X_M}{R \sin(\varphi) \cos(\theta)} \right) + b \left( \frac{R \sin(\varphi) \sin(\theta) - Y_M}{R \sin(\varphi) \sin(\theta)} \right) + c \left( \frac{R \cos(\varphi) - Z_M}{R \cos(\varphi)} \right) \tag{11b}
\]

where \( R \) is a radius, and \( \varphi \) and \( \theta \) are the elevation and azimuth, respectively. If the other point \( h \) (uncertain correspondence to point \( m \)) is measured in the left image plane (the angles \( \varphi \) and \( \theta \) differ from those in the point \( m \)), we have the following equations (for the left image):

\[
x_h - x_M = -f \left( \frac{R \sin(\varphi) \cos(\theta) - X_M}{R \sin(\varphi) \cos(\theta)} \right) + b \left( \frac{R \sin(\varphi) \sin(\theta) - Y_M}{R \sin(\varphi) \sin(\theta)} \right) + c \left( \frac{R \cos(\varphi) - Z_M}{R \cos(\varphi)} \right) \tag{12a}
\]

\[
y_h - y_M = -f \left( \frac{R \sin(\varphi) \cos(\theta) - X_M}{R \sin(\varphi) \cos(\theta)} \right) + b \left( \frac{R \sin(\varphi) \sin(\theta) - Y_M}{R \sin(\varphi) \sin(\theta)} \right) + c \left( \frac{R \cos(\varphi) - Z_M}{R \cos(\varphi)} \right) \tag{12b}
\]

To measure a point in different line feature each time, two equations are created, but two unknowns \( \varphi \) and \( \theta \) are added. In this case, when the unequal condition \( 2(N + N') > (N + N' + 2M + 1) \) or \( N + N' > 2M + 1 \) is satisfied, a unique solution can be reached (where \( N \) and \( N' \) denote the number of observed points in the left and right image planes, and \( M \) is the number of line features). If a measured point in the left image corresponds to point \( m \), the angles \( \varphi \) and \( \theta \) can be derived from the ray line \( mM \). If a measured point in the right image corresponds to point \( m \), the angles \( \varphi \) and \( \theta \) can be derived from the ray line \( hH \).

\[
X_h = R \cos(\theta) \tag{13a}
\]

\[
y_h = R \sin(\theta) \tag{13b}
\]

\[
z_h = A' R \cos(\theta) + B' R \sin(\theta) + D' \tag{14}
\]

where \( A, B, C \) are components of the normal vectors of the plane; \( D \) is a constant; \( R \) is the radius of the cylinder; \( \theta \) indicates a parameter, and \( A' = A/C, B' = B/C, D' = D/C \). If a point \( h \) in the intersection curve of the left image is observed, the collinearity equations can be formulated by the ray line \( hh \): i.e.,

\[
x_h - x_M = -f \left( \frac{R \sin(\varphi) \cos(\theta) - X_h}{R \sin(\varphi) \cos(\theta)} \right) + b \left( \frac{R \sin(\varphi) \sin(\theta) - Y_h}{R \sin(\varphi) \sin(\theta)} \right) + c \left( \frac{R \cos(\varphi) - Z_h}{R \cos(\varphi)} \right) \tag{15a}
\]

\[
y_h - y_M = -f \left( \frac{R \sin(\varphi) \cos(\theta) - X_h}{R \sin(\varphi) \cos(\theta)} \right) + b \left( \frac{R \sin(\varphi) \sin(\theta) - Y_h}{R \sin(\varphi) \sin(\theta)} \right) + c \left( \frac{R \cos(\varphi) - Z_h}{R \cos(\varphi)} \right) \tag{15b}
\]

\[
X_h = R \cos(\theta) \tag{16a}
\]

\[
y_h = R \sin(\theta) \tag{16b}
\]

\[
Z_h = A'R \cos(\theta) + B'R \sin(\theta) + D' \tag{16}
\]

If the other point \( w \) (non-correspondence to point \( h \)) in the right image plane is observed, the similar observation equations are

\[
x_w - x_M = -f \left( \frac{R \sin(\varphi) \cos(\theta) - X_w}{R \sin(\varphi) \cos(\theta)} \right) + b \left( \frac{R \sin(\varphi) \sin(\theta) - Y_w}{R \sin(\varphi) \sin(\theta)} \right) + c \left( \frac{R \cos(\varphi) - Z_w}{R \cos(\varphi)} \right) \tag{17a}
\]

\[
y_w - y_M = -f \left( \frac{R \sin(\varphi) \cos(\theta) - X_w}{R \sin(\varphi) \cos(\theta)} \right) + b \left( \frac{R \sin(\varphi) \sin(\theta) - Y_w}{R \sin(\varphi) \sin(\theta)} \right) + c \left( \frac{R \cos(\varphi) - Z_w}{R \cos(\varphi)} \right) \tag{17b}
\]

\[
X_w = R \cos(\theta) \tag{18a}
\]

\[
y_w = R \sin(\theta) \tag{18b}
\]

\[
Z_w = A'R \cos(\theta) + B'R \sin(\theta) + D' \tag{18}
\]

No matter how many points are measured on the intersection curve in the right image, only one unknown \( \theta \) is added when adding a measured point. Thus, only if the unequal condition \( 2(N + N') > (N + N' + 4) \) or \( (N + N') > 4 \) is satisfied, can the unique solution be obtained. The above mathematical models of reconstruction used
other examples such as spheroids, parabolas, hyperbolas, and tion principles to describe how to reconstruct a single primitive. Basically, other only using at least two primitives, and the two coordinate systems using the mono-tuple Boolean operator of primitives in GEMS. The model described above. That is, only translation and rotation of primitives are limited. To linearize the equation in step (4) and to solve for the parameters, we combine step (2) and step (3).

Reconstruction of Complex Industrial Objects

In the previous section, only the basic principle of recon-
structing simple industrial objects by linear features was dis-
cussed. Moreover, it is assumed that the primitive coordinate system is consistent with the object coordinate system, which is used for determining the orientation parameters of the camera. Actually, any complex industrial object can be constructed using at least two primitives, and the two coordinate systems are not consistent. That is, the geometric elements describing an object have location information except for geometric (size) information. Thus, the mathematical model has to be extended.

As mentioned above, a complex object is constructed by a mono-tuple Boolean operator of primitives in GEMS. The mono-tuple Boolean operator has rotation, translation, and scale. However, the scale is not a concern of the mathematical model described above. That is, only translation and rotation should be considered.

Assume that an industrial object is constructed by cylinder A and cylinder B. The primitive coordinate system describing cylinder B is o-xyz and describing cylinder A is O-XYZ. Moreover, we assume that the O-XYZ is also the model coordinate system. The primitive coordinate system o-xyz describing cylinder B relative to the model coordinate system O-XYZ has the following transformation:

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} =
\begin{pmatrix}
X_0 \\
Y_0 \\
Z_0
\end{pmatrix} + R
\begin{pmatrix}
X_p \\
Y_p \\
Z_p
\end{pmatrix}
\]  

(19)

where \(X_p, Y_p, Z_p\) are coordinates of any point in the primitive coordinate system, \((X_0, Y_0, Z_0)\) is location element of cylinder B in GEMS (see Figure 10), and \(R\) is a rotation matrix. Also, the point \(P\) in the primitive coordinate system satisfies the equation describing the primitive B: i.e.,

\[
\begin{align*}
X_p &= X(s) \\
Y_p &= Y(s) \\
Z_p &= Z(s)
\end{align*}
\]  

(20)

Substituting Equation 20 into Equation 19, we have

\[
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix} =
\begin{pmatrix}
X_0 \\
Y_0 \\
Z_0
\end{pmatrix} + R
\begin{pmatrix}
X(s) \\
Y(s) \\
Z(s)
\end{pmatrix}
\]  

(21)

Similarly, substituting Equation 21 into the collinearity Equation 4, we obtain a non-linear equation for reconstructing complex objects, and then we linearize the equation and solve for the unknowns by least-squares estimation (LSM). In the mathematical model for reconstructing complex objects, unknowns involve parameters describing individual primitives and coordinate system transformations (position information termed in GEMS). The number of unknowns depends on various industrial objects and various primitives.

The industrial objects constructed in GEMS are diverse, but the primitives are limited. How to recognize the primitives by GEMS-based a priori knowledge such as data structure and topologic information has been handled in Zhou (1994; 1997). As long as the primitives are recognized, we can know which mathematical model should be chosen from MLOLP for reconstruction. The procedures of reconstructing a complex industrial object are summarized as follows:

1. To recognize the primitives from 2D industrial images based on a priori knowledge of GEMS. The detailed description can be found in Zhou (1994; 1997).
2. To transform the data structure and topologic information into a prior knowledge according to recognized primitives. For example, position information of the primitive coordinate system relative to the model coordinate system, and size information (Zhou, 1994).
3. To retrieve the mathematical model for reconstructing a single primitive from MLOLP according to the result recognized by step (1).
4. To build up the mathematical model for reconstructing complex objects by combining step (2) and step (3).
5. To linearize the equation in step (4) and to solve for the parameters by LSM; and
6. To input the solved parameters into the GEMS system for comparing those solved parameters with the a priori parameters from GEMS.

Algorithm Discussions

Straight lines, curves, and planar and curved surfaces in 3D object space can be described in several forms, e.g., by parameters or implicitly. If a line is represented by the coordinates of a specific number of points, the reconstruction model degenerates to traditional stereo matching, i.e., point-to-point correspondence. Additionally, the considered straight lines or curves in object space in general correspond to straight lines or curves in image space, and the considered planar or curved surfaces in object space in general correspond to planar or curved surfaces in image space in this paper. Basically, the imaged lines can result from the perspective projection of 3D lines or from occluding contours of 3D surfaces. However, many abnormal cases can happen in a perspective projection. The following phenomena must be considered.
Lines in 2D Image Space (2D-2D)

- **Point to Point.** When a line in the right and left images is projected into a point in the left and right images, respectively, the model of reconstruction is not suitable. If this line is projected into a point in one image and is projected into a line in the other image, the model of reconstruction is suitable. However, if a plane in object space is projected into a line in one image and is projected into a plane in the other image, the model of reconstruction is suitable because a plane can be mapped into straight lines.

- **Surface to Surface.** If a surface in object space is projected into a surface in the right image, and is projected into a surface in the other image, the model of reconstruction is suitable.

3D Model Space to 2D Image Plane (3D-2D)

- **Line to Point.** If a line in object space is projected into a point in the left and right images, respectively, the model of reconstruction is not suitable. If this line is projected into a point in one image, and is projected into a line in the other image, the model of reconstruction is suitable.

- **Line to Line.** If a line in object space is projected into two lines in the left and right images, respectively, this is normal.

- **Surface to Line.** If a surface in object space is projected into a line in the right and left images, and is projected into a surface in the other image, the model of reconstruction is suitable.

- **Surface to Surface.** If a surface in object space is projected into a surface in the other image, the model of reconstruction is suitable.

### Table 1. Reconstructed Geometric Elements of a Cube

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Width (mm)</th>
<th>Height (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.V.</td>
<td>C.V.</td>
<td>T.V.</td>
</tr>
<tr>
<td>1000.0</td>
<td>999.883</td>
<td>1000.0</td>
</tr>
</tbody>
</table>

(T.V. = Theoretical Value; C.V. = Calculated Value)

### Table 2. Reconstructed Geometric Elements of a Sphere, a Cylinder, and a Spheroid

<table>
<thead>
<tr>
<th>Sphere Radius (mm)</th>
<th>Cylinder Radius</th>
<th>Cylinder Height</th>
<th>Spheroid Semi Major axis (A)</th>
<th>Spheroid Semi Medium axis (B)</th>
<th>Spheroid Semi Minor axis (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T.V.</td>
<td>C.V.</td>
<td>T.V.</td>
<td>C.V.</td>
<td>T.V.</td>
<td>C.V.</td>
</tr>
<tr>
<td>800.0</td>
<td>799.892</td>
<td>600.0</td>
<td>599.8439</td>
<td>800.0</td>
<td>800.307</td>
</tr>
</tbody>
</table>

### Table 3. Reconstructed Geometric Elements of the Intersections of a Plane and a Cylinder, a Sphere and a Sphere, and a Cylinder and a Cylinder

<table>
<thead>
<tr>
<th>Intersection of a Plane and a Cylinder</th>
<th>Intersection of a Sphere and a Sphere, unit mm</th>
<th>Intersection of a Cylinder and a Cylinder, unit mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius A'</td>
<td>B'</td>
<td>D'</td>
</tr>
<tr>
<td>T.V.</td>
<td>C.V.</td>
<td>T.V.</td>
</tr>
<tr>
<td>800.0</td>
<td>800.25</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Experiments

In order to evaluate this algorithm, we have conducted the following experiments on a Silicon Graphics workstation/Indigo 2.

Simulation Study

The simulated images were generated by back-projection. The camera parameters, including interior and exterior parameters, were designated. The industrial objects were constructed by combining CSG and B-pret, and their data structures were arranged in terms of GEMS. The size of all simulated images without any noise was 200 by 200 pixels with a pixel size of 50 μm. We divided simulations into four groups.

1. Cube: which was reconstructed using straight lines. The reconstructed geometric elements (length, width, and height) are listed in Table 1. Figures 11a and 11b are a stereo pair, and Figure 11c is its 3D reconstruction drawing.

2. Sphere, Cylinder, and Spheroid: which were reconstructed using conic curves. The reconstructed geometric elements (radius, height, semi-major axis, semi-medium axis, and semi-minor axis) are listed in Table 2. Figures 12a and 12b, 13a and 13b, and 14a and 14b are three stereo pairs for a sphere, cylinder, and spheroid, respectively, and Figures 12c, 13c, and 14c are their 3D reconstructions.

3. Intersection of a plane and a cylinder, a sphere and a sphere, and a cylinder and a cylinder: which were reconstructed using intersection curves. The reconstructed geometric elements (radius, height, and other parameters) are listed in Table 3. Figures 15a and 15b, 16a and 16b, and 17a and 17b are their stereo pairs and Figures 15c, 16c, and 17c are their 3D reconstructions.

4. Finally, a complex object was reconstructed by comprehensive use of straight lines, conic curves, and intersection curves. The reconstructed geometric elements (radius, height, and other parameters) are listed in Table 4. Figures 18a and 18b are their stereo pairs and Figure 18c is its 3D reconstruction.
Pattern Reconstruction

Three groups of patterns of industrial objects, which are representative of three scenes, were designed by the GEMS system, and then were manufactured in a factory. The patterns, divided into three groups, were imaged with a CCD camera (type: COSMICAR) located at a distance of about 880 mm. The size of the

![Figure 12. A sphere and its reconstruction.](image)

![Figure 13. A cylinder and its reconstruction.](image)

![Figure 14. A spheroid and its reconstruction (where the spheroid equation is \( \frac{x^2}{A^2} + \frac{y^2}{B^2} + \frac{z^2}{C^2} = 1 \)).](image)

![Figure 15. A plane intersecting a cylinder and their reconstruction.](image)

![Figure 16. A sphere intersecting a sphere and their reconstruction.](image)

![Figure 17. The intersection of a cylinder and a cylinder and their reconstruction (where the equation of the larger cylinder is \( X = R \cos(\theta), Y = R \sin(\theta), \) and \( Z = t \), and the equation of the smaller cylinder is \( X = R' \cos(\theta), Y = t', \) and \( Z = R' \sin(\theta) \); the central axis of the smaller cylinder is consistent with the y-axis of the larger cylinder).](image)

### Table 4. Reconstructed Geometric Elements of a Complex Surface, Consisting of the Intersections of Straight Lines, Conic Curves, and Intersection Curves

<table>
<thead>
<tr>
<th>Primitive Name</th>
<th>Parameters Name</th>
<th>D.V. (mm)</th>
<th>C.V. (mm)</th>
<th>Parameter Names</th>
<th>First Cylinder (mm)</th>
<th>Second Cylinder (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td>length</td>
<td>600.00</td>
<td>600.019</td>
<td>Radius</td>
<td>200.00</td>
<td>200.00</td>
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<tr>
<td></td>
<td>width</td>
<td>400.00</td>
<td>399.815</td>
<td>Height</td>
<td>200.00</td>
<td>200.00</td>
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<tr>
<td></td>
<td>height</td>
<td>200.00</td>
<td>199.992</td>
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<td>0.01</td>
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<td></td>
<td>up radius</td>
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<td>100.101</td>
<td></td>
<td>0.00</td>
<td>0.14</td>
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<tr>
<td></td>
<td>down radius</td>
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<td>200.203</td>
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<tr>
<td></td>
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<td>299.901</td>
<td></td>
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<td>Frustum of cone</td>
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<td>—</td>
<td>Rotate</td>
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</tr>
<tr>
<td></td>
<td>rotation parameters</td>
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<td>—</td>
<td>0.00</td>
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<tr>
<td></td>
<td>Z₀</td>
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<td>—</td>
<td>0.00</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>θ</td>
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<td>—</td>
<td>0.00</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>≈</td>
<td>0.00</td>
<td>—</td>
<td>0.00</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>radius</td>
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<td>99.989</td>
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<tr>
<td></td>
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<td>(Hole) Cylinder</td>
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<tr>
<td></td>
<td>Z₀</td>
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<td>—</td>
<td>—</td>
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<td>θ</td>
<td>0.00</td>
<td>—</td>
<td>—</td>
<td>—</td>
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</tr>
<tr>
<td></td>
<td>≈</td>
<td>0.00</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

D.V. = Design Valve, M.V. = Measurement Valve

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images was 512 by 512 pixels at 8 bits each and 256 gray levels. These industrial objects were quite complex. The procedures for reconstruction are described in the section on Reconstruction of Complex Industrial Objects. The reconstructed objects were drawn with wire frames, which were superimposed over the original images, as illustrated in Figures 19, 20, and 21.

Real Industrial Object
The final experimental data were for a real industrial object from a factory (Figure 22). We calculated its 3D coordinates for 16 vertices and cylinder parameters. True 3D coordinates were measured manually. The measured and calculated values are listed in Table 5.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Measured Value (cm)</th>
<th>Calculated Value (cm)</th>
</tr>
</thead>
<tbody>
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<td>Coordinate of center</td>
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<td>Y</td>
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</tr>
<tr>
<td>2</td>
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<td>154.5</td>
</tr>
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<td>3</td>
<td>21.0</td>
<td>154.5</td>
</tr>
<tr>
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<td>154.5</td>
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<td>12</td>
<td>91.0</td>
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<tr>
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<tr>
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<td>69.0</td>
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<tr>
<td>15</td>
<td>69.0</td>
<td>130.0</td>
</tr>
<tr>
<td>16</td>
<td>21.0</td>
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</tr>
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</table>

Conclusions
The reconstruction of GEMS-based industrial objects using line photogrammetry has been applied to space robot vision. The application of this method has the following advantages:

- The products of reconstruction are parameters which describe industrial objects in GEMS. The parameters can be directly used for drawing industrial objects based on data structure and topologic information in GEMS. Compared to a traditional DEM (raster data) product, we need not be concerned with the procedures for transformation from raster data to GEMS parameters. The near real-time processing of reconstruction is very important because the operator at the control system for the robot vision system needs to decide whether or not to send control commands to the robot, depending on the accuracy of reconstruction by comparing GEMS parameters. Thus, this algorithm, for this point, solves a real problem in the robot vision field.

- The mathematical models of reconstruction require not only 2D-to-2D correspondences, but also 2D-to-3D correspondences (but not points to points on lines or surfaces). Thus, the solution for unknowns is robust. This algorithm is not only used for line-to-line correspondences, but also for line-to-surface or surface-to-surface correspondences.

- As a by-product, the two camera positions may be separated by a large angle (see Figures 15 and 22) compared to the traditional photogrammetric pattern, i.e., the overlap degree of two images is much less than that for the conventional aerial stereo pair.

- The accuracy of reconstruction can meet the location demands of a robot in SIVE.
Acknowledgments
This work was funded by the Chinese Aerospace Industrial Department for the project named Experimental Testbed of Space Intelligent Robotic Manipulators (Space Intelligent Vision Equipment—SIVE). The colleague's encouragement, for which line photogrammetry is applied in robot vision for the solution of 3D reconstruction, from the Department of Computer Science and Technology of Tsinghua University, is appreciated. We also would like to thank those who encouraged us to develop this algorithm in the Department of Photogrammetry and Remote Sensing of Wuhan Technical University of Surveying and Mapping.

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