ANALYZING SCENE GEOMETRIES FOR STEREO PUSHBROOM IMAGERY

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ABSTRACT

Bundle Adjustment (BA) is an important step in deriving high quality terrain products from stereo pairs of satellite imagery. Most satellites use pushbroom cameras for image acquisition due to improved signal to noise ratio. The BA process is not as well constrained for pushbroom imagery as it is for frame cameras. Due to the loss of perspective along track additional degrees of freedom are present in BA. Usually Ground Control Points (GCPs) are used to provide necessary constraints. However in order to reduce cost and allow automated processing GCPs are being excluded from BA and Path Control Points (PCPs) are used instead. Without ground control, scene geometry and the technique for describing the camera’s path become more critical. In this paper we study the effects that different scene geometries have on the accuracy of the reconstructed terrain. A method to identify degrees of freedom is developed and utilized to constraints of five commonly used geometries. Theoretical results are supplemented with the numerical analysis of the geometries used in satellite and aerial imagery.

INTRODUCTION

Bundle Adjustment (BA) is an algorithm commonly utilized in attempts to improve camera position and orientation estimates for stereo imagery. This is useful as a precursor to other processes such as orthorectification, terrain extraction, and other photogrammetric processes. BA was first formulated in 1958 for use in photogrammetry (Brown, 1958). Since then it has been successfully utilized by many (for example (Orun and Natarajan, 1994, Ebner et al., 1999, Bouillion et al., 2006, Li et al., 2008) and is commonly used in photogrammetry software. A significant amount of research has been directed at reducing the computational complexity of BA, for example by utilizing sparseness of the Jacobian matrix (George and Liu, 1981, Duff et al., 1989). More recently BA has gained interest in the computer vision community and has been described in books and papers (Hartley and Zisserman, 2004, Triggs et al., 2000). While some simple cases for a lack of constraints are known in BA with frame cameras (and correspondingly for pushbroom cameras) the literature is deficient in analytical descriptions of needed constraints for BA with pushbroom cameras.

BA with pushbroom imagery is more complicated than that with frame cameras, but is increasingly being used to generate terrain models in semi-automated processing of the imagery. However, the adjustments made by BA for pushbroom imagery may have degrees of freedom (DOF) beyond those commonly known for frame cameras (absolute position and scale). These additional DOF depend on the geometric relationship between the camera paths/poses used to obtain the imagery. Furthermore, the residuals after adjustment of the camera poses do not provide any indication that the true camera poses have not been recovered.

A common approach to eliminate DOF is to include additional constraints like Ground Control Points (GCPs). However, sometimes it is impossible or impractical to add GCPs (e.g. for extraterrestrial imagery (Kirk et al., 2007) or imagery of ice sheets (Korona et al., 2009)). In those cases Path Control Points (PCPs) from navigation are often used as the sole means of control. It is important to realize that BA may not be capable of improving errors in the path measurement due to the presence of DOF.

An example of a terrain reconstruction where DOF allow significant errors to be undetected by BA is presented in Fig. 1. In this simple scenario the orientation of one camera is disturbed while the orientation of the other camera and the position (paths) of both cameras remain fixed. Then BA is applied to refine the estimate of orientation for
the disturbed camera while holding all other parameters fixed. For frame cameras this would constrain the well
known DOF in scale and absolute position and orientation. BA converges to a cost within the counting precision of
zero. However, the terrain model constructed through triangulation is a rotated, translated and warped version of the
original terrain.

In this paper a method of identifying linear DOF for a given viewing geometry is described and then analyzed
with numerical simulations. In the first section BA for frame and pushbroom cameras is formulated, along with an
extension that allows the reduction of the parameter vector for BA through the elimination of feature locations. The
second section details the mathematical formulation of the method used to identify DOF. The last section presents
results of applying this method to viewing geometries common for obtaining stereo imagery with pushbroom
cameras and is followed by numerical studies. The numerical studies indicate that because of the small field of view
(FOV) common in high resolution satellite cameras that the mathematical DOF are no more significant that the lack
of constraint posed by the small FOV. However, the numerical studies do indicate that for lower altitude, larger
FOV cameras common in aerial surveys, the viewing geometries can be important in constraining BA.

**BUNDLE ADJUSTMENT**

This work is concerned with stereo photogrammetry from pushbroom cameras whereby a digital elevation
model (DEM) is derived from two images of the same terrain captured from different vantage points. Pioneering
work was performed in this area by Hoffman et al. (Hofmann et al., 1984) and Ebner and Strunz (Ebner and Strunz,
1988). Recently this topic was covered by Tiggs et al. (Triggs et al., 2000) and Hartley and Zisserman (Hartley and
Zisserman, 2004).

Using camera pose (position and orientation) DEMs are produced through triangulation of corresponding
features identified in both images. Like all measurements, camera pose measurements are corrupted by noise.
Therefore, to improve the quality of the DEM, the camera pose estimates are adjusted to enhance scene consistency
using a process called Bundle Adjustment (BA). This section details the structure of the BA algorithm to set the
framework for discussion of the pitfalls of scene geometry in applying BA to stereo pushbroom imagery. General
BA formulation is described first. Next, a method to reduce parameter vector size by eliminating feature coordinates
is presented.

**Figure 1a.** Scene with the original terrain and terrain obtained by triangulation using the initial (disturbed)
path parameters. The latter one is shifted and rotated. Without refinement, the triangulation error is $3.1 \times 10^{40}$.

**Figure 1b.** Scene with the original terrain and that from triangulation after refinement using BA. The latter
one remains both offset and misshapen even though the residual triangulation error is only $3.1 \times 10^{-27}$. 
Formulation

BA adjusts all scene parameters including the camera positions, \( \mathbf{P}_c \), camera orientations, \( \Phi_c = [\omega \Phi_n] \), and feature locations in 3D, \( \mathbf{P}_f \), to minimize a scalar cost function. The feature locations are those associated with features identified and corresponded in two or more images, and are also known as "tie points". The cost function is typically a sum of squared errors. Individual error terms for tie points are measured in the camera's focal plane as the difference between measured location of the feature in the image and that predicted by the current state of the estimated parameters. Prediction of a feature's image location is accomplished through back-projection of \( \mathbf{P}_f \) into the camera's focal plane. Without lens distortion, the pinhole camera model describes how 3D points are imaged. With this model, the projection of \( \mathbf{P}_f \) into the camera focal plane is given by

\[
\mathbf{p}_b = \left[ u_b \ v_b \right] = \left[ \frac{\mathbf{d}}{d_z} \right] = R_{c \varphi}^{T} \left( \mathbf{P}_f - \mathbf{P}_c \right) \tag{1}
\]

where

\[
\mathbf{d} = \left[ d_x \ d_y \ d_z \right]^T = R_{c \varphi} \left( \mathbf{P}_f - \mathbf{P}_c \right)
\]

and \( R_{c \varphi} \) is a rotation matrix transforming vectors from feature \( \mathbf{P}_f \) coordinate frame to the camera coordinate frame and defined by the camera orientation \( \Phi_c \) as

\[
R_{c \varphi} = \begin{bmatrix}
\mathbf{c}_x & \mathbf{c}_y & \mathbf{c}_z & \mathbf{c}_t \\
\mathbf{s}_x & \mathbf{s}_y & \mathbf{s}_z & \mathbf{s}_t \\
-\mathbf{s}_x & \mathbf{c}_y & \mathbf{s}_z & \mathbf{s}_t \\
-\mathbf{s}_y & \mathbf{c}_x & -\mathbf{s}_z & \mathbf{s}_t
\end{bmatrix}
\]

Here \( \mathbf{c}_x \) and \( \mathbf{s}_x \) are \( \cos(\theta) \) and \( \sin(\theta) \). The back-projection error between an observed feature, \( \mathbf{p}_m \), and its prediction, \( \mathbf{p}_b \), is given by

\[
\mathbf{e} = \mathbf{p}_m - \mathbf{p}_b = \left[ \mathbf{p}_{m_1} \mathbf{p}_{m_2} \ldots \mathbf{p}_{m_n} \right] = \left[ \mathbf{e}_1 \mathbf{e}_2 \ldots \mathbf{e}_n \right] \tag{4}
\]

The scalar cost function measuring the scene consistency is the sum of squared back projection errors,

\[
\mathcal{C} = \mathbf{c}^T \mathbf{c}
\]

where, for \( m \) images with \( n \) tie points, the cost vector is

\[
\mathbf{c} = \left[ \mathbf{e}_1^T \ldots \mathbf{e}_n^T \right]^T \tag{6}
\]

where \( \mathbf{e}_i = \left[ \mathbf{e}_{i1}^T \ldots \mathbf{e}_{in}^T \right]^T \) for \( i = 1 \ldots m \). Here, for simplicity, it is assumed that all \( n \) tie points are observed in all \( m \) cameras. For \( n \) tie points and \( m \) cameras, if we define the vector of parameters to be estimated as

\[
\mathbf{k} = \left[ \mathbf{p}_{f1}^T \ldots \mathbf{p}_{fn}^T \mathbf{P}_c^T \mathbf{P}_c^T \right]^T
\]

then combining equations (1), (2), (3), (4) and (6) and plugging the result into (5) provides a scalar cost as a function of the scene parameters, \( \mathcal{C} = \mathbf{C}(\mathbf{k}) \).

Unlike frame cameras which capture an entire image in a single pose, pushbroom cameras sweep out images by traveling through space. Each image line is acquired from a slightly different pose. Therefore, pose parameters associated with each image are a function of image line number or time, \( t \). In this paper we express each camera pose parameter as a nominal path plus a time dependent offset. The offset is modeled as a second order polynomial.
in $\mathbf{F}$ whose coefficients are adjusted by BA. The camera pose is given by,

$$
[\mathbf{p}(t), \Phi(t)] =
\begin{bmatrix}
\alpha_x t^2 + \beta_x t + \gamma_x \\
\alpha_y t^2 + \beta_y t + \gamma_y \\
\alpha_z t^2 + \beta_z t + \gamma_z \\
\omega_x t^2 + \beta_x t + \gamma_x \\
\omega_y t^2 + \beta_y t + \gamma_y \\
\omega_z t^2 + \beta_z t + \gamma_z
\end{bmatrix}
$$

where the nominal trajectories $\alpha_x(t)$, $\alpha_y(t)$, $\alpha_z(t)$, $\omega_x(t)$, $\omega_y(t)$, and $\omega_z(t)$ are assumed to be known measurements provided by a navigation system. The parameter vector $\mathbf{k}$ is then expanded to include the polynomial coefficients,

$$
\mathbf{k} = [\mathbf{p}^T, \Phi^T, [\alpha_x \alpha_y \alpha_z \omega_x \omega_y \omega_z]^T].
$$

The necessary addition of these path parameters significantly decreases the constraints on the scene geometry as compared to BA for frame cameras.

Adding additional constraints such as Ground Control or Path Control Points, (GCPs or PCPs), to BA is achieved by adding elements, similar to (4), to the cost vector $\mathbf{c}$. Ground control and path control measurements have different variance than back-projection errors, therefore a weighting matrix is defined using the inverse covariance to adjust for their relative significance. The scalar cost then becomes,

$$
\mathbf{c} = \mathbf{c}^T \mathbf{W} \mathbf{c},
$$

The cost function (10) is non-linear w.r.t. the vector of parameters $\mathbf{k}$. Therefore an iterative algorithm is used to find a $\mathbf{k}$ that minimizes the cost function. The standard approach for BA utilizes the Levenberg-Marquardt (LM) algorithm which is a hybrid between gradient descent and Newton methods. At each update step the incremental adjustment to $\mathbf{k}$ in the LM algorithm is computed using

$$
\delta \mathbf{k} = -\left( \mathbf{J}^T \mathbf{W} \mathbf{J} + \lambda I \right)^{-1} \mathbf{J}^T \mathbf{W} \mathbf{c}
$$

where $\mathbf{J}$ is the Jacobian matrix and $\lambda$ is the LM damping factor, which varies during minimization and tends to zero as a minimum is approached.

In summary, BA is an iterative algorithm that seeks to minimize the cost function $\mathbf{c}(\mathbf{k})$ by varying elements of the parameter vector $\mathbf{k}$. Because the camera poses and 3D feature coordinates are elements of $\mathbf{k}$, BA can be conceptualized as simultaneous adjustment of camera positions and feature coordinates to minimize the difference between the measured feature coordinates in the images and those predicted by projecting the 3D coordinates into the focal planes of the cameras. While additional control measurements can be utilized in BA to help constrain the minimization, the inherent lack of constraints in pushbroom imagery necessitates careful consideration of the geometry used in image acquisition.

**Elimination of Features from the Parameter Vector**

The 3D positions of terrain features can be eliminated from the parameter vector for BA using triangulation. In triangulation the feature positions are found using the camera positions and orientations and the measured image coordinates of the features. If triangulation is formulated as a differentiable function of the other parameters then the feature positions become dependent variables, and can be removed from the parameter vector. Identification of the degrees of freedom by a method described in the next sections of this paper is greatly simplified by the elimination of features from the parameter vector.
Several methods exist for triangulation, including BA with the camera positions and orientations held constant, which is an iterative process. However we seek a closed form solution to triangulation. Specifically, the method used here is to take the feature location as the midpoint of the line between the closest points of approach for two lines. The two lines are defined using the camera positions and orientations and the feature image coordinates as shown in Fig. 1. The closest points of approach are determined by finding the mutual perpendicular for the two lines.

The development of the triangulation equations given here is based on an intuitive geometric description. First two vectors from the focal points of the images to the measured feature locations in the image plane are found using the following equation,

\[
\mathbf{u}_i - R_{rc_i} \left[ \begin{array}{c} p_{m_1} \\ f_i \\ \end{array} \right] - R_{rc_i} \left[ \begin{array}{c} p_{m_2} \\ f_i \\ \end{array} \right], \quad i = 1, 2
\]  

(12)

where \( R_{rc_i} \) is the transpose of \( R_{rc} \) in (3). A mutual perpendicular to \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) is given by the cross product, \( \mathbf{u}_6 = \mathbf{u}_1 \times \mathbf{u}_2 \). A vector loop can be written that uses the two camera positions, the three pointing vectors, and three to-be-determined scalars (\( \mu_1, \mu_2, \mu_3 \)) as follows:

\[
\mathbf{P}_c - \mathbf{P}_c = \mu_1 \mathbf{u}_1 + \mu_2 \mathbf{u}_6 - \mu_2 \mathbf{u}_2
\]  

(13)

Solving for \( \mu_1, \mu_2, \mu_3 \) and setting \( \mathbf{P}_f \) to be a midpoint of a line perpendicular to both \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) we get

\[
\mathbf{P}_f = \frac{\mathbf{P}_c + \mu_1 \mathbf{u}_1 + \frac{\mu_2 \mathbf{u}_2}{2}}{2}
\]  

(14)

Plugging (14) into (2) removes the dependency of the cost function on the feature locations, making it only a function of the camera poses and constants. Thus the elements of the parameter vector are reduced to the parameters associated with the camera poses.

IDENTIFYING DEGREES OF FREEDOM

In this section, an approach is developed for identifying linear degrees of freedom in the BA process that do not change the minimal cost of a solution. First the problem is stated and analyzed mathematically. Then a process is described to perform the analysis needed to identify degrees of freedom for particular scene geometries.
Mathematical Problem Statement and Method Development

We are interested in finding the set of directions, \( \mathbf{e} \), in the parameter space along which we can move from the minimum without changing the cost. In other words we wish to find the directions that satisfy

\[
G(k_{\text{min}} + \mathbf{e}) = G(k_{\text{min}}),
\]

for reasonably large \( \mathbf{e} \). An ad hoc search for such a space is not feasible. We therefore need to narrow the possibilities for degrees of freedom using some analysis of the problem.

We will denote the gradient and Hessian of \( G(k) \) at \( k_{\text{min}} \) respectively as \( \mathbf{g} \) and \( \mathbf{G} \). The curvature of \( G(k_{\text{min}} + \mathbf{e}) \) along the line \( k_{\text{min}} + \mathbf{e} \) is then \( \mathbf{e}^T \mathbf{G} \mathbf{e} \). It is well known (Fletcher, 2000) that a zero gradient and a non-negative curvature are necessary conditions for the existence of a minimum. Mathematically, these necessary conditions are

\[
\mathbf{g} = 0,
\]

and

\[
\mathbf{e}^T \mathbf{G} \mathbf{e} \geq 0, \forall \mathbf{e}.
\]

Also, it is well known (Fletcher, 2000) that a sufficient condition for the existence of a strict and isolated local minimum is

\[
\mathbf{e}^T \mathbf{G} \mathbf{e} = 0, \forall \mathbf{e}.
\]

If we are at a minimum (i.e. (16) and (17) are satisfied) then the vectors satisfying (15) necessarily lie in the space not satisfying (18). The only difference between the set of vectors satisfying (17) and those satisfying (18) are those for which \( \mathbf{e}^T \mathbf{G} \mathbf{e} = 0 \), which is a null space of \( \mathbf{G} \). It can be shown that any vector not satisfying (18) must lie in the null space defined by zero eigenvalues of \( \mathbf{G} \). Therefore the search for directions defined in (15) can be limited to vectors in the space spanned by the zero eigenvalues of \( \mathbf{G} \).

It should be noted that being in the null space of \( \mathbf{G} \) is only a necessary condition for a vector to satisfy (15) when at a minimum. This is true because (18) is only a sufficient condition not a necessary and sufficient condition for an isolated minimum. Therefore the null space must be explored further to identify degrees of freedom.

For the sum of squares cost function, the problem of finding the Hessian \( \mathbf{G} \) is significantly easier if the cost is zero. In general computing the Hessian involves finding the second partial derivatives of the cost function w.r.t. the parameters. However, if the cost is zero we only need to compute the first order derivatives associated with the Jacobian. It can be shown that for the sum of squares cost function the Hessian can be expressed as

\[
\mathbf{G} = 2J^T J + 2 \sum_{k=1}^{m} \alpha(k) \nabla^2 \alpha(k),
\]

The second term of this equation is zero if the cost is zero. The procedure we follow to search for degrees of freedom in a particular geometry is described by the following steps:

1. Form a simulated scene with true zero cost.
2. Compute the Jacobian, and Hessian (using \( \mathbf{G} = 2J^T J \)).
3. Compute the zero eigenvalues of \( \mathbf{G} \) and their corresponding normalized eigenvectors (e.g. using singular value decomposition (SVD)).
4. Test each of the directions given by these eigenvectors by moving along them from the zero cost solution while computing the cost. If the cost remains zero over a large range then consider it a degree of freedom and eliminate it from the space left to consider.
5. If the number of degrees of freedom identified is less than size of the null space minus one then it is still possible that a degree of freedom(s) lies in the set of vectors not eliminated from the search. Therefore, if possible, formulate a method to search this space in more detail.

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In this section the relationship between scene geometry and BA accuracy is investigated. First five viewing candidate geometries are presented. Then, the degrees of freedom in each of these geometries are identified employing the methods developed in the previous section. A formulation of BA with a reduced parameter space is described which is applied on simulated imagery to investigate the impact scene geometry can have on BA accuracy. The results of these simulations are presented and conclusions draw from their results.

Simulation Setup

In order to perform BA we need two or more views. For the purpose of simulations we investigated five scene geometries (presented in Fig. 2), each containing two camera paths. They describe different possibilities of obtaining
For each geometry an artificial scene is generated. The true locations of a set of well dispersed features is projected into the pushbroom images from the true camera paths, giving a scene with zero cost. The Jacobian for the reduced parameter vector, the Hessian \( \mathbf{H} = \mathbf{J}^T \mathbf{J} \), and the SVD of the Hessian are calculated. The computation is performed for both scenarios. Any eigenvector associated with an eigenvalue of zero is taken as a candidate for a degree of freedom of the geometry. To evaluate the candidate degrees of freedom, \( \xi_1 \), the camera path is perturbed from the true camera path, \( \mathbf{h} = (\mathbf{\Phi}_1, \mathbf{\Phi}_2)^T \), in the direction of the candidate. In this case the camera path is given by

\[
\mathbf{h} = (\mathbf{\Phi}_1 + \xi_1, \mathbf{\Phi}_2 + \xi_1)^T.
\]

Identified Degrees of Freedom

For each geometry an artificial scene is generated. The true locations of a set of well dispersed features is projected into the pushbroom images from the true camera paths, giving a scene with zero cost. The Jacobian for the reduced parameter vector, the Hessian \( \mathbf{H} = \mathbf{J}^T \mathbf{J} \), and the SVD of the Hessian are calculated. The computation is performed for both scenarios. Any eigenvector associated with an eigenvalue of zero is taken as a candidate for a degree of freedom of the geometry. To evaluate the candidate degrees of freedom, \( \xi_1 \), the camera path is perturbed from the true camera path, \( \mathbf{h} = (\mathbf{\Phi}_1, \mathbf{\Phi}_2)^T \), in the direction of the candidate. In this case the camera path is given by

\[
\mathbf{h} = (\mathbf{\Phi}_1 + \xi_1, \mathbf{\Phi}_2 + \xi_1)^T.
\]

### Table 1. Degrees of freedom for different geometries. Top rows correspond to holding one camera fixed, allowing another one to change pose (position and orientation). Bottom rows correspond to holding both positions fixed, while allowing both orientations to be changed.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Nr of zero eigenvalues</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single camera pose fixed</td>
<td>Nr of degrees of freedom</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Both camera positions fixed</td>
<td>Nr of zero eigenvalues</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Both camera positions fixed</td>
<td>Nr of degrees of freedom</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
where $\xi$ is a scale factor. This makes the cost a function of the scale factor, $C(\xi) = C_0 + C_1(\xi) + C_2$. If the cost remains zero over a wide range of $\xi$, then the candidate is taken as a degree of freedom for the geometry. If all of these candidates are degrees of freedom then the number of degrees of freedom is clear, and so is their orthogonal set of directions. However, if there is more than one candidate that did not pass the degree of freedom test then it is still possible for degrees of freedom to lie in the space defined by the null vectors.

MATLAB software was used in all computations. In order to increase precision Variable Precision Arithmetic (VPA) was used with 1000 decimal digits during Jacobian computation and the singular value decomposition of the Hessian matrix. This aided in distinguishing between eigenvalues that are truly zero and those that are merely small. To ensure accuracy the Jacobian was calculated using symbolically developed equations rather than a numerical method. These equations are very lengthy and cannot be presented here.

Results of analysis are presented in Table 1. Consider the first scenario. For geometry 1 three degrees of freedom were identified, namely along $Y$, $Z$, and $\Omega$. This convenient and commonly used geometry is poorly constrained. For geometry 5 a single degree of freedom, along $X$, was identified. This along path direction is coincidently the most poorly known for PCP measurements from the navigation systems of many satellites. Now consider the second scenario. Additional degrees of freedom were found again in both geometry 1 and geometry 5. For geometry 1 the degree of freedom was again found to be along $\Omega$. For geometry 5 the degree of freedom exist for simultaneous rotation along $\Omega$ in the same direction of both cameras. In summary, three degrees of freedom were found for geometry 1 and two degrees of freedom were found for geometry 5. The other geometries are mathematically constrained (i.e. a single isolated minimum exists).

### Simulations of Satellite Imagery

Computer simulations were performed to assess the influence of degrees of freedom on the accuracy of BA. A simulation was again developed for this analysis. First a terrain model consisting of 512 points was created. Then, using one of the imaging geometries and a simple camera model resembling the QuickBird camera, (Salvini et al., 2004, DigitalGlobe, 2009), artificial images were generated. At this point, error-free terrain reconstruction was possible through triangulation. Next white Gaussian noise with a standard deviation of 0.5 pixels was added to the images to simulate feature detection errors in real images. Finally colored noise was used to disturb path measurements. The standard deviation of the noise added to path measurements was 5m for position and $15e^{-6}$ rad for orientation. At this point, due to added noise, terrain reconstructed by the triangulations is significantly different from the original terrain. Ten evenly distributed path measurements were selected and used as PCPs. These artificial noisy images and PCPs provide input to the BA algorithm. This BA algorithm adjusts both camera poses and was allowed to run until convergence. RMS of the difference between BA reconstructed terrain and the original terrain defines the performance measure of the accuracy of BA. The above procedure was repeated 100 times and the average results are presented in Table 2.

In all cases the terrain reconstructed by BA is closer to the original terrain than the terrain obtained by initial triangulation and the accuracy gained through applying BA is similar for all geometries. There is no clearly visible advantage in using a mathematically constrained geometry over one that is constrained. This result appears to refute the previous analysis. To understand the discrepancy between the existence of degrees of freedom and BA’s performance additional tests were performed.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>3.76</td>
<td>3.09</td>
<td>3.02</td>
<td>2.04</td>
<td>4.44</td>
</tr>
<tr>
<td>y</td>
<td>4.24</td>
<td>2.92</td>
<td>4.19</td>
<td>4.20</td>
<td>3.93</td>
</tr>
<tr>
<td>z</td>
<td>18.47</td>
<td>15.82</td>
<td>15.74</td>
<td>16.43</td>
<td>14.53</td>
</tr>
<tr>
<td>avg</td>
<td>8.82</td>
<td>7.28</td>
<td>7.65</td>
<td>7.56</td>
<td>7.63</td>
</tr>
</tbody>
</table>

Table 2. RMS of terrain error for 100 simulation runs for satellite imagery with assumed image noise of 0.5 pixel and path noise of [5m, 5m, 5m, 15e^{-6}rad, 15e^{-6}rad, 15e^{-6}rad]
Typical satellite imagery is from high altitude with a very narrow field of view (FOV). These high aspect ratios are an additional source of ill conditioning for BA which swamp out the geometric effects we address. On the other hand pushbroom imagery from air vehicles has significantly lower aspect ratios due to both lower altitudes and wider FOV cameras. Examples of pushbroom cameras used in aerial missions include ADS40, ADS80 from Leica Geosystems (Fricker, 2001, Sandau, 2010) or High Resolution Stereo Camera (HRSC) (Neukum, 1999).

A second simulation was performed using a model of an aerial camera similar to HRSC with a FOV increased from 12 to 20 degrees. The focal length was 165mm and the pixels size was 6.5um and the flight altitude was set to 2000m. Path noise was set to 10m for the position and 5e^-3 rad for the orientation. The standard deviation of image noise was set to 0.1 pixels. These values for the path distortion are large compared to the state of the art navigation systems (Sandau, 2010). However, such systems are prohibitively expensive for some users.

The results for 100 runs comparing geometry 1 and 2 are presented in Table 3. In this case, the degrees of freedom in the scene geometry do affect the accuracy of BA. The RMS Error is shown in the first two columns. The right two columns contain the shape error. Shape error is computed as the RMS difference between the original terrain and the recovered terrain once it has been shifted and rotated to best match with the original terrain. This measure is not influenced by simple translation or rotation but only by warping as it measures shape deformation.

Reconstructed terrain from the constrained geometry 2 is twice as accurate as the geometry 1 in terms of simple RMS error. In terms of shape error, geometry 2 provides twenty times better reconstruction than geometry 1. These simulations show that geometry can affect terrain reconstruction accuracy for aerial imagery. Moreover the developed method provides a tool for analyzing any geometry and provides insight into the nature of the degrees of freedom.

Even though, we were not able to demonstrate a practical advantage of one geometry over another with aspect ratio’s typical to satellites, we were able to demonstrate the effects of geometry on the accuracy of terrain reconstruction under certain limited conditions. Another simulation, more similar to the analysis, with different perturbations on the parameters was performed. In this case, no image noise was added and only one camera pose was disturbed. The other camera’s path parameters were left undisturbed and held fixed for BA. In this setup BA was always able to reconstruct the original terrain when geometries without degrees of freedom were used. However, the reconstructed terrain was always deformed with the unconstrained geometries, even though BA cost was reduced to 0. Reasonable values of accuracy for localizing tie points impart so much noise at aspect ratios typical to satellites that these errors swamp out the improvements gained using constrained geometry.

To explore the relationship between tie point localization noise and terrain accuracy the termination condition for BA was modified to approximate image noise. Under normal conditions BA runs until it converges (i.e. no better solution can be found). In this investigation, a weaker termination condition dependent upon the cost $C$ was added. BA was stopped after it reached some predefined cost. Since the BA’s cost is an aggregate image error, a weak termination condition is similar to feature localization noise. In this case, no image noise is added, but the pose parameters were perturbed. Table 4 presents results of this simulation for geometry 1, 2 and 4. When the termination condition allows the cost $C$ to be relatively large BA exhibits no dependence on the scene constraints. As the termination condition is reduced, the performance for the constrained geometries improves. Unfortunately, to take advantage of the constraints of the scene, BA has to be able to converge to extremely small cost which on average is a fraction of the size of a pixel. Obtaining such a small cost is impossible with a real imagery, where features are identified with the accuracy up to one tenth of the pixel at best. This fact limits the practical significance of the identified degrees of freedom for the imagery obtained by the satellites.

### Table 3. RMS of terrain error for 100 simulation runs for aerial imagery with assumed image noise of 0.1 pixel and path noise of [10m, 10m, 10m, 5e^-3rad, 5e^-3rad, 5e^-3rad]

<table>
<thead>
<tr>
<th></th>
<th>Geo 1 Init BA</th>
<th>Geo 2 Init BA</th>
<th>Geo 1形状 Error</th>
<th>Geo 2形状 Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>6.08</td>
<td>5.85</td>
<td>0.24</td>
<td>0.21</td>
</tr>
<tr>
<td>y</td>
<td>6.95</td>
<td>5.53</td>
<td>2.01</td>
<td>1.56</td>
</tr>
<tr>
<td>z</td>
<td>19.35</td>
<td>15.19</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>avg</td>
<td>10.79</td>
<td>8.86</td>
<td>0.79</td>
<td>0.62</td>
</tr>
</tbody>
</table>

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This paper presented a BA formulation for pushbroom cameras. A method for identifying degrees of freedom in viewing geometries was developed. The method based on the decomposition of the Hessian matrix computed at the solution is able to identify linear directions along which the cost remains zero. The method was applied to five commonly used geometries. In two of these cases, certain degrees of freedom were found that should deleteriously affect terrain reconstruction accuracy. Numerical simulations under conditions resembling real camera systems were performed in order to verify expected negative effect of degrees of freedom on BA. It was found that for satellite imagery BA improves the terrain reconstruction similarly for all geometries, regardless of the presence of degrees of freedom in the geometry. Further analysis was able to attribute this discrepancy between theory and practice. For simulations of lower altitude aerial imagery, the geometry did affect terrain reconstruction accuracy particularly with wide FOV imagery.

**REFERENCES**


