PRECISE CALIBRATION OF FISHEYE LENS CAMERA SYSTEM AND PROJECTION MODEL

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ABSTRACT

This paper depicts a projection and camera model for fisheye lens. It also presents results of the calibration of fisheye lens made on two test fields. It was shown in this experiment that place of calibration performance (laboratory or terrain) is not important when modern calibration methods are used. It makes full using of these lens in close range photogrammetry possible. Calibration of fisheye lens camera system was made with application an software created by our team in matlab on one image. There was used camera with 14 mega pixels in this experiment.

INTRODUCTION

Fisheye lenses provide imaging a large area of the surrounding space by a single photo, sometimes more than 180 deg. They make possible to realize photo on very small distance, what in some engineering elaboration aspects may be particularly useful. Close range photogrammetry (central perspective) does not comply with fisheye image processing. The fundamental difference between a fisheye lens and classical lens is that the projection from 3D ray to a 2D image position in the fisheye lens is intrinsically non perspective. One has to take into consideration fact that not all fisheye lenses give hemispherical image. In our experiment there were used fisheye lens with focal lens 10.5 mm, which image is not hemispherical. Application of such type of fisheye lens give more possibilities of usage of images in close range photogrammetry eliminating from the image everything above FOV of 170°, and preventing simultaneously retrieval of image radius. Images were made with digital camera Kodak DCS 14n Pro f=10.5mm, with matrix 4500x3000 pixels.

Figure 1. Image realized with help of above mentioned camera base system.
Such images prevent Schwalbe’s [Schwalbe, 2005] approach to the matter, with assumption that:

\[
\frac{\alpha}{r} = \frac{90^\circ}{R} \quad \text{where} \quad r = \sqrt{x'^2 + y'^2}
\]

\(\alpha\) – angle of incidence  
\(r\) – distance between image point and optical axis  
\(R\) – image radius  
\(x', y'\) – image coordinate

**CAMERA AND PROJECTION MODEL**

Fisheye lens has a very large distortion for which the distortion polynomial used here would not converge. For such a lens the image coordinate should be represented as being ideally proportional to the off-axis angle, instead of the tangent of this angle as in the perspective projection. Then, a small distortion could be added on top of this. Furthermore, the position of the entrance pupil of a fisheye lens varies greatly with the off-axis angle to the object, therefore this variation would have to be modeled unless all viewed objects are very far away [Gendery, 2001].

The perspective projection of a pinhole camera can be depicted by the following formula:

\[
r = f \cdot \tan \theta
\]

where

\(\theta\) - angle between the optical axis and the incoming ray  
\(r\) - distance between the image point and the principal point  
\(f\) - focal length

The calibration of dioptric camera involves the estimation of an intrinsic matrix [Hartley, 2003] along with a projection model. The intrinsic matrix, which maps the camera coordinates to the image coordinates, is parameterized by principal points, focal length, aspect ratio and skewness.

A circular fisheye camera results from the size of the image plane charged coupled device (CCD) being larger than the image produced by the fisheye lens [Ho, 2005].
The existing projection models can be divided into two aspects.

fisheye image radius (r) vs. its corresponding perspective image radius (r’)
fisheye image radius (r) vs. incident angle (θ)

First aspect; r and r’ are the distances from the distortion center to the distorted image point and the corresponding perspective image point respectively. In both images, the center is the same. The adequate distances can be transformed as:

\[ r' = r + K_1 \cdot r^3 + K_2 \cdot r^5 + K_3 \cdot r^7 + ... \]  

Second aspect; fisheye lenses are habitually designed to obey one of the succeeding projections:

**equidistance projection**
\[ r = f \cdot \theta \]  

**orthogonal projection**
\[ r = f \cdot \sin \theta \]  

**equisolid angle projection**
\[ r = 2f \cdot \sin \left(\frac{\theta}{2}\right) \]  

**stereographic projection**
\[ r = 2f \cdot \tan \left(\frac{\theta}{2}\right) \]

Kannala suggests the projection model in a general polynomial form:

\[ r = k_1 \cdot \theta + k_2 \cdot \theta^3 + k_3 \cdot \theta^5 + ... \]  

Because the lens elements of real fisheye lens may deviate from precise radial symmetry and they may be inaccurately positioned causing that the projection is not exactly radially symmetric, Kannala and Brandt propose adding two distortion terms [Kannala, 2004]:

One acting in the radial direction
\[ \Delta_r(\theta, \varphi) = (k_1 \theta + k_2 \theta^3 + k_3 \theta^5) \cdot (i_1 \cos \varphi + i_2 \sin \varphi + i_3 \cos 2\varphi + i_4 \sin 2\varphi) \]  

and the other in tangential direction
\[ \Delta_r(\theta, \varphi) = (m_1 \theta + m_2 \theta^3 + m_3 \theta^5) \cdot (j_1 \cos \varphi + j_2 \sin \varphi + j_3 \cos 2\varphi + j_4 \sin 2\varphi) \]  

The distortion functions are thus separable in the variables θ, ϕ.
The projection from 3D rays to 2D image positions in a fisheye lens can be approximated by the imaginary equidistance model. Let a 3D ray from pp of the lens is specified by two angles $\theta$ and $\phi$ (figure 3).

Figure 3. (a) Camera coordination system and its relationship to the angles $\theta, \phi$. (b) From polar coordinates ($r, \phi$) to orthogonal coordinates ($u', v'$).

Together with the angle $\phi$ between the light ray reprojected to $xy$ plane and the $x$ axis of the camera centered coordinate system, the distance $r$ is sufficient to calculate the pixel coordinates:

$$u' = (u', v', I)$$

in some orthogonal image coordinate system, as

$$u' = r \cdot \cos \phi$$
$$v' = r \cdot \sin \phi$$

The complete camera model parameters including extrinsic and intrinsic parameters can be recovered from measured coordinates of calibration points by minimizing an objective function with denotes the Euclidean norm.[Bakstein, 2002].

$$J = \sum_{i=1}^{N} \left\| \hat{u} - u \right\|$$

where:
N - number of points
$\hat{u}$ - coordinates of points measured in the image
$u$ - coordinates reprojected by the camera model

**EXPERIMENTAL RESULTS**

We performed two calibration experiments. In the first experiment, the calibration points were located on the wall (figure 2). 40 points were evenly placed on building wall, with an error $m_{xy} = \pm 0.005m$ and $m_z = \pm 0.007m$. The picture to this experiment was made from the distance of 4m.

The second experiment employed a 3D calibration room with 230 GCP’s, placed in the corner room. An error of determination of coordinates amounts $m_{xy} = \pm 0.001m$ and $m_z = \pm 0.001m$ and the picture to the experiment was made from the distance of 0.35m.
We then used the Levenberg-Marquardt algorithm to minimize error with respect internal parameters.

**Table 1. Sigma naught of the adjustment, m1- model without distortion, m2- model with distortion, m3 – extended model; \( \sigma \) [pixel].**

<table>
<thead>
<tr>
<th></th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>m1</td>
<td>8.55</td>
<td>7.90</td>
</tr>
<tr>
<td>m2</td>
<td>0.32</td>
<td>0.27</td>
</tr>
<tr>
<td>m3</td>
<td>0.10</td>
<td>0.06</td>
</tr>
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**Table 2. Error of calibration fisheye lens Nikkor 10.5 mm.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_0 ) ( \mu ) m</td>
<td>( \pm 0.8 )</td>
<td>( \pm 0.7 )</td>
</tr>
<tr>
<td>( y_0 ) ( \mu ) m</td>
<td>( \pm 0.9 )</td>
<td>( \pm 0.7 )</td>
</tr>
<tr>
<td>( k_1 ) ( \mu ) m</td>
<td>( \pm 6.3 \cdot 10^{-6} )</td>
<td>( \pm 6.0 \cdot 10^{-6} )</td>
</tr>
<tr>
<td>( k_2 ) ( \mu ) m</td>
<td>( \pm 9.5 \cdot 10^{-8} )</td>
<td>( \pm 9.1 \cdot 10^{-8} )</td>
</tr>
<tr>
<td>( k_3 ) ( \mu ) m</td>
<td>( \pm 4.3 \cdot 10^{-10} )</td>
<td>( \pm 3.8 \cdot 10^{-10} )</td>
</tr>
<tr>
<td>( m_i ) ( \mu ) m</td>
<td>( \pm 1.5 \cdot 10^{-6} )</td>
<td>( \pm 1.5 \cdot 10^{-6} )</td>
</tr>
<tr>
<td>( C_i ) ( \mu ) m</td>
<td>( \pm 2.0 \cdot 10^{-3} )</td>
<td>( \pm 1.9 \cdot 10^{-3} )</td>
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</table>

**CONCLUSIONS**

In this paper, there were presented possibilities of fisheye calibration. The research were conducted on internal test (more precise) and external. The aim of such approach is demonstration of necessity of fisheye lens application in close range photogrammetry in the future. Accuracy of calibration from terrain and laboratory test, reached in the experiment certify such approach. Authors did not create a new theory in this experiment, but rested on already elaborated calibration procedures of famous authors (references). Novelty of this experiment is ascertainment that reduction of GCP’s (from 230 to 40) and test transfer (from laboratory to terrain) does not make meaningly worse calibration accuracy, and we still aim to use above mentioned images in practice.

**REFERENCES**

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