Multi-Band Wavelet for Fusing SPOT Panchromatic and Multispectral Images

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Abstract
With the development of remote sensing technology, fusion of remotely sensed images, such as SPOT panchromatic (SPOT P) with SPOT multispectral images (SPOT XS) or Landsat Thematic Mapper (TM), has become more important in order to obtain richer information in the spatial and spectral domains simultaneously. Many fusion methods have been developed based on the two-band wavelet transformation. However, due to the limitations of the transformation characteristics themselves, the two-band wavelet is not very efficient for the fusion of images whose ratio of spatial resolutions is not 2^n (n = 1, 2, 3, ...), e.g., for fusing a 10-m resolution panchromatic SPOT image and with 30-m resolution multispectral TM images. However, a recently developed new wavelet branch—multi-band wavelet—can potentially be applied to solve this problem.

In this paper, we develop a new approach for fusing SPOT P images with multispectral TM images based on multi-band wavelet transformation. First, the theoretical basis of multi-band wavelet is presented and its transformation properties are analyzed. Second, a new method for fusing a SPOT P image with multispectral images using the multi-band wavelet is proposed. Specifically, the three-band wavelet is implemented to fuse 10-m SPOT panchromatic and 30-m multispectral TM images. Third, this new method is compared with previous methods such as the two-band wavelet and IHS methods for image fusion. The proposed multi-band wavelet approach demonstrates an improvement in spatial and spectral characteristics for fusing SPOT P and multispectral TM images.

Introduction
With the development of remote sensing technology, various remotely sensed imagers—multi- and high-spectrum, multi-angle viewing, and multi-resolution—have been provided. These include one-meter resolution images such as Ikonos, large frame images, multispectral TM images, panchromatic and multispectral SPOT images, INSAR data, and many others. Each of these images has its own characteristics and contains certain types of information which may be superior spectrally or spatially. It is essential to develop advanced image fusion technologies, so that the advantages of the different remote sensing images can be integrated and to generate images with rich spatial and spectral information simultaneously.

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Such fused images will provide more detailed and accurate information, and can thus extend the areas of application, improve the reliability of their use, and increase the speed of information and feature extraction. Therefore, image fusion is an important technique to be developed, particularly when multi-sources of remote sensing images are available.

There have been many research efforts on image fusion (Carper et al., 1990; Ehler, 1991; Wald, 1999; Nunez et al., 1999; Zhang, 1999). These methods can be divided into three categories: (1) pixel-based image fusion, (2) feature-based fusion, and (3) recognition (interpretation)-based fusion. Many of these methods have been widely used, for example, pixel-based weighted image fusion, the IHS color model, the PCA method, the HLS fusion method, the COS fusion method, and the HSV (Hue, Saturation, and Value) method (Li et al., 1998; Yang et al., 1988; Carper et al., 1990; Shettigara, 1992; Zhang, 1999). Assessment of the quality of the fused images is another important issue. Wald et al. (1997) proposed an approach with criteria that can be used for evaluating the spectral quality of the fused satellite images.

One of the objectives of image fusion is to construct synthetic images that are closer to the reality they represent. According to the criteria proposed by Wald et al. (1997), the Brovey, IHS, and PCA fusion methods meet this objective (Ranchin and Wald, 2000). However, one limitation of such methods is some distortion of spectral characteristics in the original multispectral images. Fused images with such distortions may cause difficulties for further use of the image in interpretation and image analysis.

Recent developments in wavelet analysis provide a potential solution to these drawbacks. For example, Bruno et al. (1996) employed two different tools originally from signal processing: multi-resolution analysis and the two-band wavelet transformation. Sun et al. (1998) studied the fusion of multispectral sensing data based on two-band wavelet features. Nunez et al. (1999) developed an approach to fuse a high-resolution panchromatic image with a low-resolution multispectral image based on wavelet decomposition. Ranchin and Wald (2000) designed the ARSIS concept for fusing high spatial and spectral resolution images based on the multi-resolution analysis of two-band wavelet transformation. Similar studies were conducted by Yocky (1995), Ranchin et al. (1996), Zhou et al. (1998), Blanc et al. (1998), and Li et al. (1999). However, all of these developments were based on the two-band wavelet transformation.

Using the two-band wavelet transformation, an image can be decomposed into a low-frequency portion and three high-frequency portions. For example, Figure 1 is a two-band wavelet decomposition of the Lenna image.
We know that the ratio of the spatial resolutions of the two images to be fused can be one of the following two cases: (I) \(2^n\) \((n = 1, 2, 3, \ldots)\), such as two, four, eight, sixteen, etc., and (II) others, such as three, five, six, etc.

The two-band wavelet transformation, due to its decomposition characteristics, can be directly applied for images with the type (I) spatial resolution relationship. For example, one may fuse an image with 10-m resolution (e.g., SPOT P) with an image of 20-m resolution (e.g., SPOT XS) using a two-band wavelet transformation, where the ratio of spatial resolution is 2. The basic ideal of the two-band wavelet transformation, for a type (I) case, is to replace the low-frequency portion of the transformed image with a low-resolution image. These types of studies can be found, for example, in Ranchin and Wald (2000), where the authors fused images with spatial resolutions of 10 m, 20 m, and 40 m, respectively. The core of these applications is to make use of the transformation characteristics of the two-band wavelet.

For an image fusion case with a type (II) relationship, the two-band wavelet cannot be applied directly. The images need to be pre-processed first, then transformed using the two-band wavelet. In such a case, a low-resolution image (e.g., a 30-m multispectral TM image) is scaled to be the same size or half size (in length and width) as the high-resolution image (e.g., a 10-m panchromatic SPOT image) by a resampling processing, for example. However, the spectral information may be lost during such a resampling preprocess (Li et al., 1999). Therefore, it is not very efficient to apply the two-band wavelet transformation to fuse images with a type (II) relationship, for example, to fuse images with resolutions of 10 m and 30 m.

A potential new solution for fusing images with type (II) relationships will be based on the multi-band wavelet—a new branch of the wavelet, and this is identified as the focus of this study which aims to improve the image fusion effects.

The multi-band wavelet transformation has been studied in recent years. A multi-band wavelet transformation is superior to the two-band in many aspects, such as compact support and symmetry, especially in its decomposition characteristics. Now, both theoretical research (Chui et al., 1995; Bi et al., 1999; Wisutmethangoon et al., 1999) and a few application studies (Zhu, 1998; Zhu et al. 2002) have been performed with the multi-band wavelet. In the following section, the basic characteristics of the multi-band wavelet will be further introduced. It will be found, in this study, that the multi-band wavelet is very appropriate for fusing images with the type (II) relationships, i.e., where the ratio of the spatial resolution between two images is not \(2^n\) \((n = 1, 2, 3, \ldots)\).

In this paper, we aim to propose a generic image fusion method based on the multi-band wavelet, where the ratio of spatial resolution of the images is not restricted to be \(2^n\) but to any integer number. A specific discussion of fusing SPOT P with multispectral TM images is taken as an example.

The structure of this paper is as follows. The next section discusses the theoretical basis and transformation characteristics of the multi-band wavelet, and makes a comparison between the two-band wavelet and the multi-band wavelet. Then, a new image fusion approach for SPOT P and multispectral images based on the multi-band wavelet is presented. This is followed by a discussion of the image fusing experiments, including the three-band wavelet fusion of 10-m SPOT panchromatic and 30-m multispectral TM images. Next, the experimental results are analyzed. Furthermore, the proposed method is compared with the previous methods developed for image fusion, such as the two-band wavelet method and the IHS method.

**Characteristics of the Two-Band and Multi-Band Wavelet Transformations**

**The Two-Band Wavelet Transformation**

**Multi-Scale Analysis**

Wavelets are functions in a space \(L^2(\mathbb{R}) = \left\{ f(x) \right\} f^2(x) < +\infty \), determined from a basic wavelet function by dilations and translations. They are used for representing the local frequency content of functions. The basic wavelet should be
well localized in general, and the wavelet should have zero mean (Daubechies, 1992). The basic method to construct a wavelet is multi-scale analysis.

A multi-scale analysis is an increasing sequence \( \{V_j\}_{j \in \mathbb{Z}} \), which approximates \( L^2(\mathbb{R}) \), i.e.,

\[
\{0\} \subset \cdots \subset V_{-1} \subset V_0 \subset V_1 \subset \cdots \subset L^2(\mathbb{R}) ,
\]

and satisfies the following property:

\[
f(x) \in V_j \implies f(2x) \in V_{j-1}.
\]

There exists a scaling function \( \phi(x) \) in \( V_0 \) such that the set \( \{\phi(x-k)\}_{k \in \mathbb{Z}} \) is an orthonormal basis of \( V_0 \). The function \( \phi(x) \) satisfies the scaling equations

\[
\phi(x) = \sum_{k \in \mathbb{Z}} c_k \phi(2x-k),
\]

where \( Z \) is an integer set, and \( \{c_k\} \) is a set of scaling function coefficients which satisfy the following filter equation:

\[
H(z) = \frac{1}{2} \sum_{k \in \mathbb{Z}} c_k z^k.
\]

Using the scaling function, we can obtain the wavelet function \( \psi(x) \) that satisfies the following scaling equations

\[
\psi(x) = \sum_{k \in \mathbb{Z}} d_k \phi(2x-k).
\]

Two-Band Wavelet Decomposition and Reconstruction

By applying the tensor product, two-dimensional orthogonal wavelet bases can be obtained from one-dimensional wavelet bases. Hence, the two-band orthogonal wavelet decomposition and reconstruction of an image \( \{a_{n,k}\} (k, l \in \mathbb{Z}) \) can also be obtained. The decomposition formulae of two-band wavelet are

\[
a_{j+1,k,l} = \sum_{m,n} c_{m-2k,n-2l} a_{j,m,n},
\]

\[
b_{j+1,k,l} = \sum_{m,n} c_{m-2k,n-2l} b_{j,m,n},
\]

\[
b_{j+1,k,l} = \sum_{m,n} d_{m-2k,n-2l} a_{j,m,n},
\]

\[
b_{j+1,k,l} = \sum_{m,n} d_{m-2k,n-2l} b_{j,m,n},
\]

where \( j = 0, 1, 2, \ldots \).

The reconstruction formula is

\[
a_{j,k,l} = \sum_{m,n} (c_{k-2n} c_{l-2n} a_{j+1,m,n} + c_{k-2n} b_{j+1,m,n})
\]

\[
+ d_{k-2n} b_{j+1,m,n} + d_{k-2n} b_{j+1,m,n})
\]

where \( j = 0, 1, 2, \ldots \). \( \{a_{j+1,k,l}\} \) is the low-frequency portion of the \( j + 1 \) level two-band wavelet decomposition of the image \( \{a_{j,k,l}\} \), and \( \{b_{j+1,k,l}\} \) is the high frequency portion of \( j + 1 \) level. Figure 1b shows the two-band wavelet transformed image of Figure 1a. Using the two-band wavelet transformation, the image is decomposed into one low-frequency portion (the upper left portion) and three high-frequency portions (the remaining portions of the image). By applying an inverse wavelet transformation, the original image can be reconstructed.

Furthermore, we can decompose the image into many levels. Figure 2 represents the two-level orthogonal wavelet decomposition of an original image.
The decomposition formulae of $M$-band wavelet are

$$
a_{j+1,k,l} = \sum_{m} \sum_{n} c_{m-M, n-M} a_{j,m,n},$$

where $j = 0, 1, 2, \ldots$. The reconstruction formula is

$$a_{j,k,l} = \sum_{m} \sum_{n} c_{k-M+1, n-M} a_{j+1,m,n}$$

$$+ \sum_{t,s=0}^{M-1} \sum_{m} \sum_{n} d_{k-M+1, n-M} b_{j+1,m,n}$$

where $j = 0, 1, 2, \ldots$ ($a_{j,k,l}$) is the low-frequency portion of the $j+1$ level $M$-band wavelet decomposition of the image, and $b_{j+1,m,n}$ is the high-frequency portion of $j+1$ level. Hence, using the $M$-band wavelet transformation, the image is decomposed into one low-frequency portion and $(M \cdot M - 1)$ high-frequency portions. Using the inverse wavelet transformation, the original image can be reconstructed. Figure 4 gives the multi-band wavelet decomposition for the cases of $M = 3, 4, 5$, respectively.

Furthermore, Figure 5 gives the three-band wavelet transformed image of Figure 1a (where $M = 3$, $j = 0$). The coefficients of the three-band wavelet transform come from the Table 1.

From the above discussion, we can see that there are some differences between the two-band wavelet and the multi-band wavelet. These differences can be clearly seen by comparing Equation 2 with Equation 5, and Equation 3 with Equation 6. The differences in transformation characteristics, which are essential for this study, are illustrated by comparing Figure 1 with Figure 5 and Figure 2 with Figure 4. The transformation characteristics of the multi-band wavelet are used as the supporting theoretical basis for the approach developed — fusing SPOT P images and multi-spectral TM images — in this study.

### Table 1. The Coefficients of the Scaling Function and Wavelet Function of Three-Band Wavelet

<table>
<thead>
<tr>
<th>$K$</th>
<th>$C_K$</th>
<th>$d^1_k$</th>
<th>$d^2_k$</th>
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<tr>
<td>0</td>
<td>0.586101</td>
<td>-0.707106</td>
<td>-0.173494</td>
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<tr>
<td>1</td>
<td>0.9194355</td>
<td>1.414213</td>
<td>-0.272165</td>
</tr>
<tr>
<td>2</td>
<td>1.252768</td>
<td>-0.707106</td>
<td>-0.370836</td>
</tr>
<tr>
<td>3</td>
<td>0.413898</td>
<td>0.000000</td>
<td>1.39823</td>
</tr>
<tr>
<td>4</td>
<td>0.080564</td>
<td>0.000000</td>
<td>0.272165</td>
</tr>
<tr>
<td>5</td>
<td>-0.252768</td>
<td>0.000000</td>
<td>0.853908</td>
</tr>
</tbody>
</table>

The coefficients of the three-band wavelet transform come from the Table 1.
The Image Fusion Method Based on the Multi-Band Wavelet Transformation

The spatial and spectral features of a ground area appear very different on different remotely sensed images, because each remote sensor is designed to have its own imaging target(s). Spatial structure and spectral patterns can potentially be better represented if these images are fused. This may facilitate further applications of the images with more information about the ground. According to image analysis in the frequency domain, the difference in the low-frequency portion, for different images corresponding to the same ground area, is not very high. However, that for the high-frequency portion can be very high. A wavelet transformation has the characteristics of decomposition of frequency. Therefore, a wavelet transformation can be used to form a basis for image fusion.

By using a multi-band wavelet transformation, the method developed in this study can fuse images where the ratio of the spatial images can be any integer number, rather than limited to $2^n$. For simplicity, we explain the method with an example of fusing a 10-m resolution SPOT panchromatic image and three 30-m resolution multispectral TM images by applying a three-band wavelet transformation.

We now give the specific operational procedure for the proposed multi-band wavelet transformation-based image fusion approach. The operational procedure is a generic one, although two types of specific remotely sensed images were taken as an example in order to illustrate the method.

1. Both a 30-m resolution multispectral TM image and a 10-m resolution SPOT P image are geometrically registered to each other. This is performed by an image georeference processing with the aid of a digital elevation model in the study area.

2. Three new SPOT panchromatic images P1, P2, and P3 are produced, whose histograms are specified according to the histograms of the multispectral TM images M1, M2, and M3, respectively.

3. P1, P2, and P3 are decomposed into wavelet transformed images W1, W2, and W3, respectively, by applying the three-band wavelet. Each wavelet image includes one low-frequency portion and eight high-frequency portions, as illustrated in Figure 5. The image size of the low-frequency portion is the same as the multispectral TM images.

4. The low-frequency portions of wavelet transformation images W1, W2, and W3 are replaced by multispectral TM images (M1, M2, and M3), respectively. Three new images (I1, I2, and I3) are obtained.

5. The inverse wavelet transformations are carried out for I1, I2, and I3, respectively. Three new images (F1, F2, and F3) are then obtained, which reflect the spectral information of the original multispectral TM images (M1, M2, and M3).

6. F1, F2, and F3 are combined into a single fused image F. The spectral information of the original multispectral TM images is retained.

The proposed approach is a generic one. For fusing images with the spatial resolution ratio as an integer number other than 3, the approach can also be applied. For such a case, we just need to apply the corresponding multi-band wavelet transformation and inverse wavelet transformation. For example, if the ratio of spatial resolution of images is 5, we just apply a five-band wavelet transformation, instead of a three-band transformation. In general, an M-band wavelet transformation is applied to the image fusion process where the ratio of spatial resolution of the images is $M$.

Experimental Study and Analysis

An Image Fusion Experiment

As an experimental study, the above proposed image fusion method was applied to a real image fusion case. The 10-m resolution SPOT panchromatic image and three 30-m resolution multispectral TM images were fused using a three-band wavelet transformation. Figure 6 is a 10-m resolution panchromatic image. Figure 7 shows three 30-m resolution multispectral TM images. Plate 1a is the fused image for the 10-m resolution panchromatic image and the three 30-m resolution multispectral TM images using the three-band wavelet transformation. The coefficients of the scaling function and the wavelet functions for three-band wavelet transformation were based on those in Table 1. Further-
Table 2 presents a comparison of the experimental results of image fusion using the three-band wavelet, two-band wavelet, and IHS methods in terms of combination entropy, mean gradient, and standard correlation coefficient.

We now analyze the experimental results presented in Table 2, in order to find the differences between the fused images based on the multi-band wavelet (here, the three-band wavelet), the two-band wavelet, and the IHS methods.

Combination entropy is an indicator to measure the result of image fusion - the higher the combination entropy value, the better the fused image. In Table 2, the combination entropy of the three-band wavelet transformed image...
Conclusions

We have presented a newly developed method based on the multi-band wavelet transformation for fusing remote sensing images where the ratio of spatial resolution is not \(2^n\). For example, we can fuse a SPOT panchromatic image with a 10-m spatial resolution and multispectral TM images with a 30-m resolution where the spatial resolution ratio of the images is 3. Compared with the two-band wavelet transformation which is efficient in fusing images with the ratio of spatial resolution relationship of \(2^n\), the proposed multi-band wavelet method is more generic (the ratio can be any integer number) and can thus be widely applied for many more types of image fusion cases.

In this paper, an experimental study was conducted by applying the proposed method, and also other image fusion methods, for fusing a 10-m resolution SPOT panchromatic image and a 30-m multispectral TM image. A comparison of the fused image from the three-band wavelet, two-band wavelet, and IHS method was made. Based on the experimental results respecting the three indicators — the combination entropy, the mean gradient, and the standard correlation coefficient — the proposed method provides a better result than that based on the two-band wavelet and IHS methods. The result from the three-band wavelet transformation has a better visual effect for visualization and provides rich information for quantitative image analysis.

Theoretically, this study extends the wavelet-based image fusion from the two-band wavelet transformation to a multi-band wavelet transformation. With such a development, the wavelet method is applicable for more generic cases — non \(2^n\) in the spatial resolution ratio. Practically, the proposed multi-band transformation method provides a better result, both visually and quantitatively, for remote sensing fusion. The method improves the efficiency of the image fusion application, such as for automatic pattern recognition, by providing fused images with a better quality.

References


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