Modification of Pixel-swapping Algorithm with Initialization from a Sub-pixel/pixel Spatial Attraction Model

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Abstract
Pixel-swapping algorithm is a simple and efficient technique for sub-pixel mapping (Atkinson, 2001 and 2005). It was initially applied in shoreline and rural land-cover mapping but has been expanded to other land-cover mapping. However, due to its random initializing process, this algorithm must swap a large number of sub-pixels, and therefore it is computation intensive. This computing power consumption intensifies when the scale factor is large. A new, modified pixel-swapping algorithm (MPS) is presented in this paper to reduce the computation time, as well as to improve sub-pixel mapping accuracy. The MPS algorithm replaces the original random initializing process with a process based on a sub-pixel/pixel spatial attraction model. The new algorithm was used to allocate multiple land-covers at the sub-pixel level. The results showed that the MPS algorithm outperformed the original algorithm both in sub-pixel mapping accuracy and computational time. The improvement is especially significant in the case of large scale factors. Furthermore, the MPS is less sensitive to the size of neighboring sub-pixels and can still result in increased accuracy even if the size of neighbors is small. The MPS was also much less time consuming, as it reduced both the iterations and total amount of swapping needed.

Introduction
Since the launch of the first Earth observation satellite, remote sensing imagery has been utilized increasingly in many applications including land-cover analysis, environmental monitoring, mineral exploration, military surveillance, etc. A common problem associated with the application of satellite images, however, is the frequent occurrence of mixed pixels (e.g., Foody, 2004). Mixed pixels in traditional land-use and land-cover classification processes are often classified into a single land-use/land-cover type, without preserving the information that the pixel contains a mixture of multiple land-uses and covers. Soft classification techniques were introduced to avoid the loss of information by assigning a pixel to multiple land-use/land-cover classes according to the area each use/cover represents within the pixel. This soft classification technique generates a number of fractional images equal to the number of classes (Mertens et al., 2003). Unfortunately, the results from soft classification do not specify the location of each class within that particular pixel. In many practical applications, it is often desirable to know where each class is located within the pixel, in order to obtain detailed spatial patterns of land-use and land-cover.

Sub-pixel mapping (or super-resolution mapping) was then introduced (Atkinson et al., 1997; Atkinson, 1997) to achieve this desirable goal, using the information obtained from both soft and hard classification techniques. The aim of sub-pixel mapping is to determine the most suitable locations for different classes produced from a soft classification. It attempts to allocate each thematic mapping fraction to an appropriate sub-pixel location using soft classification results. Hence, sub-pixel mapping is a spatial allocation technique that transforms a soft classification into a finer scale hard classification.

Several approaches have been proposed to tackle the sub-pixel mapping issue: pixel swapping (Atkinson, 2001 and 2005), image sharpening (Foody, 1998; Gross and Schott, 1998), knowledge-based analysis (Schneider, 1993), Hopfield neural networks (Tatem et al., 2001 and 2002), de-convolution filters (Pinilla and Ariza, 2002), linear optimization (Verhoeve et al., 2002), genetic algorithms (Mertens et al., 2003), feed-forward neural networks (Mertens et al., 2004), Markov random field-based approach (Kasetkasem et al., 2005), algorithm based on sub-pixel/pixel spatial attraction models (Mertens et al., 2006), and integration of information from indicator co-kriging or indicator kriging (Boucher and Kyriakidis, 2006 and 2007).

One of the sub-pixel mapping algorithms, named pixel-swapping (PS), was first proposed by Atkinson (2001 and 2005) and tested with synthesized images. Due to its simplicity and efficiency, the PS algorithm has been used successfully for mapping shorelines in Malaysia (Muslim et al., 2006) and rural land-cover features in the Christchurch area of...
Dorset, UK (Thornton et al., 2006) with remotely sensed imagery. However, the PS is time consuming and sensitive to the number of neighboring pixels, especially while the sub-pixel scale factor is large. When the algorithm starts pixel swapping, the land-use/land-cover class proportions from soft classification are transformed into sub-pixel hard classes using random initialization within each pixel. Due to its pixel-by-pixel and iteration-by-iteration processes, this PS algorithm swaps only one pair of sub-pixels per pixel per iteration, thus taking a substantial amount of time for convergence. The computational time is more intensive when the sub-pixel scale factor is large. Random initialization of the spatial allocation of sub-pixels also affects the sub-pixel mapping accuracy. The assumption of this paper is that the initialization of the PS can be optimized to improve its computational efficiency and sub-pixel mapping accuracy. The objective of this paper is to focus on the improvement of initialization of PS, thereby reducing the computational time while improving sub-pixel mapping accuracy.

Materials and Methods

Pixel-swapping Algorithm

Pixel-swapping (PS) algorithm is an approach to sub-pixel mapping proposed initially by Atkinson (2001 and 2005). The objective was then to change the spatial arrangement of sub-pixels in such a way that the spatial correlation between neighboring sub-pixels (defined below) would be maximized.

To swap sub-pixels, the proportions of land-use/land-cover classes within each pixel are required (usually obtained from soft classification) and initialized to transform them into sub-pixel hard classes. The initialization of sub-pixel locations can affect the computing efficiency and ultimate mapping accuracy at sub-pixel level. In Atkinson (2001 and 2005) initial work, a random initialization was adopted. After spatial initialization, only the spatial arrangement of sub-pixels is allowed to change while the number of sub-pixels within each pixel and the proportions of classes remain fixed. Once initialized, optimization procedures are used to iteratively maximize spatial correlation among neighboring sub-pixels to reach a final stage of sub-pixel allocation.

The pixel-swapping algorithm comprises three basic steps in each iteration. First, for each pixel, the attractiveness of each sub-pixel with identical class is calculated as a distance-weighted function of its neighbors based on the current arrangement of sub-pixel classes, and the total attractiveness of a pixel is summed. If \( p_{ij} \) is a sub-pixel in pixel \( P_{ab} \) and \( p_k \) is one of \( p_{ij} \)’s neighboring sub-pixels, \( p_k \) may belong to \( P_{ab} \) or its neighboring pixels. The total attractiveness \( O_{pi} \) of \( P_{ab} \) is then calculated as:

\[
O_{pi} = \sum_{i=1}^{S} \sum_{j=1}^{S} O_{pi_{ij}}
\]  

(1)

where \( S \) is the scale factor for each pixel, and a pixel contains \( S^2 \) sub-pixels. \( O_{pi_{ij}} \) is the attractiveness of its \( i^{th} \) row and \( j^{th} \) column sub-pixel, calculated as a distance weighted function of its neighboring sub-pixels:

\[
O_{pi_{ij}} = \sum_{k=1}^{N} \lambda_k Z(p_{ij}, p_k)
\]  

(2)

where \( N \) is the number of neighbors and is illustrated in Figure 1, \( Z(p_{ij}, p_k) \) is the value of the class between the sub-pixel \( p_{ij} \) and its \( k^{th} \) neighbor \( p_k \). If the class value of sub-pixel \( p_{ij} \) is identical with the class value of its neighbor \( p_k \), the \( Z(p_{ij}, p_k) \) is set to 1 and otherwise is set to 0. The weighting parameter, \( \lambda_k \), is a weight calculated using the following equation:

\[
\lambda_k = \exp\left(-\frac{h(p_{ij}, p_k)}{a}\right)
\]  

(3)

where \( a \) is a non-linear parameter of the exponential model, and \( h(p_{ij}, p_k) \) is the distance between centers of sub-pixel \( p_{ij} \) and its neighboring sub-pixel \( p_k \), calculated as:

\[
h(p_{ij}, p_k) = \sqrt{(x_i-x_k)^2 + (y_i-y_k)^2}
\]  

(4)

Second, based on the attractiveness of each sub-pixel within pixel, the optimization algorithm ranks the scores on a pixel-by-pixel basis. Finally, two sub-pixels with least attractiveness and different class values are selected, and their class values are swapped if the attractiveness of pixel is increased; otherwise, no change is made. The above three-step process is repeated iteratively until a solution is reached. The process will stop when the algorithm fails to make any further improvement.

The initial algorithm used by Atkinson (2001 and 2005) was designed to work for binary class, but later was expanded to work on multiple classes by Thornton et al. (2006), Makido (2006), and Makido et al. (2007).

Initialization Based on Sub-pixel/pixel Spatial Attraction Model

The sub-pixel/pixel spatial attraction model was introduced in sub-pixel mapping by Mertens et al. (2006). Instead of iteratively optimizing the spatial correlations among sub-pixels, the spatial attraction model directly estimates the class of sub-pixels according to the class proportion of its neighboring pixels. As such, the algorithm requires no iteration to achieve the spatial allocation of sub-pixel classes. The advantage of this algorithm is that it is computationally efficient. In this study, the spatial attraction model was first used to generate the sub-pixel class map, and then this
newly generated map was used as an initialization in the pixel-swapping algorithm for subsequent mapping. The initialization process includes three basic steps. First, based on the assumption that fraction values of identical classes in neighboring locations influence each other, attraction values are calculated per class for each sub-pixel within a pixel. Hence, attraction values of different classes could be calculated for all sub-pixels within a pixel depending on their neighborhoods. Assume \( P_{ij} \) is a sub-pixel in pixel \( P_{ab} \), and \( P_k \) is one of \( P_{ab} \)'s neighbors, then the class \( m \)'s attractiveness \( o_{m,pi,j} \) for sub-pixel \( P_{ij} \) can be calculated as:

\[
o_{m,pi,j} = \sum_{k=1}^{N} \lambda_k f_m(P_k)
\]

where \( N \) is the total number of neighbors (in this study, \( N \) is set to 8, which means that it only counts the nearest ones), and \( f_m(P_k) \) is the fraction value of the \( K^{th} \) neighboring pixel \( P_k \) for class \( m \). The parameter \( \lambda_k \) is a weight factor, calculated as:

\[
\lambda_k = \exp(-h(p_{ij}, P_k))
\]

where \( h(p_{ij}, P_k) \) is the distance between centers of sub-pixel \( p_{ij} \) and its neighboring pixel \( P_k \), and can be calculated as:

\[
h(p_{ij}, P_k) = \sqrt{(X_k - x_i)^2 + (Y_k - y_j)^2}.
\]

The second step in the initialization is, according to Mertens et al. (2006), to normalize the attraction values for each class, as this can lead to better sub-pixel mapping results. Therefore, the attractiveness is normalized according to the following equation:

\[
O_{m,pi,j} = \frac{o_{m,pi,j}}{\sum_{j=1}^{S} o_{m,pi,j}}
\]

where \( O_{m,pi,j} \) is the normalized attractiveness of sub-pixel \( p_{ij} \) for class \( m \), and \( S \) is the scale factor.

Finally, the attractiveness values \( O_{m,pi,j} \) are used for the assignment of sub-pixels to appropriate classes with the assumption that the sub-pixel with the highest attractiveness is assigned first. Thus, the sub-pixel class assignment, or initialization of sub-pixel class allocation, is accomplished and subsequent sub-pixel swapping can be applied to achieve sub-pixel mapping. This modified pixel-swapping method, or MPS, which substitutes the random class initialization with a sub-pixel distribution based on sub-pixel/pixel spatial attractiveness, is thus achieved.

### Parameterization of Sub-pixel Mapping

Several parameters in SP algorithm, including scale factor \( S \), type of neighbors and non-linear parameter of distance function \( a \), are all critical and influence sub-pixel mapping accuracy. Therefore, it is necessary to test the PS and MPS algorithms with a wide range of these parameter values. In this study, five scale factors, five types of neighbors, and six different distance functions were tested and all combinations were analyzed. The selected values of these parameters are given in Table 1. Here, the type of neighbor is defined as a value of radius \( n \) and Figure 1 demonstrates the definitions of different \( n \) used in this study.

### Criteria for Algorithm Comparison and Evaluation

Four criteria variables were used, including adjusted kappa (a statistical measure of sub-pixel mapping accuracy), CPU time, number of iterations, and total swapping number to assess the performance of the algorithms (original PS and modified PS).

The accuracy assessment was measured by comparing the sub-pixel mapping results of the MPS or PS with reference images. An adjusted Kappa coefficient was calculated to evaluate the accuracy of sub-pixel mapping, which was first proposed by Mac and et al. (2003). The statistic measure is the same as the Kappa coefficient except it is calculated only for mixed pixels. Due to the contribution of pure pixels, adjusted kappa coefficient is a more sensitive and objective measure of sub-pixel mapping accuracy performance than the original Kappa coefficient.

The second performance measure is the CPU time used for different algorithms. It only counts the CPU time consumption for sub-pixel mapping computation on the same computer.

The third performance measure is the number of iterations and total amount of swapping, as they are useful indicators of algorithm performance and efficiency. Because pixel-swapping is a pixel-by-pixel, iteration-by-iteration process, the number of iterations and total swapping number will provide some idea of why different algorithms perform differently. For instance, with same set of parameters, if the number of iterations is larger, the CPU time will be higher; and if the total swapping number is low but the mapping accuracy is high, the initialization process must be effective and the efficiency of the algorithm would be better.

In the study, the algorithm was implemented in MATLAB (version 5.3) with script and all computations are conducted on a MacPro workstation with two Intel Xeon 5150 Woodcrest 2.66 GHz processors, 2 GB RAM, and Microsoft Windows® XP SP2. Every set of parameters for each algorithm runs five times, and the performance evaluation criteria variables were averaged as a statistical measure of algorithm efficiency and overall performance.

### Test Images

Two images were tested in this study. One was a binary synthesized image (Figure 2), and the other was a real land-cover map (Figure 3). Both have a size of 512 by 512. The synthesized image contained four (4) different characteristic objects as a testing image to facilitate the comparison of different algorithms. The land-cover image, which was interpreted visually from 1:10 000 panchromatic aerial photos (acquired in 2004) covering a small area in Lanxi City, Zhejiang Province, China, was used as the second image for testing practical applications of these two algorithms. And these original images were used as a reference for accuracy assessment.

Generation of fraction images from reference images is called degradation, and pixel proportions for every class in fraction images were calculated from reference images in a window of size according to the required scale factor in degradation. Fraction images were generated with five scale factors, respectively, from the synthesized and real land-cover images.

From the fraction images, the class with maximum fraction for every pixel was identified, and it was assigned to all sub-pixels located in the pixel, and then the hard classification results were derived by this approach pixel-by-pixel. Hard classification can be taken as traditional classification which classifies mixed pixels into a single land-use/land-cover type.

### Table 1. Values of Critical Parameters Used in Sub-pixel Swapping Algorithm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tested values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale factor ( S )</td>
<td>2, 4, 8, 16, 32</td>
</tr>
<tr>
<td>Type of neighbor ( n )</td>
<td>1, 2, 4, 6, 8</td>
</tr>
<tr>
<td>Non-linear value of distance function ( a )</td>
<td>0.5, 1, 2, 4, 6, 8</td>
</tr>
</tbody>
</table>
Results and Discussion

The accuracy and CPU time of sub-pixel mapping for the synthesized and real land-cover images derived from the two different initializing processes are listed in Table 2. The initializing process based on the sub-pixel/pixel spatial attraction model (SPP) has the highest adjusted kappa values for all five scale factors. More importantly, the accuracy of the resulting sub-pixel map from the SPP is far better than that of the random initialization (RANDOM) process and even better than the hard classification (HC) results. The RANDOM, in contrast, resulted in the lowest accuracy. The reason was believed that SPP made a more reasonable and accurate spatial allocation for sub-pixels based on the fraction information of its neighboring pixels in the initialization step. Due to its complicated processes, the SPP required more CPU time than RANDOM did, and time needed increased as the scale factor became larger.

Figures 2 and 3 illustrate the sub-pixel mapping results from the synthesized and land-cover images for RANDOM and SPP initializations, respectively. Based on visual assessment of the image quality, it appears that the results generated by SPP initialization are far more accurate than those from the RANDOM initialization. Obviously, the new initialization method, SPP provides far better results in the initializing mapping process.

The accuracy measurements of PS and MPS algorithms for the two test images are demonstrated in Figure 4a and 4b, respectively; the adjusted kappa coefficients for two algorithms and hard classification are illustrated in this figure. In comparison with the results of hard classification and original PS algorithm, the new MPS algorithm achieved much better accuracy for all scale factors, especially in the cases of large scale factors, due to the contribution of the new initialization procedure. For both algorithms, their adjusted kappa values monotone decrease while the scale factor $S$ increases from 2 to 32. In contrast, the original PS algorithm is very sensitive to the size of neighbors, while the MPS is much less sensitive, especially in the cases of small size of neighbors. And, the MPS is also less sensitive to the effects of value $a$, especially while scale factor is small and the size of neighbor is large. In most cases, the new MPS algorithm achieved higher accuracy, suggesting that this new MPS obviously outperformed the original one at all scale factors.

The CPU time for the two algorithms (including initialization process) is shown in Figure 5 with different combinations of parameters. Although the initialization process based on sub-pixel/pixel attractiveness model requires more computation time than the random initialization, the CPU time of the new MPS algorithm is far less than the original.
TABLE 2. ADJUSTED KAPPA VALUE AND CPU TIME FOR TEST IMAGES WITH RANDOM INITIALIZATION (RANDOM) AND INITIALIZATION BASED ON SUB-PIXEL/PIXEL MODEL (SPP)

<table>
<thead>
<tr>
<th>Scale factor</th>
<th>Adjusted kappa</th>
<th>CPU time (second)</th>
<th>Adjusted kappa</th>
<th>CPU time (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Synthesized image</td>
<td></td>
<td>Land-cover image</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HC* RANDOM SPP</td>
<td>RANDOM SPP</td>
<td>HC* RANDOM SPP</td>
<td>RANDOM SPP</td>
</tr>
<tr>
<td>2</td>
<td>0.2723 0.1412 0.9917</td>
<td>1.07 1.18</td>
<td>0.4407 0.3480 0.9792</td>
<td>1.39 3.15</td>
</tr>
<tr>
<td>4</td>
<td>0.4880 0.3184 0.9620</td>
<td>0.44 1.65</td>
<td>0.6101 0.4746 0.9385</td>
<td>0.53 2.90</td>
</tr>
<tr>
<td>8</td>
<td>0.5600 0.4067 0.9174</td>
<td>0.33 3.44</td>
<td>0.6374 0.5082 0.8812</td>
<td>0.39 4.98</td>
</tr>
<tr>
<td>16</td>
<td>0.5139 0.3560 0.7536</td>
<td>0.45 11.22</td>
<td>0.5623 0.4322 0.7606</td>
<td>0.50 12.89</td>
</tr>
<tr>
<td>32</td>
<td>0.4353 0.2891 0.4628</td>
<td>0.77 47.77</td>
<td>0.4525 0.3302 0.5745</td>
<td>0.78 55.52</td>
</tr>
</tbody>
</table>

*Hard classification

Figure 3. Sub-pixel mapping results of land-cover image with five degraded scale factors generated by two different initializing methods: (a) Random initialization (RANDOM), and (b) Initialization based on sub-pixel/pixel spatial attraction model (SPP). (Selected by the maximum adjusted kappa coefficient, solid edge represents the position of reference image. A color version of this figure is available at the ASPRS website: www.asprs.org.)

PS. Compared with the original algorithm PS, the new MPS significantly reduced the computation iterations (Figure 6) and total swapping number (Figure 7), which suggests that the new MPS overall can save computing time and improve efficiency and accuracy.

The sub-pixel mapping results from the two test images are demonstrated in Figures 8 and 9. Visual assessment of image quality reveals that the original PS algorithm yielded somewhat better results than hard classification except when the scale factor S is 32. The modified MPS method yielded much better results than the original PS method for most scale factors, especially for large scale factors. For the case of scale factor S = 32, neither the original algorithm nor the new MPS yielded satisfactory results, due to the lack of spatial information from fraction images. However, the results of sub-pixel mapping generated by the new MPS algorithm are far more reasonable than results derived from original algorithm and hard classification in most cases.
Conclusions

The pixel-swapping algorithm is a simple and efficient method for sub-pixel mapping, but for large scale factors, it is still computing intensive with low mapping accuracy. A modified pixel-swapping (MPS) algorithm was proposed in this study, by using a new initialization procedure based on sub-pixel/pixel spatial attraction model. The results demonstrated that the new MPS algorithm achieved much more accurate super-resolution images than the original PS algorithm, especially in the cases of large scale factors. Furthermore, the new MPS was demonstrated to be much less sensitive to some critical parameters of the algorithm, such as the type of neighbor and non-linear value of distance function. This new MPS, therefore, will provide better and easier ways to achieve finer resolution products from coarse remotely sensed land-use and land-cover maps. In addition to improving the accuracy of sub-pixel mapping, this new MPS algorithm reduced the computation time required for large image processing, thus improving computing efficiency. The results from this study are encouraging and promising in that the new method may...
be used to fine boundaries between classes and obtain detailed land-use/land-cover information at sub-pixel levels.

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**References**


Figure 6. Iterations in sub-pixel mapping computation (S is the scale factor, n is the type of neighbor, a is the non-linear value of distance function): (a) Synthesized image, and (b) Land-cover image.


Figure 7. Total swapping numbers in sub-pixel mapping computation (S is the scale factor, n is the type of neighbor, a is the non-linear value of distance function): (a) Synthesized image, and (b) Land-cover image.


Figure 9. Sub-pixel mapping results of land-cover image with five degraded scale factors derived by PS and MPS algorithms: (a) Hard classification, (b) Original PS algorithm, and (c) MPS algorithm. (Selected by the maximum adjusted kappa coefficient, solid edge represents the position of reference image) A color version of this figure is available at the ASPRS website: www.asprs.org.
