ELEVATION DETERMINATION BY SHADOW MEASUREMENT FROM VERTICAL MONOSCOPIC AERIAL IMAGERY

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ABSTRACT

A method is presented for determination of heights of vertical objects from non-orthorectified, monoscopic, vertical imagery using trigonometric calculations on cast shadows. If time of day of exposure is unknown, derivation of solar elevation from the Nautical Almanac or online equivalents (using celestial navigation sight reduction techniques) is explained.

INTRODUCTION

From time to time, it is necessary to use remote sensing techniques to determine the heights above the ground of objects such as smokestacks, radio towers and tree canopies . The classical method for accurately measuring the heights of vertical objects has been stereo photogrammetry. Under emergency conditions, however, it may not be possible to plan and execute a photo mission and circumstances may require the use of recent non-stereoscopic imagery, or even archival, hard copy paper prints. Photointerpretive techniques can be applied instead to identify and measure vertical obstructions by the measurement of their cast shadows; it is not always necessary for the imagery to be fully orthorectified, particularly in flat terrain. This monograph will briefly review these techniques and then consider the case when the time of photographic exposure is unknown and it is necessary to determine it from the imagery itself.

SHADOW MEASUREMENT

The determination of the height of a vertical object is accomplished by measuring the length of its shadow and multiplying that value by a constant derived from the angular height of the sun above the horizon (Hc). The analyst must ensure that certain conditions are met; the photograph must be properly scaled and oriented, that is, rectified, and those parameters must be known. The shadow must also fall on flat ground, and (for non-orthorectified images) the relief of the terrain should be such that the elevation differences across the image are small compared to the height of the imaging platform above the ground. This is a situation frequently encountered in my home state of Florida, where the lack of relief often makes orthorectification unnecessary.

The height of any vertical object is given by the following equation:

<u>Object Height = Shadow Length x tan (Hc)</u> **(Correcting for Sun Elevation)**

The tangent function is easily executed from many pocket calculators, even by the most trigonometrically challenged among us. Consequently, the key value required for this calculation is the determination of the solar altitude Hc at the time of the exposure by using the Nautical Almanac.

CALCULATIONS BY HAND

At all times, the sun's altitude will be constantly changing throughout the day and must be determined using the geographic position (Latitude and Longitude), date, and time of the exposure. The quantities required from the Almanac are the *Greenwich Hour Angle* (GHA) and *Declination* (Dec) for the sun at the date and *Universal Time* (UT) the image was acquired. This information is also available online at the US Naval Observatory website:

http://aa.usno.navy.mil/data/docs/celnavtable.html

This web page will calculate solar altitudes and azimuths but the manual procedures for calculating these values is also presented below; it is recommended that the user be familiar with them in the event the online Almanac/calculator is not available. They are explained in detail, with examples, in the US Nautical Almanac, *Sight Reduction Procedures*. Throughout this paper, the same nomenclature is adopted as in the Almanac.

a) Determine UT of the exposure by adding 1 hour to the Standard Time for every time zone west of the Prime Meridian. Subtract an hour if Daylight Savings Time was in effect at the time of the photography. Go to the appropriate Almanac page and and look up the solar GHA and Dec for that UT, using the gray pages in the back of the Almanac to interpolate for times between even hours.

b) For that UT and date, determine the sun's *Local Hour Angle* (LHA). Recall that W Longitudes and S Latitudes are negative.

LHA = GHA + Lon

c) Calculate the intermediate values S and C

S = sin (Dec)

C = cos (Dec) x cos (LHA)

d) The solar altitude in degrees is calculated as follows, all student and engineering/scientific calculators are equipped to calculate INVERSE trigonometric functions, that is, given the number, what is the angle corresponding to it?

Hc = INVsin (S x sin (Lat) + C x cos (Lat)) **(Determining Sun Elevation)**

CALCULATING THE UNKNOWN EXPOSURE TIME

If the time of the exposure is not known, it can still be determined from the observed azimuth of the shadow. In other words, you are using the object to be measured as a sundial! The Almanac *Sight Reduction Procedures* has instructions for calculating the computed azimuth, Zn, so continue iterating with different times until you have bracketed the suspect time by generating Zn values slightly larger and smaller than your measured azimuth. Interpolate between the two corresponding times to get the UT for the azimuth value you measured.

The observed azimuth of a shadow is its angle, measured clockwise from true north. The shadow of a light pole points to the sun, so the azimuth is measured from the shadow of the top of the pole to the point where it touches the bottom of the pole. Make sure you measure the angle carefully in an area where both ends of the shadow are visible and where it falls on flat ground. If you do not have an angle measuring tool in your GIS software, determine the deltas in northings and eastings and take the inverse tangent of the ratio of the two. For example, suppose the coordinates of the two end points of a shadow pointing to a southeasterly sun are measured:

- Tail of shadow (top of light pole): N = 10000, E = 5000.
- Head of shadow (base of pole): N = 9960, E = 5030.

The shadow appears to be approximately 50 units in length. Consider these the endpoints of the hypotenuse of a right triangle with its right angle vertex due south of the tail and due west of the head, located at N = 9960, E = 5000. The tangent of the angle at the tail is defined as the ratio of opposite side to adjacent side, or 30/40 = 0.75. Using a pocket calculator or tables, take the inverse tangent of 0.75 and the result is approximately 37 degrees. This means that the angle of the shadow is roughly 37degrees E of S, corresponding to an observed azimuth of 143 degrees. If the map projection the photography is rectified to does not have a grid north equal to true north, the azimuth must be corrected to reflect this difference.

The general procedure to calculate the exact solar azimuth Zn for any moment of time at a particular location is as follows, first determining the intermediate values X and A :

X = (S x cos (Lat) - C x sin (Lat)) / cos (Hc)
If X > +1 then X = +1
If X < -1 then X = -1
A = INVcos (X)

If LHA > 180, then <u>Zn = A</u> **(Determining solar azimuth)**
Otherwise, <u>Zn = 360 - A</u>

In this example, common sense tells you that the time of the exposure was some time in mid-morning, so pick a reasonable estimate, say 10:00 AM local time, convert to UT and calculate Hc and Zn using the Almanac. If your calculated azimuth is W of your measured one, your estimate was too late; if your calculated azimuth is E of your observed, your estimate is too early. Continue these iterations until your calculated and measured azimuths are close enough for your purposes, and use that corresponding solar altitude as Hc in your height calculations.

SOURCES OF ERROR

If the date of the photography is not known with precision, it still may be possible to achieve acceptable results if it can be narrowed to within several days. An inspection of the Almanac reveals how quickly solar GHA and Dec values change during the estimated time period for corresponding times on different days. Dec values vary over a range of 47 degrees, most rapidly during late March and September, where they are altering by about a third of a degree per day. During late June and December, the Declination is essentially static. Greenwich Hour Angle varies at an almost constant 15 degrees per hour year round, but it's value at any particular hour never strays by more than 4 degrees from the average over the years; GHA and Dec for any hour on a given date are not the same from year to year, but they are usually close. Consequently, Zn and Hc tend to repeat for the same hour and date from year to year. An error in Dec will result in the largest altitude errors around the noon hour, while an error in GHA will have the most effect on shadows in early morning and late afternoon photography.

As a result, if you have imagery available and can establish the date it was acquired to within several days, you can always determine the time, and hence the solar altitude, with the procedure described above. Using the USNO website calculator, or the expressions above, it is wise to experiment with different dates to determine the possible error which may be introduced by estimating an incorrect date for image acquisition. This error will vary widely depending on the date, time and latitude of the original imagery. It is also important to keep in mind that it is not possible to measure shadow lengths on an image with high precision, so it may not be necessary to determine the sun elevation to a very high level of accuracy.

GLOSSARY

Hc - Height computed - Degrees above the horizon of a celestial body at a specific place and time, as determined from the Nautical Almanac.
Azimuth - Compass direction, measured clockwise from N, 0-360 degrees.
Zn - Computed Azimuth - Azimuth as computed from the Nautical Almanac for a specific place and time.
Dec - Declination - Degrees N or S of the Equator of a celestial body**.**
GHA - Greenwich Hour Angle - Angle measured W from the Prime Meridian to a celestial body's meridian.
LHA - Local Hour Angle - Angle measured W from the observer's meridian to a celestial body's meridian.
Meridian - An imaginary line running from pole to pole through the observer and his zenith.
Prime Meridian - The Meridian passing through Greenwich, UK.
UT- Universal Time, also known as Greenwich Mean Time (GMT), the Standard Time in Greenwich, UK.