Calibrating Film and Digital Sensors for Today’s Remote Sensing Business

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With special thanks to Don Light who could not attend but provided many of the workshop slides

ASPRS Annual Meeting
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Workshop # 173
Workshop Agenda

7:45 – 8:00  Introduction and Welcoming Remarks
8:00 – 9:00  What is camera calibration and why is it necessary? (Munjy)
9:00 – 9:30  An overview of different architectures for digital sensors and their geometry: (Abdullah)
   ✓ Framing Sensors
   ✓ Push-broom sensors
9:30 – 11:00 Procedures and mathematical models employed in calibrating digital sensors; (Munjy & Abdullah)
   ✓ Framing Sensors (Munjy)
   ✓ Push-broom sensors (Abdullah)

(10:00 – 10:15  Break)
11:00 – 11:15 Self calibration techniques as practiced today in the aerial imaging industry; (Abdullah)
11:15 – 11:35 Effects of GPS/IMU on calibration (Abdullah)
11:35 – 11:50 Design and illustration of indoor and in situ calibration fields; (Munjy & Abdullah)
11:50 – 11:55 Special considerations to UAS-based sensor operations (Munjy & Abdullah)
11:55 – 12:00 Typical sensor calibration reports; (Abdullah)
12:00 – 12:05 Agencies and companies providing sensor calibration; (Abdullah)
12:05 – 12:10 Commercially available software for sensor calibration. (Munjy & Abdullah)
12:10 – 12:15 Q&A
What is camera calibration and why is it necessary?

Dr. Munjy
Why Is Calibration Necessary?

- Calibration is necessary to determine exact values for the camera’s geometry. The calibration parameters are used to correct the raw x,y-measurements from an image so that the corrected x,y-measurements represent true to-scale values of the object space.

- Precision Photogrammetry requires a calibrated camera.

All Cameras have some lens distortion.
What is Camera Calibration?

**Camera Calibration:** The process of measuring and comparing to an accurate standard to determine accurate values defining the camera’s (geometry) Interior orientation parameters.

Calibration parameters for **film** cameras are:

1. Focal length (f) or Camera Constant (C) \( f \) or \( C \)
2. Point of symmetry \( x_p, y_p \)
3. Radial distortion, (Gaussian) \( K_1, K_2, K_3 \)
4. Radial Distortion (Balanced) \( K_0', K_1', K_2', K_3' \)
5. Balanced focal length \( C_b \)
6. Decentering distortion \( P_1, P_2 \)
7. \( x, y \)-coordinates of 4 to 8 fiducial marks \( x, y \)
Calibration Parameters for Digital Cameras

1. Focal length \( f \) or \( C \)
2. Point of Symmetry \( x_p, y_p \)
3. Radial distortion (Gaussian) \( K_1, K_2, K_3 \)
4. Radial distortion (Balanced) \( K_o', K_1', K_2', K_3' \)
5. Balanced focal length \( C_b \)
6. Decentering distortion \( P_1, P_2 \)
7. Non-square pixels \( B_1, B_2 \)
8. Pixel size \( x \)
9. Number of pixels in the array \( L \times W \)
Perfect Lens

- Such perfect lenses are very rare
- Very good lenses have lens elements that significantly reduce distortion

Ref: https://photographylife.com/what-is-distortion
Radial Lens Distortion

Barrel Distortion

- When straight lines are curved inwards in a shape of a barrel, this type of aberration is called “barrel distortion”.
- Commonly seen on wide angle lenses.
- Straight lines are visibly curved inwards, especially towards the extreme edges of the frame.

Ref: https://photographylife.com/what-is-distortion
Radial Lens Distortion
Pincushion Distortion

• Pincushion distortion is the exact opposite of barrel distortion – straight lines are curved outwards from the center.

• This type of distortion is commonly seen on telephoto lenses, and it occurs due to image magnification increasing towards the edges of the frame from the optical axis. T

• Straight lines appear to be pulled upwards in the corners

Ref: https://photographylife.com/what-is-distortion
Radial Lens Distortion
Wavy/Moustache/complex Distortion

• The nastiest of the radial distortion types is mustache distortion, which I sometimes call “wavy” distortion.
• It is basically a combination of the barrel distortion and pincushion distortion.
• Straight lines appear curved inwards towards the center of the frame, then curve outwards at the extreme corners.

Ref: https://photographylife.com/what-is-distortion
Variation of Lens Distortion with Focal Length

Ref: Automatic Camera Calibration in Close Range Photogrammetry, C Fraser, ASPRS
Radial Distortion profile for each channel

Ref: Automatic Camera Calibration in Close Range Photogrammetry, C Fraser, ASPRS
Radial Lens Distortion at the Edges

- The problem of extrapolation of the radial distortion correction function beyond the largest radial distance (RD) to any image point. A1 and B1 are modelled distortion profiles corresponding to maximum encountered radial distances of 10.5mm and 12.7mm, respectively.

Ref: Automatic Camera Calibration in Close Range Photogrammetry, C Fraser, ASPRS
Decentering/Tangential Distortion

Perfect Lens

- Illustration of a “perfect” lens (not that one exists)
- Light rays that enter a perfect lens are bent normally, per original lens design, converging correctly

https://photographylife.com/what-is-a-decentered-lens
Decentering/Tangential Distortion
Decentered Lens

• When a lens element is decentered, its axis is no longer aligned with the principal axis of the lens.
• This results in light rays getting bent differently, which can drastically reduce the sharpness of the image and geometric accuracy.

Ref: https://photographylife.com/what-is-a-decentered-lens
Decentering/Tangential Distortion
Test Chart

- Take a look at the postal stamps on the left side of the frame and compare them to the ones on the right – they appear noticeably softer.
- The sharpness of the lens quickly degrades right off the center of the frame to the left.
- The borders of the squares are very soft.
- The right side does not suffer from sharpness loss as badly as the left side.

Ref: https://photographylife.com/what-is-a-decentered-lens
Image Coordinate System and Interior Orientation

- Image x,y must be referenced to the Pt of symmetry.
- Use the correct focal length (f).
- Correct x,y-image coordinates for Lens Distortions.
Geometry of a Frame Photograph

Elements of Exterior Orientation (6)
Notice with 20 um of error in the x,y measurement, $\sigma_h$ goes from 1.88 ft to 6.26 ft

Conclusion:
Metric cameras win more contracts
An overview of different architectures for digital sensors and their geometry

Dr. Abdullah
Cameras by our forefathers...
Aerial Mapping Cameras of Today

Film

Courtesy Intergraph

Digital

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Digital Sensor Geometry Types

- Push Broom
- Panoramic
- Framing
- Oblique
Digital Modular Camera (DMC) by Intergraph

- Field of view 69.3° × 42°
- Panchromatic 13.824 x 7.680 pixel
- 4 optics f = 1:4.0 / 120mm
- Multi spectral 2.048 x 3.072 pixel
- 4 channels RGB & NIR
- 4 optics f = 1:4.0 / 25mm
- Shutter, aperture variable
- Flight data storage 840 GB = >2200 images
- Frame rate 2 sec / image
- Radiometric resolution 12 bit
- Weight (camera only) < 80 kg
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Multi-camera Head.
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4 overlapping images

image mosaicking
- apply camera calibration parameters
- apply platform calibration
- tie point check
- robust adjustment
- projection to virtual perspective
- fusion with color composite

tie point area
<table>
<thead>
<tr>
<th>Compatibility</th>
<th>RMK D</th>
<th>RMK DX</th>
<th>DMC II\textsubscript{140}</th>
<th>DMC II\textsubscript{230}</th>
<th>DMC II\textsubscript{250}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensors</td>
<td>Multi-spectral sensor, including 4 camera heads, RGB, and NIR</td>
<td>Multi-spectral sensor, including 5 camera heads, RGB, NIR, and PAN</td>
<td>Multi-spectral sensor, including 5 camera heads, RGB, NIR, and PAN</td>
<td>Multi-spectral sensor, including 5 camera heads, RGB, NIR, and PAN</td>
<td>Multi-spectral sensor, including 5 camera heads, RGB, NIR, and PAN</td>
</tr>
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<td>Flight Management Systems</td>
<td>Compatible with Z/I Inflight and Z/I Track</td>
<td>Compatible with Z/I Inflight and Z/I Track</td>
<td>Compatible with Z/I Inflight and Z/I Track</td>
<td>Compatible with Z/I Inflight and Z/I Track</td>
<td>Compatible with Z/I Inflight and Z/I Track</td>
</tr>
<tr>
<td>Data Storage</td>
<td>SSD-based airborne data storage</td>
<td>SSD-based airborne data storage</td>
<td>SSD-based airborne data storage</td>
<td>SSD-based airborne data storage</td>
<td>SSD-based airborne data storage</td>
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<td>Post Processing</td>
<td>Compatible with DMC post-processing software</td>
<td>Compatible with DMC post-processing software</td>
<td>Compatible with DMC post-processing software</td>
<td>Compatible with DMC post-processing software</td>
<td>Compatible with DMC post-processing software</td>
</tr>
<tr>
<td>Upgrade Path</td>
<td>Upgrade path to DMC II family</td>
<td>N/A</td>
<td>Upgrade path to DMC II 230 and 250</td>
<td>Upgrade path to DMC II 250</td>
<td>N/A</td>
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## DMC Sensors

<table>
<thead>
<tr>
<th>DMC</th>
<th>RMK D</th>
<th>RMK DX, DMC II〈140</th>
<th>DMC II〈230</th>
<th>DMC II〈250</th>
</tr>
</thead>
</table>

![Image Size Comparison](image.png)

17,216x14656

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DMC® II 250 Camera System

MODULAR AND COMPATIBLE
DMC II 250 is compatible with all existing peripheral devices used for RMK TOP, DMC, and RMK D, which include Z/I Mission planning software, Z/I Inflight sensor management system, solid state disks (SSD) storage cartridges, Readout Station, T-AS mount and Z/I Mount. In addition, a new adapter plate for the new generation of Z/I Imaging cameras allows you to use a wide range of different inertial measurement unit (IMU) sensors. You can easily upgrade your RMK D into a DMC II 250 by installing the PAN camera head.

![Image of DMC II 250 Camera System]

Figure 1: The DMC II camera design is an evolution of the proven DMC camera technology.

### DMC II 250 Technical Specifications

<table>
<thead>
<tr>
<th>Feature</th>
<th>Value</th>
<th>Comment</th>
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<tbody>
<tr>
<td>Pixel across track (1)</td>
<td>17216</td>
<td></td>
</tr>
<tr>
<td>Pixel along track (1)</td>
<td>14656</td>
<td></td>
</tr>
<tr>
<td>Fov across track</td>
<td>46.6 °</td>
<td></td>
</tr>
<tr>
<td>Fov along track</td>
<td>40.2 °</td>
<td></td>
</tr>
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<td>Focal length</td>
<td>112 mm</td>
<td></td>
</tr>
<tr>
<td>Gsd@500m</td>
<td>2.5 cm</td>
<td></td>
</tr>
<tr>
<td>B/h</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Pixel size</td>
<td>5.6 μm</td>
<td></td>
</tr>
<tr>
<td>Number of camera heads</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Pan : color resolution</td>
<td>1:3,2</td>
<td>PAN 16 readouts, MS 2 readouts</td>
</tr>
<tr>
<td>Frame rate</td>
<td>1.7 sec</td>
<td></td>
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<tr>
<td>Color channels</td>
<td>R,G,B, NIR</td>
<td></td>
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<td>Resolution per pixel</td>
<td>14 bit</td>
<td></td>
</tr>
<tr>
<td>Fmc</td>
<td>yes</td>
<td>via TDI</td>
</tr>
<tr>
<td>Cod dynamic range</td>
<td>&gt;67 dB</td>
<td></td>
</tr>
<tr>
<td>Onboard storage</td>
<td>1.5 Tbyte</td>
<td>1350 images</td>
</tr>
<tr>
<td>Weight</td>
<td>66 kg</td>
<td>Including storage</td>
</tr>
<tr>
<td>Power consumption</td>
<td>350 W</td>
<td>Including storage</td>
</tr>
<tr>
<td>Altitude non-pressurized</td>
<td>8000 m</td>
<td></td>
</tr>
<tr>
<td>Operating temperature</td>
<td>-20°C - 40°C</td>
<td>(Electronic inside the aircraft: 0° - 40° C)</td>
</tr>
</tbody>
</table>

(1) Number of pixels of the processed image
Multi-Head Sensors
UltraCamLp

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Multispectral Image

RGB/IR bands merged with pan scene

RGB

NIR

A+B

Full Scene PAN
UltraCamXp

- 17,310 PAN pixel across
- Collects PAN, R, G, B, NIR
- 1:3 PAN sharpening
- 2 seconds frame rate
- 6,600 images storage, in-flight exchangeable
- 100mm PAN focal length
- Monolithic geometry & radiometry
UltraCamXp Wide Angle

- 17,310 PAN pixel across
- Collects PAN, R, G, B, NIR
- 1:3 PAN sharpening
- 2 seconds frame rate
- 6,600 images storage, in-flight exchangeable
- 70mm PAN focal length
- Monolithic geometry & radiometry

- Allows same GSD at lower flight altitude
- Enables small scale mapping with lower altitude aircrafts

- Offered in parallel to the UltraCamXp as an additional sensor head
- Same technical concept as UltraCamXp
ULTRACAM Eagle

20,010 x 13,080 pixels

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UltraCamXp Wide Angle
RGB, 10cm GSD

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UltraCamXp Wide Angle
CIR, 10cm GSD

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UCL, 10cm GSD, Graz
### Operational Parameters
**UCXp & UCXp WA**

<table>
<thead>
<tr>
<th>Camera</th>
<th>GSD</th>
<th>Altitude</th>
<th>Width</th>
<th>Max.Speed(*)</th>
<th>Max. Frontlap(***)</th>
</tr>
</thead>
<tbody>
<tr>
<td>UCXp</td>
<td>5cm</td>
<td>838m</td>
<td>866m</td>
<td>220kts</td>
<td>78%</td>
</tr>
<tr>
<td>UCXp WA</td>
<td>5cm</td>
<td>583m</td>
<td>866m</td>
<td>220kts</td>
<td>78%</td>
</tr>
<tr>
<td>UCXp</td>
<td>25cm</td>
<td>4,188m</td>
<td>4,328m</td>
<td>1,099kts</td>
<td>96%</td>
</tr>
<tr>
<td>UCXp WA</td>
<td>25cm</td>
<td>2,917m</td>
<td>4,328m</td>
<td>1,099kts</td>
<td>96%</td>
</tr>
<tr>
<td>UCXp</td>
<td>50cm</td>
<td>8,375m</td>
<td>18,655m</td>
<td>2,198kts</td>
<td>98%</td>
</tr>
<tr>
<td>UCXp WA</td>
<td>50cm</td>
<td>5,833m</td>
<td>18,655m</td>
<td>2,198kts</td>
<td>98%</td>
</tr>
</tbody>
</table>

- 

- 

- 

- 

- (*): max. speed calculated for 60% frontlap
- (**) : max. frontlap calculated at a survey speed of 120 kts
Panoramic Sensors

106° sweeping field of view
A 3 - Twin Stepping Frame Camera System

Focal length = 300 mm

Image courtesy, Vision Map

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Mechanism of the Multi-head and Stepping Cameras
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Principle of Single Push Broom Scanner
Push Broom Scanners: Stereo Coverage

Flight Direction

Three-Line Scanner

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**ADS40 Multiple Look Angles Capability**

1. **Backward scene**
   - Composed of backward view lines

2. **Nadir scene**
   - Composed of nadir view lines

3. **Forward scene**
   - Composed of forward view lines
Calibration Process for Leica ADS Sensor

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Procedures and Mathematical Models employed in Calibrating Digital **Framing** Sensors

Dr. Munjy
Methods of Camera Calibration

In general, camera calibration may be classified into one of three basic categories:

1. Laboratory methods  Multicollimators, Gonimeters, Cages
2. Field methods      Ground control point ranges, targets

The calibration parameters are generally referred to as "elements of interior orientation."

These elements are needed in order to determine accurate spatial information from x,y-image coordinates.
CCD Array, I,J-Coordinate System, and Camera Geometry
Geometry of a Vertical Frame Photograph

The positive has a right relationship with the Ground Coordinate System.

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Calibrator at USGS

Fig. 1. Schematic diagram of one collimator bank.

Fig. 2. Target positions that appear on glass plate.

Fig. 3. Reticle/resolution target.
### RES-1
Clear lines on an opaque chrome background (2.0 OD), optical glass substrate
50 x 50 x 1.5 mm

### RES-2
Black lines on heavy white gloss paper
50 x 50 mm

<table>
<thead>
<tr>
<th>Line Pairs per millimeter</th>
<th>Group Number</th>
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</thead>
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<tr>
<td>Element Number</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>1.12</td>
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<tr>
<td>3</td>
<td>1.26</td>
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<tr>
<td>4</td>
<td>1.41</td>
</tr>
<tr>
<td>5</td>
<td>1.59</td>
</tr>
<tr>
<td>6</td>
<td>1.78</td>
</tr>
</tbody>
</table>

- Line pairs/mm = LP
- Line width (mm) = 1/(2LP)
- Space width (mm) = 1/(2LP)
- Line length = 5(line width)

AF 51 Resolution Targets and Table for reading lp/mm on Film Cameras.

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Image Coordinate System and Interior Orientation

All xp, yp image-measurements must be referenced to this Principal Point, then distortion corrections and refinements can be applied to x, y.

(Also Point of symmetry, (xp, yp) or (xs, ys))
Simultaneous Multiframe Analytical Calibration Model - SMAC

\[
x - x_p = f \left[ \frac{A \lambda + B \mu + C v}{D \lambda + E \mu + F v} \right] + x \left[ K_1 r^2 + K_2 r^4 + K_3 r^6 \right] + \left[ 1 + P_3 r^2 \right] \left[ P_1 (r^2 + 2x^2) + 2P_2 xy \right]
\]

\[
y - y_p = f \left[ \frac{A' \lambda + B' \mu + C' v}{D' \lambda + E' \mu + F' v} \right] + y \left[ K_1 r^2 + K_2 r^4 + K_3 r^6 \right] + \left[ 1 + P_3 r^2 \right] \left[ 2P_1 xy + P_2 (r^2 + 2y^2) \right]
\]

<table>
<thead>
<tr>
<th>Image</th>
<th>Projective equations</th>
<th>Radial distortion</th>
<th>Decentering distortion</th>
</tr>
</thead>
</table>

\(x, y\) Measured plate coordinates with respect to the photo coordinate system
\(\lambda, \mu, v\) Orientation elements relating image points to corresponding collimator points
\(x_p, y_p\) Coordinates of principal point
\(K_1, K_2, K_3\) Coefficients of radial distortion
\(f\) Focal length
\(P_1, P_2, P_3\) Coefficients of decentering distortion
\(A, B, C, A', B', C', D, E, F\) orientation matrix elements which are functions of three independent angles
\(\alpha, \omega, \kappa\) referred to an arbitrary \(X, Y, Z\) frame in object space
\(\mathbf{r}\) Radial distance referred to the principal point
Wild Heerbrugg: U3 Diapositive Printer

- Uses correction plates to correct for:
  - Lens distortion
  - Earth curvature
  - Atmospheric refraction
Small format Digital Camera FOVs as shown in **Blue** do not image enough control targets to solve the calibration equations.
Select 3 points in each photo center. Measure x,y-coord.
Goniometer for Calibrating Linear Array Sensors

The operator points the T1 telescope to graduations on the p-scale along the array of pixels and observes the corresponding angles $r$. The radial optical distortions are computed from the differences $\Delta p = C_0 \tan (r)$. Where $C_0$ is the best known value for the focal length (principal distance).
Composite of All Three Panels, Black 1, Red 2, and Blue 3

Scale: 1 inch = 2 ft on Cage

Front Panel 1 black, Mid Panel 2 Red, Rear Panel 3 Blue

Cage size: (12 W x 9 H x 8 D) Ft

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Attach black curtains in the back to provide target contrast
Hyperfocal Distance (Hp) (meters) for Cameras

Focused at Infinity for Calibration

<table>
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<tr>
<th>F#</th>
<th>4</th>
<th>5.6</th>
<th>8</th>
<th>11</th>
<th>16</th>
<th>22</th>
<th>32</th>
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<td>f (mm)</td>
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<td>0.12</td>
<td>0.09</td>
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<td>0.04</td>
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<td>11</td>
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<td>0.22</td>
<td>0.16</td>
<td>0.12</td>
<td>0.09</td>
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<td>0.10</td>
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<td>32.08</td>
<td>22.94</td>
<td>16.08</td>
<td>11.72</td>
<td>8.08</td>
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<td>4.08</td>
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<td>50.10</td>
<td>35.81</td>
<td>25.10</td>
<td>18.28</td>
<td>12.60</td>
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<td>72.12</td>
<td>51.55</td>
<td>36.12</td>
<td>26.30</td>
<td>18.12</td>
<td>13.21</td>
<td>9.12</td>
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<td>11.53</td>
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<td>24.64</td>
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<td>115.67</td>
<td>82.67</td>
<td>57.91</td>
<td>42.16</td>
<td>29.03</td>
<td>21.16</td>
<td>14.59</td>
</tr>
</tbody>
</table>

\[
C = 0.05 \text{ mm}
\]

\[
Hp = \frac{f^2}{FC} + f
\]

Where:
- \( f \) = lens focal length
- \( F = F\# = \frac{f}{D} \)
- \( C = \text{Circle of Confusion} \approx 0.050 \text{ mm} \)

RIT DL 6-24-2012

Camera Calibration DL
1. Focus lens at Infinity.
2. Hyper focal distance = \( \frac{f^2}{F\# \times C} \)
3. C is Circle of confusion \( \sim 72 \mu m \)

**Camera Positions and Control Point Cage**

Roll the camera 90° ± about the optical axis at Positions 1 and 3

* High
* Mid
* Low
Geometry of a Frame Photograph

Elements of Exterior Orientation (6)
Derivation of Collinearity Equations

Photo Eq DL

Calibrating Film and Digital Sensors for Today's RS Business
The B Matrix is the Observation Equations for a 69 x 45 matrix.

3 Photo strip
9 Pass Points (9 tie Pts)
3 Aux data 6x6 matrices from POS
3 Control Points No 1,6 & 7 (X,Y,Z)

<table>
<thead>
<tr>
<th>No of Unknowns</th>
<th>No of observation Eq</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Photos x 6 = 18</td>
<td>69 &gt; 45</td>
</tr>
<tr>
<td>9 Pts x 3 = 27</td>
<td></td>
</tr>
<tr>
<td>Total = 45</td>
<td></td>
</tr>
</tbody>
</table>

No of Observation Equations

<table>
<thead>
<tr>
<th>Photo</th>
<th>Pts</th>
<th>2 x Pts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photo I</td>
<td>6</td>
<td>2 x 6 = 12</td>
</tr>
<tr>
<td>Photo II</td>
<td>9</td>
<td>2 x 9 = 18</td>
</tr>
<tr>
<td>Photo III</td>
<td>6</td>
<td>2 x 6 = 12</td>
</tr>
<tr>
<td>Total = 42</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Photo I 6 B' = 6
Photo II 6 B' = 6
Photo III 6 B' = 6
Ctr Pts 1,6,7 9 B'' = 9

Total No of Eq is 42 + 27 = 69
Normal Equations $N$ for 3 photo strip $45 \times 45$.

$3 \times 6 = 18$ orientation elements.

$9$ tie Pts $\times 3 = 27$ unknowns.

3 tie points are also control pts 1, 6 & 7 ($X,Y,Z$).

Matrices for Least Sqs

$[B^TB]\Delta = [B^TF^o]$  

or with Weights $[W]$

$[B^TWB]\Delta = [B^TWF^o]$  

$[W] = 1/\sigma^2$
expanded extended collinearity equations

Expanded form

\[
x - dx_p - \frac{x_m}{c} dc + K_1 x_m r^2 + K_2 x_m r^4 + K_3 x_m r^6 + P_1 \left(3 x_m^2 + y_m^2\right) + 2 P_2 x_m y_m = -c \frac{X'}{Z'}
\]
\[
y - dy_p - \frac{y_m}{c} dc + K_1 y_m r^2 + K_2 y_m r^4 + K_3 y_m r^6 + 2 P_1 x_m y_m + P_2 \left(3 y_m^2 + x_m^2\right) = -c \frac{Y'}{Z'}
\]

where

\[
x_m = x - x_p
\]
\[
y_m = y - y_p
\]

We can now express these extended collinearity equations in matrix form as

\[
A_1 \delta_1 + A_2 \delta_2 + A_3 \delta_3 + w = 0
\]

where

\[
\begin{pmatrix}
dx \\
dy
\end{pmatrix}_3 = A_3 \delta_3
\]
Taylor Series Expansion to Create an Observation Equation from the Collinearity Equation

\[ x_p - x_o = -f \left[ \frac{m_{11}(X_p - X_{o1}) + m_{12}(Y_p - Y_{o1}) + m_{13}(Z_p - Z_{o1})}{m_{31}(X_p - X_{o1}) + m_{32}(Y_p - Y_{o1}) + m_{33}(Z_p - Z_{o1})} \right] \]

\[ y_p - y_o = -f \left[ \frac{m_{21}(X_p - X_{o1}) + m_{22}(Y_p - Y_{o1}) + m_{23}(Z_p - Z_{o1})}{m_{31}(X_p - X_{o1}) + m_{32}(Y_p - Y_{o1}) + m_{33}(Z_p - Z_{o1})} \right] \]

\[
F_0 + \left( \frac{\partial F}{\partial \omega} \right)_0 d\omega + \left( \frac{\partial F}{\partial \phi} \right)_0 d\phi + \left( \frac{\partial F}{\partial \kappa} \right)_0 d\kappa + \left( \frac{\partial F}{\partial X_L} \right)_0 dX_L + \left( \frac{\partial F}{\partial Y_L} \right)_0 dY_L \\
+ \left( \frac{\partial F}{\partial Z_L} \right)_0 dZ_L + \left( \frac{\partial F}{\partial X_A} \right)_0 dX_A + \left( \frac{\partial F}{\partial Y_A} \right)_0 dY_A + \left( \frac{\partial F}{\partial Z_A} \right)_0 dZ_A = x_a \quad (D-9) \]

\[
G_0 + \left( \frac{\partial G}{\partial \omega} \right)_0 d\omega + \left( \frac{\partial G}{\partial \phi} \right)_0 d\phi + \left( \frac{\partial G}{\partial \kappa} \right)_0 d\kappa + \left( \frac{\partial G}{\partial X_L} \right)_0 dX_L + \left( \frac{\partial G}{\partial Y_L} \right)_0 dY_L \\
+ \left( \frac{\partial G}{\partial Z_L} \right)_0 dZ_L + \left( \frac{\partial G}{\partial X_A} \right)_0 dX_A + \left( \frac{\partial G}{\partial Y_A} \right)_0 dY_A + \left( \frac{\partial G}{\partial Z_A} \right)_0 dZ_A = y_a \quad (D-10) \]
Alignment of Lens Elements

Misalignment of Camera lenses leads to Decentering Distortion

Calibrating Film and Digital Sensors for Today’s RS Business

Cameras DL
A Multiframe Analytical Calibration Model

\[
x - x_p = c \left[ \frac{m_{11}(X - X_c) + m_{12}(Y - Y_c) + m_{13}(Z - Z_c)}{m_{31}(X - X_c) + m_{32}(Y - Y_c) + m_{33}(Z - Z_c)} \right] + \frac{x}{r} \left[ K_1 r^3 + K_2 r^5 + K_3 r^7 \right] + \left[ P_1 r^2 + 2x^2 + 2P_2 xy \right] + B_1 x + B_2 y
\]

\[
y - y_p = c \left[ \frac{m_{21}(X - X_c) + m_{22}(Y - Y_c) + m_{23}(Z - Z_c)}{m_{31}(X - X_c) + m_{32}(Y - Y_c) + m_{33}(Z - Z_c)} \right] + \frac{y}{r} \left[ K_1 r^3 + K_2 r^5 + K_3 r^7 \right] + \left[ 2P_1 xy + P_2 r^2 + 2y^2 \right]
\]

<table>
<thead>
<tr>
<th>Image</th>
<th>Projective equations</th>
<th>Radial distortion</th>
<th>Decentering distortion</th>
<th>Dif-Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x, y)</td>
<td>Measured image coordinates</td>
<td>(K_1, K_2, K_3)</td>
<td>Gaussian coefficients of radial distortion</td>
<td></td>
</tr>
<tr>
<td>(x_p, y_p)</td>
<td>Coordinates of principal point</td>
<td>(K'_0, K'_1, K'_2, K'_3)</td>
<td>Coefficients of balanced radial distortion</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>Focal length</td>
<td>(P_1, P_2), (r)</td>
<td>Coefficients of decentering distortion</td>
<td>Radial distance referred to (x_p, y_p)</td>
</tr>
<tr>
<td>(m_{11}, m_{12}...m_{33})</td>
<td>Orientation matrix elements which are functions of three independent angles Alpha, Elev., Roll referred to an arbitrary (X, Y, Z) frame in object target space</td>
<td></td>
<td></td>
<td>Coefficients of image axis affinity and non-orthogonality (rarely significant)</td>
</tr>
<tr>
<td>(X, Y, Z)</td>
<td>Object space target points</td>
<td>(X_c, Y_c, Z_c)</td>
<td></td>
<td>Camera station coordinates</td>
</tr>
</tbody>
</table>

Calibrating Film and Digital Sensors for Today's RS Business
Collinearity Equation Augmented for Analytical Self-Calibration

\[
x_a = x_o - \bar{x}_a (k_1 r_a^2 + k_2 r_a^4 + k_3 r_a^6) - (1 + p_3^2 r_a^2) [p_1 (3\bar{x}_a^2 + \bar{y}_a^2) + 2p_2 \bar{x}_a \bar{y}_a] - f^r \frac{r}{q}
\]
\[
y_a = y_o - \bar{y}_a (k_1 r_a^2 + k_2 r_a^4 + k_3 r_a^6) - (1 + p_3^2 r_a^2) [2p_1 \bar{x}_a \bar{y}_a + p_2 (\bar{x}_a^2 + 3\bar{y}_a^2)] - f^s \frac{s}{q}
\]

where

- \(x_a, y_a\) = measured photo coordinates related to fiducials
- \(x_o, y_o\) = coordinates of the principal point
- \(\bar{x}_a = x_a - x_o\)
- \(\bar{y}_a = y_a - y_o\)
- \(r_a^2 = \bar{x}_a^2 + \bar{y}_a^2\)
- \(k_1, k_2, k_3\) = symmetric radial lens distortion coefficients
- \(p_1, p_2, p_3\) = decentering distortion coefficients
- \(f\) = calibrated focal length
- \(r, s, q\) = collinearity equation terms as defined for Eqs. (D-7) and (D-8)
Two Photo Stereo Ortho Cell
Not sufficient GCPs for rigorous calibration.
Notice that each observation is stacked under its appropriate unknown including the 8 Camera Calibration Parameters.

<table>
<thead>
<tr>
<th>Photos</th>
<th>Points</th>
<th>Cal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega, \phi, \kappa, X^c, Y^c, Z^c$</td>
<td>$X, Y, Z$</td>
<td>$k_1, k_2, k_3, p_1$</td>
</tr>
</tbody>
</table>

Not sufficient GCPs for rigorous calibration. Notice that each observation is stacked under its appropriate unknown including the 8 Camera Calibration Parameters.

6 Auxiliary Orientation Observations per Photo.

3 Aux $X, Y, Z$ Observations per each GCP.

[69 x 53] Equations MC and DL Cam Cal
Normal Equations (N) for 3-photos and 8 Calibration Parameters. Example, not sufficient for rigorous camera calibration. Notice for [N] the 8 calibration observations stack along the bottom of the [N] matrix.

The 8 observations for the calibration, will add 8 unknowns to the square [N] matrix.

<table>
<thead>
<tr>
<th>Photos</th>
<th>Points</th>
<th>Cal</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>II</td>
<td>III</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>I</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>I</td>
<td>3</td>
<td>4</td>
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<tr>
<td>III</td>
<td>II</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>III</td>
<td></td>
</tr>
</tbody>
</table>

Normal Equations Matrix $B^T B$ for a Three Photo Strip with Calibration Parameters.
where \( r \) is the radial distance (from the principal point) and the \( k_i \)'s are the coefficients of distortion. The single coefficient \( k_1 \) is sufficient to account for the distortion of most simple lenses over their usable field. However, we have found that modern highly corrected lenses are likely to require the fifth and even the seventh order coefficients. Three coefficients of the expansion have proven adequate for all ballistic cameras encountered to date.

In the event that it is desired to enforce a specified principal distance in the plate reduction, the distortion function must assume the form

\[
(4.2) \quad \delta' = k_0' r + k_1' r^3 + k_2' r^5 + k_3' r^7 + \ldots.
\]

If \( c \) denotes the principal distance associated with \( \delta \) and \( c' = c + \Delta c \) denotes the principal distance associated with \( \delta' \), it follows from (4.1) that

\[
(4.3) \quad \delta' = \left( 1 + \frac{\Delta c}{c} \right) \delta + \frac{\Delta c}{c} r,
\]

from which

\[
(4.4) \quad k_0' = \frac{\Delta c}{c}, \quad k_1' = \left( 1 + \frac{\Delta c}{c} \right) k_1, \quad k_2' = \left( 1 + \frac{\Delta c}{c} \right) k_2, \quad \text{etc.}
\]

This emphasizes that a distortion function is meaningful only when its associated principal distance is specified. The term \( k_0' r \) in (4.2) is equivalent to a constant scale factor. Therefore, one cannot arbitrarily carry both \( k_0 \) and \( c \) as unknowns in a plate reduction, for both parameters perform precisely the same function in the model; to do so would lead to an indeterminate set of normal equations. Accordingly, if the principal distance is carried as an unknown, the associated distortion function must be of the form (4.1); the form (4.2) may be used only if an arbitrary value of the principal distance is enforced. It is essentially immaterial which approach is taken, for the results of the one can be transformed to correspond to the other.
Procedures and Mathematical Models employed in Calibrating Digital **Push Broom** Sensors

Dr. Abdullah
Calibration Process for Leica ADS Sensor

Forward

PAN FW
+27.4°

0°

Backward

RGB NIR

-16°

RGB NIR

PAN BW
-14.2°

PAN

2nd Tetrachroid

1st Tetrachroid

Calibrating Film and Digital Sensors for Today's RS Business
Push-Broom: Sensor Modeling

- With the movement of the aircraft three image strips are constructed synchronously.
- Each image strip is formed from adjacent scan lines along the flight track.
- Each scan line of the Three Line Scanner (TLS) is collected at different instant of time.
- Each scan line is treated like a traditional frame photo image.
- Each scan line owns an independent set of exterior orientation.

Line Camera: Scene Acquisition

\[ \text{Time of exposure} = f(\text{image/row number within the scene}) \]
- Image-to-ground transformation in TLS is based on the collinaerity equation.
- The TLS such as ADS80 is equipped with an IMU of medium accuracy.
- Good mathematical sensor model is needed to improve the time-dependent orientation elements of the TLS trajectory by photogrammetric triangulation.
- The math model combine the high short-term accuracy of the IMU with the high global accuracy of the GPS.
Calibrating Film and Digital Sensors for Today's RS Business

ADS: CCD in Focal Plane

PANF27

PANF02

RED, GREEN, BLUE, NIR (Nadir)

RED, GREEN, BLUE, NIR (Backward 16)

Calibrating Film and Digital Sensors for Today's RS Business
Why we need calibration for ADS?

Leica Requirement on Calibration

- “SH81/SH82 calibration should be verified at the beginning of each flying season and during the flying season after half a year of operation.”

- Various factors will affect the Leica ADS80 system, in particular:
  - Operating hours
  - Environmental conditions in use and storage
  - Care in use, handling and storage

<table>
<thead>
<tr>
<th>Component</th>
<th>Type of job</th>
<th>Operation hours</th>
<th>Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole Leica ADS80 system</td>
<td>Preemptive technical and calibration check</td>
<td></td>
<td>1 - 2 years</td>
</tr>
</tbody>
</table>
Parameters Effected by Camera Calibration

**Internal Calibration Parameters**

- Lens focal length – can be fixed to certain values
- Focal plan Geometry: CCD Positions – updated by calibration
- Orientation of the optical system with respect to IMU, misalignment – Measured by calibration
  - They are highly stable and static
  - They are calibrated during manufacturing and do not change, may be slightly

**External parameters** – Fairly dynamic

- Position of the camera in geocentric frame – Measured by ABGPS
- Orientation of the camera in geocentric frame – Measured by IMU
The collinearity equation represent the condition that the points on the ground \( A \), its image \( a \) and lens focal point \( L \) lie on a straight line.

\[ X_L, X_L, L_L = \text{Camera perspective center} \]

\( O = \) projection of perspective center on image (principal point)

\( X, Y, Z = \) ground point position

\( x, y = \) point position on image

\( m_{ii} = \) photo orientation matrix

\( f = \) camera lens focal length

\[
x = -f \left[ \frac{m_{11}(X - X_L) + m_{12}(Y - Y_L) + m_{13}(Z - Z_L)}{m_{31}(X - X_L) + m_{32}(Y - Y_L) + m_{33}(Z - Z_L)} \right]
\]

\[
y = -f \left[ \frac{m_{21}(X - X_L) + m_{22}(Y - Y_L) + m_{23}(Z - Z_L)}{m_{31}(X - X_L) + m_{32}(Y - Y_L) + m_{33}(Z - Z_L)} \right]
\]
Calibrating Film and Digital Sensors for Today's RS Business

Optical distortion

Optical center

Focal plane

distortion

Virtual focal plane

Terrain
the (x, y) coordinates for each CCD in the focal plane depend on the chosen value of the focal length f.

Leica fixes the focal length f to 62.7mm and corresponding pixel size to approximately 6.5µm.

(x, y) and f determine the direction of a line of sight with respect to the optical sensor.
Line Camera: Scene Acquisition

- Time of exposure = f (image/row number within the scene).
Factors Effecting Calibration: GPS Antenna Offset

Offset Between Airborne Antenna and Camera

- Offset of the antenna from the exposure station must be known in the image coordinate system.

- Offsets can be measured by conventional surveying. A millimeter level accuracy can be achieved very easily. The aircraft must be jacked above the wheels and leveled during this process.

- Offset varies with aircraft heading and tilt.
Factors Effecting Calibration: GPS Antenna Offset

- Determination of the Spatial Offset
  - Offsets can usually be measured using traditional surveying methods. A millimeter level accuracy can be achieved. The aircraft must be jacked above the wheels and leveled during this process.
  - Offset varies with aircraft heading and tilt.
Factors Effecting Calibration: Time Offset

WILD RC-20 Mid-Exposure Pulse

COMMENT:
- Max time difference between:
  - GPS mid-exposure pulse and actual center of exposure is
    - 52 us (microseconds), independent of the shutter speed used
- Mean time difference to be expected (approximately)
  - 35 us (microseconds)
- Mid exposure signal performance applies to all RC20 lens cones with software release 2.2 (upgrading of earlier SW-version possible)

Calibrating Film and Digital Sensors for Today's RS Business
Effects of GPS and IMU on the Calibration of Digital Push Broom Sensors

Dr. Abdullah
IMU Role in ADS Exterior Orientation Determination

- During the flight, the IMU computes in real time the orientation between its body frame \((X^bY^bZ^b)\) and the local geographic frame (Navigation Frame North, East, Down).

- Self Calibration is used to model:
  - The bias in the computed orientation
  - The drift in time
GPS Drift Parameters

- Drift error in each coordinate is modeled as:

\[
\Delta x = a_x + b_x (t - t_o)
\]
\[
\Delta y = a_y + b_y (t - t_o)
\]
\[
\Delta z = a_z + b_z (t - t_o)
\]
GPS Drift Parameters

- The GPS coordinates are introduced as additional observations via the following equation:

$$
\begin{bmatrix}
X \\
Y \\
Z \\
\end{bmatrix}_A +
\begin{bmatrix}
VX \\
VY \\
VZ \\
\end{bmatrix}_A =
\begin{bmatrix}
X \\
Y \\
Z \\
\end{bmatrix}_P +
M^T
\begin{bmatrix}
dX \\
dY \\
dZ \\
\end{bmatrix}_A +
\begin{bmatrix}
ax \\
ay \\
az_k \\
\end{bmatrix}_k +
\begin{bmatrix}
bx \\
by \\
bz_k \\
\end{bmatrix}_k (t - t_0)
$$

Where:

dx, dy and dz offsets between camera and receiver
ax, ay,…, bz drift parameters for k th strip.
Facts on the Roles of IMU in ADS Calibration

- Inertial Measurement Unit (IMU) Measures camera orientation in local geographic frame
- The IMU is rigidly mounted to the optical sensor.
- The orientation between the optical sensor and the IMU is determined by the camera manufacturer during camera manufacturing.
- This orientation can vary slightly with time or operation by the amount of misalignment.
- The misalignment values need to be determined during re-calibration.
ADS sensor without IMU
Calibrating Film and Digital Sensors for Today’s RS Business

ADS Calibration purpose

**Assumption:** Problems in the internal camera parameters, those are
- IMU misalignment
- Focal plane geometry (CCD lines geometry)

**Goal:** more accurate values for these parameters to be computed before using the camera in production
Calibration Procedure

1. Collect multiple images over an area from different perspective and different altitudes;

1. Establish dense tie points network between all of these imagery in order to establish the constraining condition;

1. Bundle adjustment, compute and analyze residuals

2. Self Calibration: Compute the camera internal parameters that satisfy all the geometrical constraints;

1. Update camera files

GCP are used for verification ONLY
Calibration Flight: Practical Design

Fly over controlled area back and forth in two orthogonal directions from at least two different altitudes (four is preferred).

High altitude

Low altitude
Flight Plan for Leica’s 4 Altitudes Design
Tie/Pass Points – typical auto-correlated points distribution
Facts about observations

- **Redundancy**: Number of equations should be more than unknown parameters

- **Random Error**: It is not possible to perfectly satisfy all geometrical constraints because of deviation due to the fact that matched points have limited precision

- **Random Error**: There will always be a residual error that can not be eliminated but we strive to find the parameters that minimize this residual error
Modeling of Residual Error

- After collecting all image residuals, we fit a polynomial function to the cloud of points (for x, and for y residuals)
- We modify the position of CCD cells using this polynomial function
There is a higher risk that the bundle adjustment does not converge (or converge slowly) if there are correlated parameters.

Therefore we have to fix some parameters among f, IMU misalignment ($r_{xc}$, $r_{yc}$, $r_{zc}$) and IMU bias and drift ($p_{x0}$, $p_{x1}$, $p_{y0}$, $p_{y1}$).

Usually the overall shape of one CCD line is fixed in the adjustment.
**ADS40 Processing Workflow: ORIMA Orientation Fixes**

- The GPS and IMU data, which is measured at high rates during image acquisition, yields a continuous position and attitude of the ADS40.
- During the triangulation process, we want to update this continuous stream of data based on the principles of least squares bundle adjustment.
- In order to do that, “orientation fixes” at regular intervals along the flight path of the push broom scanner are used.
Orientation fixes are characterized by:

- An orientation fix is the orientation of the sensor at a certain time
- Geometrically best conditions are obtained when the distance between two fixes equals the short base
- The time interval between two fixes depends on the gyro (IMU) quality
- The six orientation parameters for each fix are updated by the triangulation process; each fix is identified by the unique time
- One scene (image) has multiple orientation fixes
The relation between the orientation parameters, the orientation fixes and the GPS/IMU observations

Typically, the GPS/IMU sensors will have a systematic offset to the actual sensor head. This systematic offset between the true orientation and the GPS/IMU observations is compensated and computed by additional parameters within the self-calibration process of the triangulation.
The relation between the orientation parameters, the orientation fixes and the GPS/IMU observations

After the adjustment, the orientation of the GPS/IMU is updated by piecewise interpolation from the orientation fixes, which were computed in the bundle adjustment.
Mathematical Model for TLS Aerotriangulation

- The mathematical model describes the transformation of a point from the ground system to the orientation fixes.

The ground to sensor transformation is characterized by:
- Points can be measured at any location
- Each projected point falls in between two orientation fixes
- The transformation from ground to sensor is expressed as a function of the two neighbouring orientation fixes
The image coordinates are expressed as a function of the ground point \((\mathbf{P}_i)\) and the orientation parameters of the two neighbouring orientation fixes \((k)\) and \((k+1)\).

\[
\begin{align*}
  x_{ij} &= F_{ij}(X_i, Y_i, Z_i, X_k, Y_k, Z_k, \omega_k, \varphi_k, \kappa_k, \\
  &\quad X_{k+1}, Y_{k+1}, Z_{k+1}, \omega_{k+1}, \varphi_{k+1}, \kappa_{k+1}) \\
  y_{ij} &= G_{ij}(X_i, Y_i, Z_i, X_k, Y_k, Z_k, \omega_k, \varphi_k, \kappa_k, \\
  &\quad X_{k+1}, Y_{k+1}, Z_{k+1}, \omega_{k+1}, \varphi_{k+1}, \kappa_{k+1})
\end{align*}
\]

The orientation parameters \((x_j, \kappa_j)\) are introduced into the well known collinearity equations to transform the point from the ground system to the sensor system.

\[
\begin{bmatrix}
  x_{ij} \\
  y_{ij} \\
  -f
\end{bmatrix}
= \lambda_{ij} \mathbf{R}(\omega, \varphi, \kappa)_{ij}
\begin{bmatrix}
  X_i - X_j \\
  Y_i - Y_j \\
  Z_i - Z_j
\end{bmatrix}
\]
Design and illustration of indoor and in situ calibration fields

Dr. Munjy
### Methods of Camera Calibration

In general, camera calibration may be classified into one of three basic categories:

<table>
<thead>
<tr>
<th></th>
<th>Laboratory methods</th>
<th>Multicollimators, Gonimeters, Cages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Field methods</td>
<td>Ground control point ranges, targets</td>
</tr>
<tr>
<td>2</td>
<td>Stellar methods</td>
<td>Star coordinates.</td>
</tr>
</tbody>
</table>

The calibration parameters are generally referred to as "**elements of interior interior orientation**."

These elements are needed in order to determine accurate spatial information from x,y-image coordinates.
USGS Optical Collimator for Calibrating Aerial 9” x 9” Film Cameras.

- Collimator contains 57 targets spanning 94 degrees wide FOV
- The USGS calibrates about 120 film cameras per year.
Digital Camera Calibration Cage at Visual Intelligence

Calibrating Film and Digital Sensors for Today's RS Business
Camera Stand Tripod by Calument

Calibrating Film and Digital Sensors for Today’s RS Business
Control Point Cage: EROS Data Center

Control Point Cage and Moveable Tripod
Calibrating Film and Digital Sensors for Today’s RS Business
RIT CIS Control Point Cage VNIR & TIR Targets (12W x 9H x 8D)ft
Center of Target Cage: Visible targets are white and Thermal targets show a black dot (Resistor) in the center that generates heat so it will be imaged by the TIR cameras.
Night Time Imaging with Visible, SWIR and LWIR Cameras

Thermal Infra-Red Cameras are Temperature Sensors

Calibrating Film and Digital Sensors for Today's RS Business
Digital Camera Calibration Target Cage
(12 ft W x 10 ft H x 8 ft D)
The UltraCam calibration laboratory is equipped with a three dimensional target which contains 367 targets. The size of the cage volume is 8.4 m x 2.4 m x 2.5 m.
Composite of All three panels, Black 1, Red 2, and Blue 3

Front Panel 1 black, Mid Panel 2 Red, Rear Panel 3 Blue

Cage size: (12 W x 9 H x 8 D) Ft
Attach ½ inch plywood painted flat black to rear panel
Composite of All Three Panels, Black 1, Red 2, and Blue 3

Scale: 1 inch = 2 ft on Cage

Front Panel 1 black, Mid Panel 2 Red, Rear Panel 3 Blue

Cage size: (12 W x 9 H x 8 D) Ft

Attach black curtains in the back to provide target contrast
Taut Line Method of Camera Calibration

1. Hang Mono-filament lines ~ 10 from a ceiling with weights on the bottom near the floor. Keep the lines tight and straight.
2. Take six pictures from ~ - 30° to ~ + 30° angles filling the camera’s field of view as much as possible with each exposure.
3. The deviation of the imaged lines from the taut straight lines is the basis for modeling the distortion inherent in the camera lens. (Sin curve)
4. Solving for the focal length by this method is open to question because of insufficient depth difference in the target lines.

This concept of camera calibrations was a dinner conversation with Duane Brown Circa 1970

~ 10 Lines
Calibrating With Chessboards

- A chessboard certain properties that makes it good for calibrating purposes:
  - There exist points on this plane that you can identify uniquely.
  - You can extract these points very easily.
  - These points physically lie in straight lines, no matter how they appear on camera.

Ref: http://aishack.in/tutorials/calibrating-camera-theory/
Posing the Chessboard

- You take several pictures of the chessboard.
- Each picture in a different orientation.
- These orientations should vary a lot. Or you might get a bad estimate of the various parameters.
- A possible chessboard orientation could be like this:

Ref: http://aishack.in/tutorials/calibrating-camera-theory/
Automated Camera Calibration

Ref: Automated Target Free Camera Calibration, C. Fraser, et al., ASPRS 2014 Annual Conference,
Automated Camera Calibration

- Configuration for 25,000 point ‘Sandstone’ self-calibration network for Nikon D200 SLR

Ref: Automated Target Free Camera Calibration, C. Fraser, et. al, ASPRS 2014 Annual Conference.
UAV Self-Calibration

- Configuration for 20,000 point Hessigheim UAV self-calibration network for the Canon IXUS 100 IS

Ref: Automated Target Free Camera Calibration, C. Fraser, et. al, ASPRS 2014 Annual Conference,
Flight Design (4 x 6) Images for Bore Sight and Self Calibration Comps

GSD (H/f x pix size) at “d” must be large enough to see on the imagery.

GCPs should be in the forward Stereo overlap area.

Strip 1

Strip 4

Ground Control Point (X, Y, Z)
GPS Photo Center
Direction of Flight

Panels for GCPs
Special Consideration for UAS Imaging

The new Phan3 4K features a powerful 4K camera for
• GPS-Assisted Hover
• Wi-Fi Video Downlink
• Customizable Remote Controller
Design and illustration of in situ calibration fields

Dr. Abdullah
USGS Quality Assurance Plan: Manufacturer Certification Program

TYPE Certification: A team from USGS and partner members visits the manufacturer of digital sensor system and learn about:

- Design
- Development
- Testing
- Manufacturer’s intended operational constraints and required support

- It is intended to ensure that the sensor system made by the manufacturer have been designed to reliably, repeatedly, and routinely deliver an output product of consistent quality “verification of the manufacturer claim."
- It is not imply that each separate sensor system within certification type class will deliver identical data characteristics.
In SITU Accuracy and Image Quality Evaluation:
Manufacturer fly the sensor over the USGS controlled field and send the ortho-photo to the USGS for evaluation
USGS Certification Field, Sioux Falls

Available Data:
- Ground Controls for Multi Scale
- Lidar Terrain Model

Calibrating Film and Digital Sensors for Today's RS Business
Typical sensor calibration reports

Dr. Abdullah
Calibration Certificate

Digital Mapping Camera (DMC)

DMC Serial Number: DMC01-0006

CBU Serial Number: 01000006

For

Aero Metric, Inc.
420 Technology Parkway
Sheboygan, WI 53083

USA

Calibrating Film and Digital Sensors for Today's RS Business
DMC Calibration

Figure Image Overview (Pan Camera)
Calibration Protocol
DMC01 - 0006

Camera Orientation PAN-Cameras (Burn-In Flight 05.11.2003)

<table>
<thead>
<tr>
<th>Camera (Serial Number)</th>
<th>X [m] (Accuracy)</th>
<th>Y [m] (Accuracy)</th>
<th>Z [m] (Accuracy)</th>
<th>Omega [Deg] (Accuracy)</th>
<th>Phi [Deg] (Accuracy)</th>
<th>Kappa [Deg] (Accuracy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAN1 (00109379)</td>
<td>0.064 (0)</td>
<td>-0.079 (0)</td>
<td>1000 (0)</td>
<td>17.946 (0.001)</td>
<td>10.055 (0.001)</td>
<td>87.058 (0.001)</td>
</tr>
<tr>
<td>PAN2 (00109393)</td>
<td>-0.064 (0)</td>
<td>-0.079 (0)</td>
<td>1000 (0)</td>
<td>17.918 (0.001)</td>
<td>-10.211 (0.001)</td>
<td>93.182 (0.001)</td>
</tr>
<tr>
<td>PAN3 (00109392)</td>
<td>-0.064 (0)</td>
<td>0.079 (0)</td>
<td>1000 (0)</td>
<td>-17.967 (0.001)</td>
<td>-10.043 (0.001)</td>
<td>-93.355 (0.001)</td>
</tr>
<tr>
<td>PAN4 (00109388)</td>
<td>0.064 (0)</td>
<td>0.079 (0)</td>
<td>1000 (0)</td>
<td>-17.896 (0.001)</td>
<td>10.202 (0.001)</td>
<td>-87.077 (0.001)</td>
</tr>
</tbody>
</table>

The data is connected to the virtual projection center of the virtual image. The above Platform calibration values are initial values and are be liable to slight fluctuations between project images and between different projects. The position is fix and error free. The rotation axes of the angles are (in this order)

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Omega</td>
<td>x-Axis</td>
</tr>
<tr>
<td>Phi</td>
<td>y-Axis</td>
</tr>
<tr>
<td>Kappa</td>
<td>z-Axis</td>
</tr>
</tbody>
</table>

The results of the Platform calibration were generated with DMC Postprocessing SW (PPS), Version 5.4, from Intergraph Z/I Imaging photogrammetric product suite.

Calibrating Film and Digital Sensors for Today’s RS Business
Aerotriangulation Results (Burn-In Flight 05.11.2003)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photo Scale</td>
<td>1:5000</td>
</tr>
<tr>
<td>Flying Height [m]</td>
<td>600 AGL</td>
</tr>
<tr>
<td>Flying Altitude [m]</td>
<td>1200 AMSL</td>
</tr>
<tr>
<td>Run-Spacing [m]</td>
<td>580.6</td>
</tr>
<tr>
<td>Base-Length [m]</td>
<td>184.3</td>
</tr>
<tr>
<td>Number of Exposures</td>
<td>88</td>
</tr>
<tr>
<td>Side-lap [%]</td>
<td>30</td>
</tr>
<tr>
<td>End-lap [%]</td>
<td>60</td>
</tr>
<tr>
<td>Terrain Height [m]</td>
<td>600</td>
</tr>
<tr>
<td>Number of strips</td>
<td>4</td>
</tr>
<tr>
<td>Photos in one strip</td>
<td>4 x 22 N-S</td>
</tr>
<tr>
<td>Photos Used</td>
<td>88</td>
</tr>
<tr>
<td>Control Points Used</td>
<td>16</td>
</tr>
<tr>
<td>Check Points Used</td>
<td></td>
</tr>
<tr>
<td>GSD [cm]</td>
<td>6</td>
</tr>
</tbody>
</table>

Statistic results:

Matching results: 0 Weak Areas – covered with clouds

Whole Block  88 exposures used
Whole Block  0 exposures not used
Whole Block  Sigma relativ: 1.556 um
Whole Block  Sigma absolut: 1.683 um

PhotoS Triangulation Options:

- Adjustment Mode: Absolute
- Precision Computation: Enabled
- Error Detection: Disabled
- Camera Calibration: Disabled
- Self-Calibration: Disabled
Geometric Calibration Protocol

Calibration Parameters for single camera head

<table>
<thead>
<tr>
<th>Camera Type</th>
<th>DMC-Panchromatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Focal Length</td>
<td>0.12 m</td>
</tr>
<tr>
<td>Serial Number</td>
<td>00109393</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Param</th>
<th>Adjusted</th>
<th>Std.dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_0$</td>
<td>-6.459E-05</td>
<td>1.851E-06</td>
</tr>
<tr>
<td>$Y_0$</td>
<td>5.056E-05</td>
<td>6.371E-06</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>-0.0003769</td>
<td>2.217E-06</td>
</tr>
<tr>
<td>$K_1$</td>
<td>0.3664</td>
<td>0.05449</td>
</tr>
<tr>
<td>$K_2$</td>
<td>-1.178</td>
<td>48.61</td>
</tr>
<tr>
<td>$K_3$</td>
<td>-0.46640</td>
<td>12670</td>
</tr>
<tr>
<td>$P_1$</td>
<td>0</td>
<td>1E-31</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.0005563</td>
<td>0.0001214</td>
</tr>
<tr>
<td>$B_1$</td>
<td>6.267E-05</td>
<td>1.534E-05</td>
</tr>
<tr>
<td>$B_2$</td>
<td>1.956E-05</td>
<td>7.333E-06</td>
</tr>
</tbody>
</table>

Adjusted Focal length = 0.12 + dc ~0.1196231 [m]
Residuals in Image after Adjustment

Max Residual [μm]: 2.1
Threshold [μm]: 8.5

Remarks:
The images after the post processing are distortion free. For interior orientation parameters of the DMC virtual image see section: "Calibration Parameter of the virtual images".
The calibration model is explained in the section “Calibration Model” at the end of this documentation.
ADS40 Calibration Certificate

This certificate is valid for

- Sensor Head: SH52
  - Serial Number: 30102
- Control Unit: CU40
  - Serial Number: 31102

Calibration certificate issued on 29 January 2007

Certificate and calibration data ID: 870107_30101_070129-1

Inspector

Muzaffer Adigüzel

Leica Geosystems AG
Heinrich-Wild-Strasse
9435 Heerbrugg
Switzerland

Document code 870107
Results of geometrical calibration

Calibrated apparent pixel coordinates for all sensor lines are contained on the calibration file attached to this certificate. File: 30102-070129-1.zip

Stereoc lines

<table>
<thead>
<tr>
<th>A-lines</th>
<th>PANF27A</th>
<th>PANF02A</th>
<th>PANH14A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration method</td>
<td>Estimation of additional parameters in simultaneous bundle adjustment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sigma sought of bundle adjustment</td>
<td>3.8 micron</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean local redundancy</td>
<td>&gt; 0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Accuracy of calibrated apparent pixel coordinates</td>
<td>±1.0 micron</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Final bundle adjustment result after elimination of tie point blunders and before introduction of ground control:

IMU misalignment

Misalignment results in [rad]:

- $\omega = -0.000113222 \pm 0.0000093631$
- $\phi = +0.00061228 \pm 0.0000125149$
- $\kappa = +0.0008318302 \pm 0.0000224956$
CAMERA CALIBRATION REPORT

PROJECT DETAILS
Camera: Illini xmv1100c
Filename: C:\Documents and Settings\camera\Desktop\06.03.08 CAGE TEST 2\18June08\jpg18Ju
Calibration Date: 19/06/2008 14:44pm

METRIC CALIBRATION PARAMETERS
Resolution = 4096 x 2672 pixels
Pixel width = 0.0090mm, Pixel height = 0.0090mm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal distance</td>
<td>c = 89.155mm</td>
<td>0.030mm</td>
</tr>
<tr>
<td>Principal point offset in x-image coordinate</td>
<td>xp = -0.224mm</td>
<td>0.016mm</td>
</tr>
<tr>
<td>Principal point offset in y-image coordinate</td>
<td>yp = -0.293mm</td>
<td>0.011mm</td>
</tr>
<tr>
<td>3rd-order term of radial distortion correction</td>
<td>K1 = -1.45375e-005</td>
<td>4.3006e-007</td>
</tr>
<tr>
<td>5th-order term of radial distortion correction</td>
<td>K2 = 1.79284e-008</td>
<td>2.2552e-009</td>
</tr>
<tr>
<td>7th-order term of radial distortion correction</td>
<td>K3 = 1.49583e-011</td>
<td>3.5197e-012</td>
</tr>
<tr>
<td>Coefficient of decentering distortion</td>
<td>P1 = 4.4071e-006</td>
<td>6.943e-007</td>
</tr>
<tr>
<td>Coefficient of decentering distortion</td>
<td>P2 = 1.5606e-006</td>
<td>4.717e-007</td>
</tr>
<tr>
<td>Differential scaling between x &amp; y</td>
<td>B1 = 5.6049e-005</td>
<td>9.542e-006</td>
</tr>
<tr>
<td>Non-orthogonality between x &amp; y axes</td>
<td>B2 = -1.7746e-005</td>
<td>8.160e-006</td>
</tr>
</tbody>
</table>

STANDARD CORRECTION EQUATION
The corrected image coordinates \(x_{\text{corr}}\) & \(y_{\text{corr}}\) can be calculated from the measured coordinates \(x_{\text{meas}}\) & \(y_{\text{meas}}\) by using the formulas:

\[
x = x_{\text{meas}} - x_p
\]
\[
y = y_{\text{meas}} - y_p
\]

\(x\) and \(y\) are now with respect to the principal point,

\[
r^2 = x^2 + y^2
\]
\[
dr = K_1 r^3 + K_2 r^5 + K_3 r^7
\]
\[
x_{\text{corr}} = x_{\text{meas}} - x_p + x \cdot dr + P_1 (r^2 + 2x^2) + 2P_2 xy
\]
\[
y_{\text{corr}} = y_{\text{meas}} - y_p + y \cdot dr + P_2 (r^2 + 2y^2) + 2P_1 xy
\]
CAMERA CALIBRATION REPORT

GAUSSIAN RADIAL DISTORTION CORRECTION PROFILE (dr)
For principal distance c, Gaussian radial distortion correction dr (microns) is
given for any radial distance r (mm) as:
\[ dr = K1 r^3 + K2 r^5 + K3 r^7 \]
correction dx = x \cdot \frac{dr}{r}
correction dy = y \cdot \frac{dr}{r}

<table>
<thead>
<tr>
<th>VALUE</th>
<th>STANDARD ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>c = 89.155mm</td>
<td>0.030mm</td>
</tr>
<tr>
<td>K1 = -1.45375e-005</td>
<td>4.3006e-007</td>
</tr>
<tr>
<td>K2 = 1.79284e-008</td>
<td>2.2552e-009</td>
</tr>
<tr>
<td>K3 = 1.49583e-011</td>
<td>3.5197e-012</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>r(mm)</th>
<th>dr(microns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0</td>
</tr>
<tr>
<td>2.00</td>
<td>-0.1</td>
</tr>
<tr>
<td>4.00</td>
<td>-0.9</td>
</tr>
<tr>
<td>6.00</td>
<td>-3.0</td>
</tr>
<tr>
<td>8.00</td>
<td>-6.8</td>
</tr>
<tr>
<td>10.00</td>
<td>-12.6</td>
</tr>
<tr>
<td>12.00</td>
<td>-20.1</td>
</tr>
<tr>
<td>14.00</td>
<td>-28.7</td>
</tr>
<tr>
<td>16.00</td>
<td>-36.7</td>
</tr>
<tr>
<td>18.00</td>
<td>-41.7</td>
</tr>
<tr>
<td>20.00</td>
<td>-39.8</td>
</tr>
<tr>
<td>Gauss (um)</td>
<td>Balanced (um)</td>
</tr>
<tr>
<td>-----------</td>
<td>--------------</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>16.7</td>
</tr>
<tr>
<td>5</td>
<td>33.1</td>
</tr>
<tr>
<td>6</td>
<td>58.2</td>
</tr>
<tr>
<td>7</td>
<td>93.8</td>
</tr>
<tr>
<td>8</td>
<td>141.8</td>
</tr>
<tr>
<td>9</td>
<td>203.7</td>
</tr>
<tr>
<td>10</td>
<td>280.1</td>
</tr>
<tr>
<td>11</td>
<td>370.6</td>
</tr>
</tbody>
</table>

**Gaussian and Balanced Focal length: Radial Distortion Values.**

Gaussian f must be changed to Balanced f’ also.
**BALANCED RADIAL DISTORTION CORRECTION PROFILE**

For 'balanced' principal distance $c_b$, radial distortion correction $dr$ (microns) is given for any radial distance $r$ (mm) as:

$$dr = K0 \cdot r + K1 \cdot r^3 + K2 \cdot r^5 + K3 \cdot r^7$$

- $c_b = 89.353$ mm
- $K0 = 2.22219e-03$
- $K1 = -1.45698e-05$
- $K2 = 1.79682e-03$
- $K3 = 1.49916e-01$

<table>
<thead>
<tr>
<th>$r$ (mm)</th>
<th>$dr$ (microns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0</td>
</tr>
<tr>
<td>2.00</td>
<td>4.3</td>
</tr>
<tr>
<td>4.00</td>
<td>8.0</td>
</tr>
<tr>
<td>6.00</td>
<td>10.3</td>
</tr>
<tr>
<td>8.00</td>
<td>10.9</td>
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<td>10.00</td>
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</tr>
<tr>
<td>14.00</td>
<td>2.4</td>
</tr>
<tr>
<td>16.00</td>
<td>-1.3</td>
</tr>
<tr>
<td>18.00</td>
<td>-1.8</td>
</tr>
<tr>
<td>20.00</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Distortion profile is 'balanced' ($dr = 0.0$) about a radial distance of $r = 15.2$ mm.
CAMERA CALIBRATION REPORT

GAUSSIAN RADIAL DISTORTION PLOT  [dr shown in micrometres]

DECENTRING DISTORTION PLOT  [P(r) shown in micrometres]

(If present, — — — indicates the maximum radial distance encountered in the self-calibration.)
BALANCED RADIAL DISTORTION PLOT  [dr shown in micrometres]

Radial Distance (r)

(-- indicates the maximum radial distance encountered in the self-calibration.)
<table>
<thead>
<tr>
<th>SMAC</th>
<th>FotoG</th>
<th>Bouquet Camera Calibration Toolbox</th>
<th>Track-Eye / TEMA</th>
<th>VRTwo</th>
<th>Australis / iWitness</th>
<th>Compass</th>
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<tbody>
<tr>
<td>USGS</td>
<td>Vexcel Corp</td>
<td>camera or target centered</td>
<td>Image Systems</td>
<td>Cardinal</td>
<td>Photometrix</td>
<td>Owner Unknown</td>
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<tr>
<th>Exterior Orientation</th>
<th>X Y Z</th>
<th>Yaw</th>
<th>Pitch</th>
<th>Roll</th>
<th>Kappa</th>
<th>Roll</th>
<th>Swing = Kappa + 180</th>
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</thead>
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<tr>
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<td></td>
<td></td>
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<td></td>
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<tr>
<td>Y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Omega</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phi</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kappa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</table>

<table>
<thead>
<tr>
<th>Radial Distortion</th>
<th>k&lt;sup&gt;0&lt;/sup&gt;</th>
<th>k1</th>
<th>k2</th>
<th>k3</th>
<th>k&lt;sup&gt;0&lt;/sup&gt;</th>
<th>k1</th>
<th>k2</th>
<th>k3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-k&lt;sup&gt;0&lt;/sup&gt;</td>
<td>-K1</td>
<td>-K2</td>
<td>-K3</td>
<td>k&lt;sup&gt;0&lt;/sup&gt;</td>
<td>-K1</td>
<td>-K2</td>
<td>-K3</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Decentering Distortion</th>
<th>p&lt;sup&gt;1&lt;/sup&gt;</th>
<th>p&lt;sup&gt;2&lt;/sup&gt;</th>
<th>PPOX</th>
<th>Xp</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-P1</td>
<td>-P2</td>
<td></td>
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<table>
<thead>
<tr>
<th>Principal Point</th>
<th>xp = 0</th>
<th>cc(1) - xsize/2 - 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>yp</td>
<td>cc(2) - ysize/2 - 0.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Focal Length</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>fc(1)</td>
</tr>
<tr>
<td></td>
<td>fc(2)</td>
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<table>
<thead>
<tr>
<th>Skew</th>
<th>not used</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>alpha_c</td>
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<table>
<thead>
<tr>
<th>Image Origin</th>
<th>center</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>upper left</td>
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</tbody>
</table>

---

Camera Calibration Software: Reference, NASA’s Johnson Space Center

Calibrating Film and Digital Sensors for Today’s RS Business
Commercially Available Calibration Software

- Australis
- ORIMA
- BINGO
- Intergraph AAT
- Inpho Match AT
- Trimble/Applanix POS Cal
- Others
Questions and Discussions
Time Permission

IMU Integration with Aerial Sensors

Dr. Abdullah and Dr. Munjy
Fundamentals of INS

- **Inertia** is the tendency of bodies to maintain constant linear and rotational velocity, unless disturbed by external forces or torques, respectively.

- An **Inertial Reference Frame** is a coordinate frame in which Newton’s laws of motion are valid. These frames are not rotating nor accelerating.

- **Inertial Sensors** measure linear and angular velocity rates (both are vector valued variables).

- **Accelerometers** are sensors for measuring acceleration along its input axis.

- **Gyrosopes** (or simply gyros) are sensors for measuring rotation rate.
Integration of INS & GPS

• **GPS Advantages**
  - Provides position and velocity data
  - Maintains accuracy over long period and distance
  - Globally available system
  - Inexpensive for user
  - Easy to operate & process

• **INS Advantages**
  - Autonomous system that does not require external aids or visibility; can operate in tunnels (or underpass) and underwater
  - Immune to jamming and is inherently undetectable
  - Inherently well suited for integrated navigation; ideal for vehicular navigation, guidance and control
Integration of INS & GPS

- **GPS Disadvantages**
  - High frequency data is noisy with low accuracy
  - Weak signal is liable to interference and jamming
  - Needs line of sight; liable to signal interruption
  - Non-autonomous system; high precision requires relative positioning and post-processing

- **INS Disadvantages**
  - Precise high frequency data errors increase rapidly with time
  - Heavy cost of acquisition, operation and maintenance
  - Precision systems are still large in size and heavy
  - Higher power requirements
  - Heat dissipation needs
Kalman Filter Basics
Kalman Filter Recursive Solution

- **What is a Kalman Filter?**
  - A filter is anything that separates one thing from another.
    Example: An electrical analog or digital filter to separate voice signal from data over phone lines.

- **Objective:**
  - To separate one noise-like signal from another.
Computing Roll and Pitch Using Gyros and Accelerometers

- The optimal means of computing roll and pitch is to combine accelerometers with gyros such as in an IMU.
- The integrated gyro errors are controlled by low-pass filtering the accelerometer output to determine absolute tilts.
- Low-pass filtering action can be achieved using a Kalman filter.
Heading Computation

- Heading is initially determined by observing the component of Earth rate in the horizontal gyros.
- The vertical gyro is then integrated over time to determine change in heading.
Computation of Ground Accuracy
(1st order approx)

Position error due to Roll and Pitch error (roll/pitch < 10 deg)

\[ \Delta x_p = H \tan \Delta \theta \]
\[ \Delta y_R = H \tan \Delta \phi \]
\[ \Delta z_R, \Delta z_p = \frac{W}{2} \tan \Delta \phi = H \tan \beta \tan \Delta \phi \]

where:

- \( H \) = height
- \( W \) = swath width
- \( \beta \) = FOV/2
- \( \Delta \theta \) = pitch error
- \( \Delta \phi \) = roll error
Solving for Bore Sight Angles ($d\omega$, $d\phi$, $dk$)

Guidelines:

1. Flights must image a Control Point Range ~ (20 GCPs)
2. Fly two or more strips. Four Strips x five Images (20) is sufficient.

Items to be accomplished:

1. The POS (IMU) Program outputs the $\omega$, $\phi$, $\kappa$ for each image.

2. Run a Block Adjustment Program fitting to the 20 or more GCPs imaged in the flight. Output $\omega$, $\phi$, $\kappa$ for each image.

Apply the computed Bore Sight Angles (3) to correct each set of orientation angles in the project to make the X,Y,Z more accurate.
Bore Sight corrections (3 angles) change the computed POS X, Y, Z-coordinates to more closely agree with the surveyed GCP X, Y, Z. Therefore accuracy is increased.

**Airborne Position & Orientation System**

Apply the 3 correction angles to each image “O”

Computed Position - True ground is the POS error to be corrected to closer match the GCPs.
Solving for Alignment Angles \([dM] = (d\omega, d\phi, dK)\) when using a 
Position and Orientation System (POS)           By Don Light

1. An orientation matrix \([M]_{33}\) has the same 9 values (Direction Cosines) regardless 
of the definition of the angles, or the order of their rotation if 
it points to the same object.

2. Therefore; the 9 values from \([M]\) can be used to compute the Omega, Phi, Kappa 
(O,P,K) angles for that frame’s orientation using Wolf 2\textsuperscript{nd} Edition 1983, p613, Eq C-23.

3. The appropriate quadrant and each angles correct value can be determined using 
the table that follows Equation C-23. Proper choice of the quadrant is essential.

Solving for the Alignment angles, dOmega, dPhi, dKappa; d (O, P, K) to bring 
the 3 POS angles into agreement with the bundle block (AT) derived truth set.

1. Fly images and perform a block triangulation with three or four strips of 
approximately 5 to 10 frames each holding to ground control points (GCP). This 
provides from 15 to 40 frames where each frame will have an attitude derived matrix 
\([M]_{AT}\); therefore, an Omega, Phi, Kappa for each frame in the 4 strips becomes the 
truth set. Then derive one matrix from the average values.

2. Find the average for each angle set and now we have the average value for O, and 
average P, and the average for K based on the aerial triangulation (AT) fit to the GCPs. 
This average value matrix for the angles is the truth set to be used in the math below.

3. Find the Applanix IMU angles and make an O, P, heading (K) out of each frames 
Position and Orientation System (POS) output. Find the average, so we have average 
O, average P, and average K derived from the POS.

4. Construct the matrix \([M]_{AT}\) using the average values obtained for O, P, K which 
results in a 3x3 matrix \([M]_{AT}\) as given in (1) above.

5. Construct the matrix \([M]_{POS}\) from the O, P, K given by the Applanix POS same as in 
Paragraph 3 above. Now the \([M]_{POS}\) differs from the \([M]_{AT}\) by a delta amount which we 
will call \([dM]\). This \([dM]\) is the alignment Matrix that we seek via the following matrix 
mathematics. The \([dM]\) brings the POS angles into alignment with the \([M]_{AT}\)

Let \([M]_{POS} [dM] = [M]_{AT}\) \([dM]\) = is the unknown boresight matrix. \hspace{1cm} (1)

Multiply both sides of (1) on the left by \([M]^{-1}_{POS}\) which yields a unit matrix, then

\([dM] = [M]^{-1}_{POS} [M]_{AT}\). \hspace{1cm} [M]_{POS} \text{ is an orthogonal Matrix; therefore, } [M]^{-1}_{POS} = [M]^{T}_{POS}.

Conclusion: \([dM]\) is the unknown Alignment Matrix, also called the Boresight Matrix.
Northbound Flights for Calibration over the RIT Campus.  

<table>
<thead>
<tr>
<th>Frame ID</th>
<th>POS IMU Degrees omega</th>
<th>POS IMU Degrees phi</th>
<th>POS IMU Degrees kappa</th>
<th>Aero-Triangulation from LPS (AT) Degrees omega</th>
<th>Degrees phi</th>
<th>Degrees kappa</th>
</tr>
</thead>
<tbody>
<tr>
<td>MWIR_14.jpg</td>
<td>-0.38333</td>
<td>0.01551</td>
<td>-1.37091</td>
<td>-0.3378</td>
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<td>-1.403</td>
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<tr>
<td>MWIR_15.jpg</td>
<td>-0.05515</td>
<td>-0.52008</td>
<td>-0.86033</td>
<td>0.0291</td>
<td>-0.5344</td>
<td>-1.0487</td>
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<tr>
<td>MWIR_16.jpg</td>
<td>0.34907</td>
<td>-0.77569</td>
<td>0.73222</td>
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<td>-0.8041</td>
<td>0.5387</td>
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<td>MWIR_17.jpg</td>
<td>0.69616</td>
<td>0.87906</td>
<td>2.23158</td>
<td>0.7955</td>
<td>0.8462</td>
<td>1.9131</td>
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<tr>
<td>MWIR_18.jpg</td>
<td>0.2992</td>
<td>-2.73436</td>
<td>0.48231</td>
<td>0.3493</td>
<td>-2.781</td>
<td>0.2322</td>
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<tr>
<td>MWIR_19.jpg</td>
<td>0.14461</td>
<td>-0.81277</td>
<td>-1.387</td>
<td>0.1401</td>
<td>-0.8323</td>
<td>-1.698</td>
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<tr>
<td>MWIR_20.jpg</td>
<td>-0.21591</td>
<td>0.84816</td>
<td>-0.72019</td>
<td>-0.2139</td>
<td>0.8305</td>
<td>-0.9309</td>
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<tr>
<td>MWIR_21.jpg</td>
<td>-0.06208</td>
<td>0.30106</td>
<td>0.67217</td>
<td>-0.0841</td>
<td>0.2276</td>
<td>0.4963</td>
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<tr>
<td>MWIR_22.jpg</td>
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<td>-0.27846</td>
<td>0.79814</td>
<td>0.1543</td>
<td>-0.361</td>
<td>0.7013</td>
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<tr>
<td>MWIR_23.jpg</td>
<td>0.36682</td>
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<td>-0.85701</td>
<td>0.3129</td>
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<td>-0.892</td>
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<tr>
<td>MWIR_39.jpg</td>
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<td>2.07326</td>
<td>-1.46781</td>
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<td>-1.5789</td>
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<tr>
<td>MWIR_40.jpg</td>
<td>0.09952</td>
<td>1.16867</td>
<td>-1.04573</td>
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<tr>
<td>MWIR_41.jpg</td>
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<td>0.07921</td>
<td>-1.21865</td>
<td>0.2436</td>
<td>0.0439</td>
<td>-1.5715</td>
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<tr>
<td>MWIR_42.jpg</td>
<td>0.45211</td>
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<td>-2.33625</td>
<td>0.6062</td>
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<tr>
<td>MWIR_43.jpg</td>
<td>0.18717</td>
<td>0.8688</td>
<td>-4.46563</td>
<td>0.3356</td>
<td>0.8223</td>
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<td>MWIR_44.jpg</td>
<td>0.55993</td>
<td>2.53802</td>
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<td>0.6945</td>
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<td>MWIR_45.jpg</td>
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<td>0.88306</td>
<td>-4.00756</td>
<td>0.8088</td>
<td>0.8191</td>
<td>-4.1924</td>
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<td>MWIR_46.jpg</td>
<td>0.45114</td>
<td>0.05881</td>
<td>-4.07063</td>
<td>0.5578</td>
<td>-0.0591</td>
<td>-4.2358</td>
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<td>MWIR_47.jpg</td>
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<td>-1.09503</td>
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<td>0.70628</td>
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<td>-3.2339</td>
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<tr>
<td><strong>Averages</strong></td>
<td>0.200448</td>
<td>0.029382</td>
<td>-1.46418</td>
<td>0.2843571</td>
<td>-0.02741</td>
<td>-1.6572286</td>
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**RIT Campus Control Point Range**

<table>
<thead>
<tr>
<th>Sum of 21</th>
<th>Ave of 21</th>
<th><strong>Deg</strong></th>
<th><strong>Deg</strong></th>
<th><strong>Deg</strong></th>
<th><strong>Deg</strong></th>
<th><strong>Deg</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>4.20941</td>
<td>0.2004481</td>
<td>0.61702</td>
<td>-30.7477</td>
<td>5.9715</td>
<td>-0.5756</td>
<td>-34.8018</td>
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<tr>
<td>-30.7477</td>
<td>0.2004481</td>
<td>-0.5756</td>
<td>-34.8018</td>
<td>5.9715</td>
<td>-0.5756</td>
<td>-34.8018</td>
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<tr>
<td>-34.8018</td>
<td>-0.5756</td>
<td>-34.8018</td>
<td>5.9715</td>
<td>-0.5756</td>
<td>-34.8018</td>
<td>5.9715</td>
</tr>
</tbody>
</table>

Average of 21 angles from a North flight MWIR DL
Collinearity Equations

\[ x_p - x_o = -f \frac{m_{11}(X_p - X_o) + m_{12}(Y_p - Y_o) + m_{13}(Z_p - Z_o)}{m_{31}(X_p - X_o) + m_{32}(Y_p - Y_o) + m_{33}(Z_p - Z_o)} \]

\[ y_p - y_o = -f \frac{m_{21}(X_p - X_o) + m_{22}(Y_p - Y_o) + m_{23}(Z_p - Z_o)}{m_{31}(X_p - X_o) + m_{32}(Y_p - Y_o) + m_{33}(Z_p - Z_o)} \]

\[ M = M_\kappa M_\phi M_\omega \]

\[ M = \begin{bmatrix} \cos \phi \cos \kappa & \cos \omega \sin \kappa + \sin \omega \sin \phi \cos \kappa & \sin \omega \sin \kappa - \cos \omega \sin \phi \cos \kappa \\ -\cos \phi \sin \kappa & \cos \omega \cos \kappa - \sin \omega \sin \phi \sin \kappa & \sin \omega \cos \kappa + \cos \omega \sin \phi \sin \kappa \\ \sin \phi & -\sin \omega \cos \phi & \cos \omega \cos \phi \end{bmatrix} \]
### 21 frames S to N for Boresights.

<table>
<thead>
<tr>
<th>Degrees</th>
<th>Pos Data</th>
<th>AT Data</th>
<th>AT - POS</th>
<th>[M] is a 3,3 Orientation matrix for 3 angles.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average Deg</td>
<td>Average Deg</td>
<td>Averages</td>
<td>Let $[M]<em>{pos} [dM] = [M]</em>{AT}$, then $[dM] = [M]<em>{pos} [M]</em>{AT}$</td>
</tr>
<tr>
<td>Omega</td>
<td>0.2004481</td>
<td>0.2843571</td>
<td>0.083909</td>
<td></td>
</tr>
<tr>
<td>Phi</td>
<td>0.0293820</td>
<td>-0.0274095</td>
<td>-0.0567915</td>
<td></td>
</tr>
<tr>
<td>Kappa</td>
<td>-1.4641760</td>
<td>-1.6572286</td>
<td>-0.1930526</td>
<td></td>
</tr>
</tbody>
</table>

Gray values are the only inputs

| MPOS | 0.999673365  | -0.025549961 | -0.000602035 |
| O-Matrix | 0.025551907  | 0.999667425  | 0.003484227 |
|        | 0.000512813  | -0.003498472 | 0.999993749 |
| MPOS-1 | 0.9999734    | 0.0255519    | 0.00051281  |
| O-Matrix | -0.02555    | 0.9996674    | -0.0034985 |
|        | -0.000602    | 0.0034842    | 0.9999937  |

Same as MPOS-1 above

| MAT | 0.999581613  | -0.02892208  | 0.000334651 |
| O-Matrix | 0.028920059  | 0.999569349  | 0.004974706 |
|        | -0.00047839  | -0.004962947 | 0.99998757 |
| MPOST | 0.9996734    | 0.0255519    | 0.00051281  |
| O-Matrix | -0.02555    | 0.9996674    | -0.0034985 |
|        | -0.000602    | 0.0034842    | 0.9999937  |

$[M_{POS-1}][M_{POS}]$ is a unit Matrix

$[dM] = [M_{POS-1}][M_{MAT}]$

$[dM]$ (Delta Matrix) = Direction Cosines of Bore Sight Angle's O-matrix

### Results

<table>
<thead>
<tr>
<th>Degrees</th>
<th>Rigorous Solution Deg</th>
<th>Solution by Subtraction</th>
<th>Difference</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Omega</td>
<td>0.083811042</td>
<td>0.083909</td>
<td>9.7958E-06</td>
<td>0.001</td>
</tr>
<tr>
<td>Phi</td>
<td>-0.056115598</td>
<td>-0.0567915</td>
<td>-0.0006759</td>
<td>0.012</td>
</tr>
<tr>
<td>Kappa</td>
<td>-0.193250173</td>
<td>-0.1930526</td>
<td>0.00019757</td>
<td>-0.001</td>
</tr>
</tbody>
</table>

$M(3, 2) = -\sin(\text{omega}) \times \cos(\text{phi})$

$M(3, 1) = \sin(\text{phi})$

$M(2, 1) = -\cos(\text{phi}) \times \sin(\text{kappa})$

RMS error = 0.00047815

Subtraction is close.

Don Light 8-9-2010
N6371D: Ground Control Offsets X,Y meters
(cool colors => EW flight lines, hot colors => NS flight lines)
References listed by the year.
Calibrating Film Cameras Then and Digital Cameras Now (2009) at M7VI:


End of References