PERFORMANCE EVALUATION FOR AERIAL IMAGES AND AIRBORNE LASER ALTIMETRY DATA REGISTRATION PROCEDURES

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ABSTRACT

Photogrammetry is one of the traditional methods and sources of obtaining digital surface models (DSMs). Airborne Laser Altimetry (also widely known as LIDAR — Light Detection and Ranging), on the other hand, is a newer, highly automated, still improving and an accurate method providing coordinate measurements. The two methods deliver complementary surface information where the disadvantages of one method can be compensated by the advantages of the other method, if they are used together, in order to extend the range and the utilization of the information gathered due to the data fusion that can occur. This paper describes the accurate co-registration between the two data sets which takes place through a 3D transformation. This co-registration is the prerequisite step for the fusion of the two data sets. The theoretical framework of the algorithm which is presented is based on the minimization of the distances between points of one surface to surface patches of the other surface, parallel to the corresponding surface normals as per the research of Schenk et al. (2000). In this research, the performance of this algorithm is evaluated on the sets which would be used to create the surface patches and on the processing levels of data sets. Moreover, the paper includes the description of the available data of the Espoonlahti area. The entire area is covered by a block of aerial images and a 3D laser point cloud taken in the same time period. Also, the results and the effects on registration using images and laser point cloud are described. The aims of the research were to investigate the geometric stability of the transformation, the analysis of the results, the effects on the registration and the accuracy of the derived parameters through an own developed and self-executable software script using real data and under real conditions.

INTRODUCTION

For many years, Photogrammetry, has been providing accurate coordinate measurements through the stereoscopic method based on its well known principles. Laser Altimetry, on the other hand, is becoming the prime method for large scale acquisition of elevation data due to its capability to directly measure 3D coordinates of a huge number of points. Several countries are currently using Laser Altimetry for creating or updating dense digital surface models (DSMs). There are many advantages of Laser Altimetry and this research exploits one of the most promising ones, which is its ability to enhance the quality of information obtained by photogrammetric means. Laser Altimetry can provide measurements in areas where traditional photogrammetric techniques encounter problems due to occlusions or shadows. Although Laser Altimetry has many benefits, it also has limitations due to its lack of thematic information recording and due to calibration errors that may occur during data acquisition. Therefore some
questions can arise: How can the advantages of both these different elevation data sources be exploited? How can two point sets which are irregularly distributed thereby not having pairs of conjugate points between them be co-registered? This procedure is usually referred to as fusion of the two data sets and exploitation of their advantages. This approach has been suggested recently by many researchers (Ackermann, 1999, Baltsavias, 1999, Brenner, 1999, Csatho et al., 1999, Toth and Grejner-Brzezinska, 1999, Vosselman 1999, Postolov et al., 1999, Habib and Schenk, 1999). Therefore, the first step for it is the registration which is the main subject of the present research. It is known that registration is not a specific problem to the laser scanner domain. The problem is more general and many researchers have been involved with this subject.

Registration is an increasingly interesting research topic. Most experiments have been carried out with a view to testing accuracy and reliability of DEM matching methods and it is thought that efficiency is directly related to successful automation and increased accuracy. The most common methods for determining the orientation parameters between two data sets are based on conjugate points. The Iterative Closest Point (ICP) algorithm (Besl and McKay, 1992, Chen and Medioni, 1992) was considered one of the most popular methods for many years. However, the ICP algorithm assumes that one point set is a subset of the other. Thus, this method is not applicable when using airborne laser data as the laser measurements refer to a footprint, and not to a specific point identifiable on ground (Baltsavias, 1999). Neither conjugate points between laser and photo sets can be obtained.

Registration of point clouds derived from different sources and by different methods which represent the same object surface, should be defined as a surface matching problem. Gruen (1985) first regarded the issue of the surface patch matching as a straight extension of Least Squares Matching (LSM). There have been some studies on the absolute orientation of stereo models using DEMs as control information. This work is known as DEM matching. It was first proposed by Ebner and Mueller (1986), and Ebner and Strunz (1988) describing a solution that is based on interpolating the data to a grid. Rosenholm and Torlegard (1988) have used DEMs as unique information for stereomodel orientation in a modified similarity transformation relying on DEM slopes. This method estimates the 3D similarity transformation parameters between two DEM patches, minimizing the least square differences along the z-axis. A few applications of DEM matching have been reported (Karras and Petsa, 1993, Xu and Li, 2000).

In order to provide a high density laser point data set over an extended surface area, many parallel overlapping laser strips are needed. Obviously, this is another area where a registration task has to be used. This registration of sequential laser strips has been a major area of research (Burman, 2000, Maas 2000). Gruen and Akca (2004), and Akca (2004) extended the Least Square Matching theory (Gruen, 1985) to Least Squares 3D Surface Matching (LS3D), estimating the 3D transformation parameters between two or more arbitrarily oriented 3D surface patches. It is known that most of the techniques for 2.5DEM surface matching have a limitation value especially in close range applications but also in urban areas where the slopes are extremely steep. This new matching technique LS3D is considered to have overcome this problem deriving its strength from the LSM concept and offers high level of flexibility for any kind of 3D surface matching.

Another similar method for registering surfaces acquired using different methods, in particular Laser Altimetry and Photogrammetry, has been presented. In a research by Schenk et al. (2000), two different problems involved in the general task of surface matching are clearly mentioned. Namely, the "correspondence" problem which is the establishment of a relationship of surface features (e.g. points, lines) between the two surfaces and the "transformation" problem which is the determination of a set of transformation parameters. In this comparative study of surface matching algorithms of Schenk et al. (2000) two main mathematical methods, assumed suitable to determine optimal transformation parameters between two sets in different reference systems and without identical points, were presented. Postolov et al. (1999) implemented the first mathematical method minimizing the remaining differences along the z-axis (min z) of one of the reference systems and dividing the 3D transformation into a sequence of planimetry and elevation transformations. Habib and Schenk (1999) have implemented the second mathematical method minimizing the distance (min D) between a point of one surface parallel to the normal of a surface patch of the other surface concentrating on the solution of the "correspondence - matching" problem. A voting scheme, based on the Modified Hough transformation, to analyze the parameter space was proposed.

In the present research, a registration algorithm between aerial images and laser point clouds is evaluated. This research after implementing an algorithm for the co-registration between the two data sets, which takes place through a 3D transformation, assesses the results under real conditions and with real data. The algorithm is based on the minimization of the distances between points of one surface and surface patches of the other surface, parallel to the corresponding surface normals (Schenk et al., 2000). The 3D transformation estimates the parameters through a Gauss Markov model. This was carried out by developing a suitable software script within Matlab programming environment. The data, which will be described below in detail, includes a laser 3D point cloud and a 3D point set which has been derived by photogrammetric means. The problem should be defined as a surface matching problem due to the use of different methods and devices from which the 3D point sets were derived. The transformation...
parameters between the surfaces, both of which contain irregularly distributed points, were determined without being interpolated to a regular grid. Instead a TIN model was produced. These parameters represent a 3D transformation, and include scale, translations and rotations. The mathematical method, which is based on the minimization of the distances, presents better adjustment when the surface is derived from a man-made area (urban) where the slopes are steeper. Also, the results, the effects on registration using images and laser point cloud are described including proposals for future research. An analysis of the results was investigated in order to present the correlation between the number of TINs, which represent the control surface, and the accuracy of the derived parameters.

In section 2, the data which is used in this research is described. Section 3 outlines the proposed implementation of the algorithm for the DTM matching, the mathematical model and remarks about TINs. In section 4, the experimental results using Laser and Model points as well as the analysis of the results are presented. Section 5 includes the conclusions and recommendations for future work.

DATA DESCRIPTION

The data, owned by the Finnish Geodetic Institute, is a part of a EuroSDR (European Spatial Data Research) test which took place in 2004 having the objective of Building Extraction (Kaarinen et al., 2005). The test area is called Espoonlahti and is located in Espoo, a city about 15km west of Helsinki with high-rise buildings and terraced houses.

Acquisition time. The stereo model which has been used belongs to a block of 18 aerial images in two strips. The part in which this research is focused (Figure 5) includes 27871 laser points from an approximate total number of 1.6 million laser points (Figure 2). Images and Laser scanning have been produced in the same time period in order to avoid capturing changes that may have happened in the area (e.g. buildings, environmental conditions). The description of Aerial Images and Laser scanner data are shown in Table 1 and Table 2.

| Photos | 3058-8945 and 3058-8946 |
| Date | 26th of June 2003 at 7:40 UTC |
| Camera | RC-30 |
| Lens | 153.59 mm |
| Calibration date | 22nd of November 2002 |
| Flying height, scale | 860 m, 1:5300 |
| Pixel size | 14 microns |

| Acquisition | 14th of May 2003 |
| Instrument | Toposys Falcon |
| Flight altitude | 400 m |
| Pulse frequency | 83000 Hz |
| Field of View | 7.15 degrees |
| Measurement density | 10-20 per m² |
| Swath width | 100 m |
| Mode | First pulse |

Data formats. Aerial images, with orientation parameters and ground point coordinates are in TIFF-format. Laser scanner data is in ASCII format (x,y,z point data).


Figure 1 shows a part of an Aerial Image on which the corresponding Laser data area is highlighted. In Figure 2 the laser data is depicted.
For this test, a Z/I ImageStation digital photogrammetric workstation was employed in order to produce a full restitution, which includes segments of the buildings, points on the roofs and points on the ground, through a photogrammetric project whereby manual measurements were taken. The exterior orientation of the aerial images is a result of a photogrammetric triangulation with GPS/INS support. As far as the mean standard deviation of exterior orientation parameters is concerned it was better than 5cm and 30° for the translations and rotations, respectively. The accuracy of the measurements should also be in the same rate, therefore, in the projects 5cm standard deviation was used taking into account the specific conditions during the restitution (visual quality of the target). All experiments were carried out using an own self-developed Matlab code that runs under Microsoft Windows XP.

THE IMPLEMENTATION OF THE REGISTRATION ALGORITHM

As already mentioned, the first data set P={p1, p2, …, pn} was derived from a photogrammetric restitution and includes elevation points and segments – breaklines. The second data set Q={q1, q2, …, qm} includes the laser point set. The points in the two sets were irregularly distributed (n≠m) thereby there are no pairs of conjugate points between the two sets. Avoiding interpolation of the data sets to a regular raster grid, in order not to introduce errors due to the interpolation, both data sets were treated as point data (x,y,z) (Morgan and Habib, 2002, Baltsavias, 1999).

Having set as ultimate goal to find optimal transformation parameters between the surfaces P and Q, observation equations based on the difference in the distances between points of one surface and surface patches of the other surface, parallel to the corresponding surface normals, were used (Schenk et al., 2000). This mathematical method is assumed suitable in steep surfaces with high gradients as is the present data. The transformation parameters between the surfaces, both of which contain irregularly distributed points, were determined without being interpolated to a regular raster grid. Instead a TIN model was produced (Habib and Schenk, 1999), on which more details are given in the next subsection.

Initially one of the two data sets should be considered as reference. In order to make the proper choice the following considerations were taken into account. Laser Altimetry is a technology in which several sensors are integrated to obtain 3D coordinates of points. It makes use of precise GPS instruments for the position of the sensor, inertial navigation system (INS) for determining the attitude of the sensor and narrow laser beams for determining the range between the sensor and the target points (Morgan and Habib, 2002). On the other hand, as it is known, traditional Photogrammetry has given invaluable results for many decades. Nowadays the reliability of both techniques is similar. However, for the present study, points chosen and measured by a skilled operator in a digital photogrammetric workstation were assumed to have higher reliability. Considering this issue, the surface generated by photogrammetric means P was chosen as the control reference system while the surface generated by the laser
scanning (experimental surface) Q was registered allowing the surfaces to be transformed to a common coordinate system (Figure 4).

Moreover, to check the algorithm, a few tests inverting the task of the surfaces were evaluated. In those tests the laser points surface Q obtained the role of the control reference system and the surface generated by photogrammetric means P was registered.

**Triangulated Irregular Networks (TINs)**

2D Delaunay triangulation has been mostly used for 2.5D surfaces, whose analytic function is described in the explicit form $z = f(x,y)$. This formulation has several problems in the matching of solid 3D surfaces due to the fact that these points are structured to shape triangles within the 2D convex hull of the data. Only two sets of coordinates, namely, x and y, are used for this purpose and the spatial relationships among points are limited to their projected distances on the x-y plane (Morgan and Habib, 2001, Balis et al., 2003). As a result, two points with the same x and y coordinates but with different z coordinate are not able to take part in a 2D Delaunay triangulation successfully. Thus, the triangles which are required to represent the 3D digital surface model fail to be constructed. Therefore, the above is a considerable limitation of using 2D Delaunay for city models generation (man-made constructions, building walls). Morgan and Habib (2001) have proposed a region-growing algorithm to overcome this limitation.

In order to check the results, firstly an accurate model of the area was created by the available software (digital photogrammetric workstation) through points and breaklines. This is depicted in Figure 3. In addition, having as a goal the development of a self executable script by Matlab software, more processing in the P data set that is produced by photogrammetric means was considered necessary in order to produce a denser triangulation and to overcome the 2D Delaunay’s algorithm limitation. As far as the evaluation procedure is concerned significant specifications have been assumed:

- Linear interpolation of 20cm, 30cm, 40cm, 50cm along breaklines to produce more points and a denser triangulation, correspondingly.
- Movement of 1mm of the points presenting the lower z value (e.g. the lower edge between the two edges of a wall on a building) in order to overcome the limitation of the 2D Delaunay algorithm.

In the next section all the tests, the steps of the processing, the combinations and the results are presented in detail.

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**Figure 3.** TINs generated by Autocad Land software.
Mathematical Model

Assuming both the data sets as point clouds \( P (x_{pi}, y_{pi}, z_{pi}) \) (\( p_i=1,..., n \)) and \( Q (x_{qi}, y_{qi}, z_{qi}) \) (\( q_i=1,..., m \)) produced by different methods, they must be transformed into a common system. To express the geometric relationship between them, a 7-parameter transformation is used minimizing the distance between a point of \( Q \) surface and a TIN surface patch of \( P \) surface. In equation 1, points of surface \( Q \) are transformed into the system \( P \) of the control surface.

\[
\begin{bmatrix}
x_p \\
y_p \\
z_p
\end{bmatrix} = c \cdot R \cdot 
\begin{bmatrix}
x_q \\
y_q \\
z_q
\end{bmatrix} + 
\begin{bmatrix}
t_x \\
t_y \\
t_z
\end{bmatrix}
\]

(1)

Where \( R (\omega, \phi, \kappa) \) is the orthogonal rotation matrix (equation 2), \( t_x \), \( t_y \), \( t_z \) are the elements of the translation vector and \( c \) is the scale factor.

Since the functional model is non-linear, it is solved using an iterative least-squares adjustment.

\[
R = 
\begin{bmatrix}
\cos \phi \cos \kappa & \cos \phi \sin \kappa + \sin \phi \cos \kappa & \sin \phi \sin \kappa - \cos \phi \sin \kappa \cos \kappa \\
-\cos \phi \sin \kappa & \cos \phi \cos \kappa - \sin \phi \sin \kappa \sin \kappa & \sin \phi \cos \kappa + \cos \phi \sin \phi \sin \kappa \\
\sin \phi & -\sin \phi \cos \phi & \cos \phi \cos \phi
\end{bmatrix}
\]

(2)

To perform least squares estimation, equation 1 must be linearized by Taylor expansion (regarding the 7 parameters: \( t_x \), \( t_y \), \( t_z \), \( c \), \( \omega \), \( \phi \), \( \kappa \)), creating the equation 3, in matrix notation. Using the stochastic Gauss-Markov model, related to a linear combination of the parameters, the observations are assumed as non-correlated. Having a standard deviation of \( \sigma = 0.05 \) m, the solution of equation 3 is produced by the equation 4. In equation 4, \( W \) is the diagonal weight matrix of the observations, while the best estimation of the vector \( \hat{x} \) of the parameters is given by equation 5. To make it clear, in Figure 4 the surface patch of the control surface \( P \) can be defined by 3 points \( (p_m, p_k, p_i) \) and one point of \( Q \) point cloud has to be transformed to the closer surface patch. Let the projection of \( q_i \) (\( x_{qi}, y_{qi}, z_{qi} \)) point to the surface patch be the \( q_i' \) (\( x_{qi}', y_{qi}', z_{qi}' \)).

\[
A \delta x = \delta \ell + v
\]

(3)

\[
\delta x = (A^T WA)^{-1} A^T W \delta \ell
\]

(4)

\[
\hat{x} = x^o - \delta x
\]

(5)

Where, \( A \) is the design matrix, which includes as many rows as the number of observation equations that are created corresponding to the number of points in \( Q \) point cloud and as many columns as the number of parameters, namely equal to 7. Moreover, \( \delta x \) is the vector of the corrections of the approximation values \( x^o \) of the unknown parameter vector \( x \), \( \delta \ell = \ell - \ell^o \) is the second part of the observation equation and \( v \) the residual vector. The vector \( \delta \ell \) is calculated by the subtraction of the right part from the left part of equation 1 using the approximation values \( x^o \).

\[
A = \begin{bmatrix}
y_{pm} & Z_{pm} & 1 \\
y_{pk} & Z_{pk} & 1 \\
y_{p'} & Z_{p'} & 1
\end{bmatrix}
\]

(6)

\[
B = \begin{bmatrix}
x_{pm} & Z_{pm} & 1 \\
x_{pk} & Z_{pk} & 1 \\
x_{p'} & Z_{p'} & 1
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
x_{pm} & y_{pm} & 1 \\
x_{pk} & y_{pk} & 1 \\
x_{p'} & y_{p'} & 1
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
x_{pm} & y_{pm} & Z_{pm} \\
x_{pk} & y_{pk} & Z_{pk} \\
x_{p'} & y_{p'} & Z_{p'}
\end{bmatrix}
\]
\[ x_{q'} = x_q - \frac{Ax_q - By_q + Cz_q - D}{A^2 + B^2 + C^2} A \]
\[ y_{q'} = y_q + \frac{Ax_q - By_q + Cz_q - D}{A^2 + B^2 + C^2} B \]
\[ z_{q'} = z_q - \frac{Ax_q - By_q + Cz_q - D}{A^2 + B^2 + C^2} C \] (7)

**Figure 4.** Point qi is a point of surface Q that is transformed to surface patch of P surface as qi'. The shortest distance from qi to the surface is used for determining the 3D transformation.

As this is a non-linear problem it is clear that for the first iteration initial approximation values for the unknown parameters \( x' \) are needed. For this, a known matching method from the literature, (e.g. Habib and Schenk, 1999) is suggested.

**EXPERIMENTS AND RESULTS**

As stated in section 3, at first, the surface P generated by photogrammetric means was chosen as the control reference system while the surface Q generated by the laser scanning (experimental surface) was registered allowing the surfaces to be transformed to a common reference system. The results of the tests performed creating TINs by photogrammetric points are presented in Table 3 (Projects 1-5).

The test area is a part of a city which consists of buildings, trees, bare ground, and paved roads. The evaluation has the intention to test the algorithm in these real conditions. The test area which is depicted in Figure 5, covers 2000m² approximately.
While the number of the laser points in this area is 27871 and it remains constant, the number of photogrammetric points varies due to the dependence on the linear interpolation level. Results are shown in Table 3.

All projects had been performed through iterative least square procedure. The initial approximation values of the unknown parameters, had been set equal to zero (0) i.e. for the rotations and the translations. For all projects the corresponding value for the scale was equal to 1. However, some different initial approximation values were also put into practice, using values for the rotations in the range of 1 – 2 degrees and for the translations in the range of 0 - 5 meters. These projects were also converged in the two first iterations as all the other projects.

In order to avoid arithmetic problems, due to the necessary number of digits of the coordinates, all points in both data sets (P and Q) were referred to the common Gravity Center of the two point clouds, according to well known numerical analysis rules. However, for practical purposes, in projects 5 and 10 (with the notation "NOT Center of Gravity") this conversion was not applied. From the results, it is evident, that this conversion is needed.

### Table 3. Results of Registration between Laser and Photogrammetric Restitution (control surface)

<table>
<thead>
<tr>
<th>Project</th>
<th>Processing Level of Data Derived by Photogrammetric means</th>
<th>Control Surface</th>
<th># of TINs</th>
<th># of photo points</th>
<th># of Laser points</th>
<th>Abs Mean value of residuals (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Breakline Interpolation 20cm Movement 1mm</td>
<td>Photo Restitution</td>
<td>13950</td>
<td>8320</td>
<td>27871</td>
<td>0.9286</td>
</tr>
<tr>
<td>2</td>
<td>Breakline Interpolation 20cm</td>
<td>Photo Restitution</td>
<td>10390</td>
<td>8320</td>
<td>27871</td>
<td>1.2796</td>
</tr>
<tr>
<td>3</td>
<td>Without processing</td>
<td>Photo Restitution</td>
<td>733</td>
<td>1224</td>
<td>27871</td>
<td>7.0722</td>
</tr>
<tr>
<td>4</td>
<td>Movement 1mm</td>
<td>Photo Restitution</td>
<td>1353</td>
<td>1224</td>
<td>27871</td>
<td>4.5634</td>
</tr>
<tr>
<td>5</td>
<td>Breakline Interpolation 20cm Movement 1mm - NOT Center of Gravity</td>
<td>Photo Restitution</td>
<td>473</td>
<td>1224</td>
<td>27871</td>
<td>10.1012</td>
</tr>
</tbody>
</table>
Figure 6. A project after the convergence. Photogrammetric restitution (highlighted triangles) depicts
the control surface. Laser points are shown by red circles.

In order to gain more experience, the role of the two surfaces was inverted. In those tests the laser points surface Q obtained the role of the control reference system and the surface P generated by photogrammetric means was registered. The results of the tests performed creating TINs by laser points are presented in Table 4 (Projects 6-10).

Table 4. Results of Registration between Laser (control surface) and Photogrammetric Restitution

<table>
<thead>
<tr>
<th>Project</th>
<th>Processing Level of Data Derived by Photogrammetric means</th>
<th>Control Surface</th>
<th># of TINs</th>
<th># of photo points</th>
<th># of Laser points</th>
<th>Abs Mean value of residuals (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Breakline Interpolation 20cm</td>
<td>Laser</td>
<td>55642</td>
<td>8320</td>
<td>27871</td>
<td>0.0645</td>
</tr>
<tr>
<td></td>
<td>Movement 1mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Breakline Interpolation 20cm</td>
<td>Laser</td>
<td>55642</td>
<td>8320</td>
<td>27871</td>
<td>0.0653</td>
</tr>
<tr>
<td>8</td>
<td>Without processing</td>
<td>Laser</td>
<td>55642</td>
<td>1224</td>
<td>27871</td>
<td>0.0604</td>
</tr>
<tr>
<td>9</td>
<td>Movement 1mm</td>
<td>Laser</td>
<td>55642</td>
<td>1224</td>
<td>27871</td>
<td>0.0554</td>
</tr>
<tr>
<td>10</td>
<td>Breakline Interpolation 20cm Movement 1mm - NOT Center of Gravity</td>
<td>Laser</td>
<td>4341</td>
<td>1224</td>
<td>27871</td>
<td>0.7945</td>
</tr>
</tbody>
</table>

An experimental curve based on the results of the tests, in which the control surface was the Photogrammetric one, was fitted, in Figure 7. In this Figure the correlation between the number of TINs and the Abs-Mean of residuals (mm) is indicated. The Abs-Mean of residuals was chosen as a more rigorous standard instead of the Mean of residuals. In Figure 8 one can also see the residuals of the fitting. The curve is an exponential curve of the form which can be described as: $f(x) = \frac{695.5}{x^{0.7}}$. From both Figures 7 and 8 it is clear that the 2nd project does not fit in the experimental curve so accurately as the other projects. This project had been produced by the registration of the laser data set over the control photogrammetric surface while the points of control surface had not been moved so that not to overcome the limitation of the 2D Delaunay’s algorithm (as it is described in the TINs subsection). Moreover, it should be noted that several projects were also put into practice, using movement values in the range of 1mm – 5mm. No difference was noticed in the number of TINs so that to make it clear that the movement of 1mm was able to overcome the limitation.
Figure 7. Number of TINs vs ABS-mean of residuals (mm).

Figure 8. Residuals of the fitting.

Points evaluated by restitution have also been interpolated by 30cm, 40cm, 50cm in the projects 11-16, respectively, in order to the correlation between the densification (proportionate to the number of TINs) and the Abs-Mean of residuals of the solution. The results are shown in Tables 5 and 6.

Table 5. Results of Registration between Laser and Photogrammetric Restitution (control surface) Different level of Linear Breakline Interpolation

<table>
<thead>
<tr>
<th>Project</th>
<th>Processing Level of Data Derived by Photogrammetric means</th>
<th>Control Surface</th>
<th># of TINs</th>
<th># of photo points</th>
<th># of Laser points</th>
<th>Abs Mean value of residuals (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Breakline Interpolation 20cm Movement 1mm</td>
<td>Photo Restitution</td>
<td>13950</td>
<td>8320</td>
<td>27871</td>
<td>0.9286</td>
</tr>
<tr>
<td>14</td>
<td>Breakline Interpolation 30cm Movement 1mm</td>
<td>Photo Restitution</td>
<td>9333</td>
<td>5724</td>
<td>27871</td>
<td>1.2270</td>
</tr>
<tr>
<td>15</td>
<td>Breakline Interpolation 40cm Movement 1mm</td>
<td>Photo Restitution</td>
<td>7027</td>
<td>4437</td>
<td>27871</td>
<td>1.4741</td>
</tr>
<tr>
<td>16</td>
<td>Breakline Interpolation 50cm Movement 1mm</td>
<td>Photo Restitution</td>
<td>5743</td>
<td>3699</td>
<td>27871</td>
<td>1.6460</td>
</tr>
</tbody>
</table>
In Table 5 the number of TINs is conversely proportional to the value of the interpolation as the number of photogrammetric points is reduced while the Interpolation distance is increased. In contrast, in Tables 4 and 6 the number of laser points is constant as also the number of TINs.

In Tables 4 and 6, where the laser points surface Q is the control reference system, one can perceive that the Abs-Mean of residuals is systematically better than in Tables 3 and 5 respectively. This is a result of multiple number of laser points, therefore multiple TINs, in comparison with the number and TINs of photogrammetric points.

Table 6. Results of Registration between Laser (control surface) and Photogrammetric Restitution

<table>
<thead>
<tr>
<th>Project</th>
<th>Processing Level of Data Derived by Photogrammetric means</th>
<th>Control Surface</th>
<th># of TINs</th>
<th># of Photo points</th>
<th># of Laser points</th>
<th>Abs Mean value of residuals (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Breakline Interpolation 20cm Movement 1mm</td>
<td>Laser</td>
<td>55642</td>
<td>8320</td>
<td>27871</td>
<td>0.0645</td>
</tr>
<tr>
<td>11</td>
<td>Breakline Interpolation 30cm Movement 1mm</td>
<td>Laser</td>
<td>55642</td>
<td>5724</td>
<td>27871</td>
<td>0.0632</td>
</tr>
<tr>
<td>12</td>
<td>Breakline Interpolation 40cm Movement 1mm</td>
<td>Laser</td>
<td>55642</td>
<td>4437</td>
<td>27871</td>
<td>0.0642</td>
</tr>
<tr>
<td>13</td>
<td>Breakline Interpolation 50cm Movement 1mm</td>
<td>Laser</td>
<td>55642</td>
<td>3699</td>
<td>27871</td>
<td>0.0621</td>
</tr>
</tbody>
</table>

The a posteriori uncertainties of the unknown parameters are in the following range: $1 \mu m - 9 \mu m$ for $\hat{\sigma}_u$, $\hat{\sigma}_v$, $\hat{\sigma}_w$, $0.1$ppm –$3$ppm for scale and $0.06^{cc}-5^{cc}$ for $\hat{\sigma}_{uu}$ $\hat{\sigma}_{uv}$ $\hat{\sigma}_{vw}$. The above values depend on the processing level of points.

CONCLUSIONS

In this research, a self-executable software script was created for the registration of two surfaces in order to evaluate its capability under real conditions. The two surfaces were defined by irregularly distributed points, while the point density was different in each one. Registration of these two surfaces is the prerequisite step for fusion. By fusing the two surfaces, the digital surface model (DSM) provided from Aerial Laser Altimetry is being improved. The algorithm used for the registration is based on the minimization of the distances between surface patches of the control surface and points of the surface which is registered (experimental surface), parallel to the corresponding surface normals.

As far as the initial approximation values are concerned no difference was perceived at the time of the convergence (all projects converged after the first two iterations) and similar values for the parameters were accomplished.

There were two processing steps of photogrammetric points which are: the Breakline Interpolation and the Movement. According to the results, the Interpolation produced a denser TIN model of the photogrammetric points, and the Movement (of 1mm of the points that present the lower z value e.g. the lower edge between the two edges of a wall on a building) proved to be sufficient in order to overcome the 2D Delaunay’s algorithm limitation creating more triangles and a more accurate TIN model. An exponential curve of the $f(x)=a \cdot x^b$ form can describe the correlation between the number of TINs and the Abs-Mean of residuals.

Therefore, the method can be applied to a variety of co-registration problems between point data sets. Future studies will include comparison of this algorithm with a new algorithm which will be evaluated through more practical tests.

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REFERENCES


