Mapping Matters

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Your Questions Answered
The layman's perspective on technical theory and practical applications of mapping and GIS

Question: Please explain Sigma as an indicator of accuracy.

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Dr. Abdullah: Before I answer the question, I would like to elaborate on the nature of random error distribution and the concept of the probability. In most experiments that include repetitive measurements of the same variable, when performed carefully, there is a certain expectancy of the outcome of the measurement. For example, if you toss a coin 100 times and count how many times it comes up heads, you will find the probability of such an outcome is always around 50%. However, if you repeat such an experiment 100 times randomly (not forcing the outcome to be heads or tails), the results will always be close to but not exactly 50%, such as 48%, 51%, or 55%, but almost never 24% or 85%. This random, yet clustered, distribution is what we call the normal distribution of the natural occurrence of things in nature. If we plot the results of the 100 outcomes of the experiment, it will result in the well-known bell-shape curve (see Figure 1).

The concept of normal distribution is important in science, as the question always arises as to what makes the new findings or results trustworthy or significant. In statistics, when we talk about the significance of the results we use the unit of significance or standard deviation (σ). The standard deviation, or sigma, measures the disparity of the result or error (as we are always concerned about measuring the errors in lidar data) from the mean value of such errors. Let us now try to understand such a statement in terms of the normal distribution curve (Figure 1).

In Figure 1, there is always a chance that 68% of the errors will fall within the standard deviation value of ±1σ (between –σ and + σ). In other words, if the project specification calls for lidar data with an accuracy of one sigma equal to15 cm and the client tested the accuracy of the data with 20 ground checkpoints, then the client should find 68% or more of the individual error values from the 20 points fall within 15 cm, with a possibility that 32% of the checkpoints will have an error in excess of 15 cm. Similarly, there is a chance that 95% of the individual error values fall within ±2σ or 30 cm.

The standard deviation, or sigma, is an indicator of how well the measurements fit each other. An individual error with a value of 3σ indicates that such a point does not fit the model properly, as it is far from the mean and so on. A large sigma value indicates that the individual errors fluctuate widely around the mean, while smaller sigma values mean the errors are close to each other as well as the mean. This conclusion can also be derived by looking at the bell curve. A flattened bell curve shape indicates widely dispersed errors around the mean, while a narrow bell shape indicates a close proximity of the errors to each other and the mean. The deviation is how far an individual error is from the mean. For the coin example, an experiment with an outcome of 48% means a deviation of 2 from the mean or the most probable value of 50%. We can also use the following formulas to calculate the standard deviation σ for the lidar error example:

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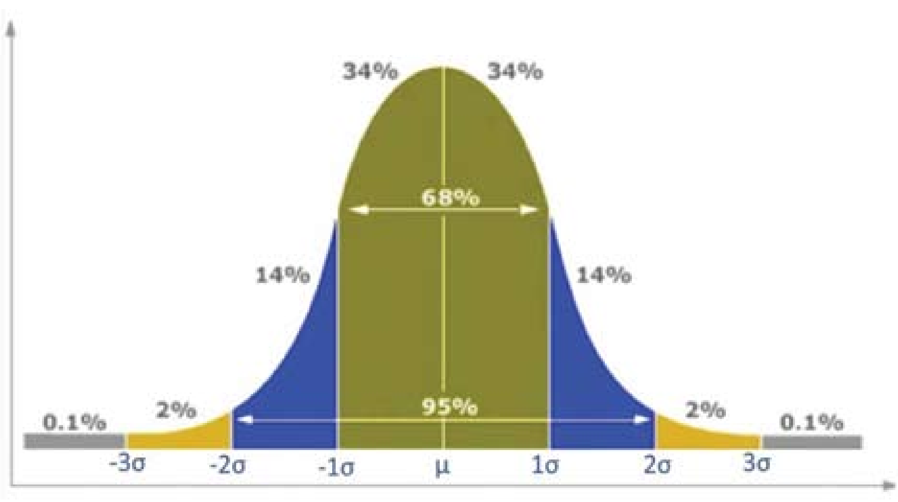

Figure 1. The normal distribution curve showing a typical bell shape curve (modified from http://theeducatedsociety.com/tag/bell-curve/).

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Deviation (d) = Individual error (v) – arithmetic mean of errors (μ)

where,

$$\text{Arithmetic mean } (\mu) = \frac{\sum \text{errors}}{\text{number of data points (n)}}$$

As for the computation of the standard deviation, we use the following formula:

$$\sigma = \sqrt{\sum d^2 / (n-1)}$$

where,
σ is sigma or the standard deviation
μ is the arithmetic mean of the sample
v is the error in a point
d is the deviation in a point
n is the number of checkpoints

Here, I would like to elaborate on the terms "standard deviation" (sigma) and "accuracy". Standard deviation, as we discussed earlier, shows us how the errors in the dataset fit together and how they are distributed around the mean. Therefore, it is a measure of precision, as the mean could have an error in the shape of bias, while the standard deviation still seems small and acceptable. In this case the Root Mean Squares Error (RMSE) is more suitable to stand on the accuracy or errors in a dataset; however, if we assume that all biases are removed from the dataset, then the sigma and the RMSE will be equal, and they both can be used as a measure of accuracy. Table 1 presents a typical accuracy calculation using 23 checkpoints.

From Table 1 we notice the standard deviation for the Easting coordinates is 0.17 m while the RMSE is 0.18 m. Let us now introduce -0.20 m for each of the measured Easting. Table 2 shows the results of the new calculations containing the bias.

It is obvious the standard deviation does not reveal any problem with the dataset unless we examine the new average of the dataset, -0.26 m, which is artificially inflated due to the introduced bias of -0.20 m. From the new value of the RMSE of 0.31 m, however, we can easily spot the problem in the dataset. Therefore, we need to be very careful in analyzing the accuracy of a dataset. Without understanding these statistical concepts and what affects them, we could derive a wrong conclusion in evaluating the accuracy of a geospatial dataset.

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Table 1. Error computations without bias.

Point ID	Measured Values			Surveyed Values			Residuals		
	Easting (E)	Northing (N)	Elevation (H)	Easting (E)	Northing (N)	Elevation (H)	ΔE (Easting)	ΔN (Northing)	ΔH (Elevation)
	meter	meter	meter	meter	meter	meter	meter	meter	meter
1	359584.354	5142449.934	477.027	359584.314	5142450.004	477.098	0.040	-0.070	-0.071
2	359872.330	5147939.180	412.406	359872.290	5147939.250	412.384	0.040	-0.070	0.022
3	359893.029	5136979.824	487.222	359892.989	5136979.894	487.190	0.040	-0.070	0.032
4	359927.144	5151084.129	393.571	359927.264	5151083.979	393.561	-0.120	0.150	0.010
5	372737.074	5151675.999	451.305	372736.844	5151675.879	451.318	0.230	0.120	-0.013
6	373282.550	5137216.753	466.579	373282.320	5137216.633	466.567	0.230	0.120	0.012
7	383714.817	5135495.256	452.954	383714.587	5135495.136	452.933	0.230	0.120	0.021
8	383908.812	5148006.957	492.029	383908.932	5148006.807	492.067	-0.120	0.150	-0.038
9	384026.003	5153418.148	488.902	384025.773	5153418.028	488.890	0.230	0.120	0.012
10	397996.280	5171300.737	440.296	397996.380	5171300.782	440.322	-0.100	-0.045	-0.026
11	400459.859	5179949.119	361.217	400459.979	5179948.969	361.268	-0.120	0.150	-0.051
12	400758.695	5167097.114	543.830	400759.007	5167097.107	543.816	-0.312	0.007	0.014
13	400794.774	5188864.498	263.666	400795.086	5188864.491	263.719	-0.312	0.007	-0.053
14	411418.398	5177915.793	525.953	411418.458	5177915.713	525.920	-0.060	0.080	0.033
15	413983.652	5170924.872	546.530	413983.752	5170924.917	546.574	-0.100	-0.045	-0.044
16	416177.444	5185743.870	320.442	416177.756	5185743.863	320.467	-0.312	0.007	-0.025
17	435497.773	5180055.008	345.719	435497.833	5180054.928	345.664	-0.060	0.080	0.055
18	435725.456	5165270.316	468.886	435725.556	5165270.361	468.892	-0.100	-0.045	-0.006
19	435979.436	5175221.390	443.009	435979.496	5175221.310	443.026	-0.060	0.080	-0.017
20	439669.309	5188155.815	190.855	439669.621	5188155.808	190.813	-0.312	0.007	0.042
21	448111.544	5184558.142	190.447	448111.664	5184557.992	190.458	-0.120	0.150	-0.011
22	450709.272	5164362.790	433.864	450709.372	5164362.835	433.851	-0.100	-0.045	0.013
23	452302.471	5175490.022	226.278	452302.531	5175489.942	226.230	-0.060	0.080	0.048
						Number of check points	23	23	23
						Mean	**-0.06**	**0.05**	**0.00**
						StDEV	0.17	0.08	0.03
						RMSE	**0.18**	**0.09**	**0.03**

Table 2. Error computations with the added bias of -0.20 m

Mean	**-0.26**	**0.05**	**0.00**
StDEV	0.17	0.08	0.03
RMSE	**0.31**	**0.09**	**0.03**