The Limit of Parallax Perception

EVERT W. JOHNSON. *Assistant Professor, Department of Forestry, Agricultural Experiment Station, Alabama Polytechnic Institute, Auburn, Alabama*

ABSTRACT: *A mathematical proof is presented for the statement that differential parallaxes smaller than 0.001 inch cannot be detected by average photo-interpreters. From this it is implied that instrumental graduations finer than 0.001 inch do not necessarily increase the precision of spot height measurement.*

FROM time to time arguments are heard concerning the relative merits of different instruments for measuring spot elevation differences on vertical aerial photographs. On one side are individuals such as S. H. Spurr[4] and E. J. Schlatter[3], who champion simple instruments of the parallax wedge type, while opposing them are many workers who are of the opinion that single-dot instruments provide better results.

One of the arguments that one often hears in support of the single-dot instruments, particularly among workers who do not have have an extensive background in the fundamentals of stereoscopy, is that, all else being equal, the much finer least readings of the single-dot instruments ensures more precise results. On the face of things this is logical. However, as Spurr[4] states in his text:

> "Regardless of the type of measuring stereoscope employed, the ability of an observer to measure parallax is determined by the sensitivity of his eyes to minute changes in convergence. The average observer can detect differences in parallax of 0.002 of an inch; the highly skilled observer can detect differences of about 0.001 of an inch."

Thus, even though the parallax wedge is graduated in only 0.002 inch units, theoretically, when used by an average operator, it should not be less precise than a parallax bar graduated in 0.01 mm.

Unfortunately, the reasoning behind Spurr's statement is not given. In an attempt to obtain an explanation, a search for information was made in the literature. This search yielded several hints but in no case was the reasoning carried through. Consequently, the problem was analyzed independently. The solution is presented here, not as an original contribution, but rather as an explanation for an otherwise puzzling statement.

A human sees stereoscopically by the simultaneous viewing of an object or field of view by a pair of normal eyes. Since the eyes are separated from one another each has a different perspective view. When these different views are superimposed in the viewer's brain, a sensation of depth results[1,6]. If both images were identical, no stereoscopic effect would be noticed, even if both images were superimposed. The differences in the two images can be measured on the focal plane by the use of coordinate systems based on the principal points or optical centers of the images and the line connecting the two principal points [5,6]. These differences or displacements on the focal plane are manifestations of changes in the parallactic angle θ (Figure 1). Von Gruber[6] has related changes in eye-object distances to changes in the parallactic angle using the following approximation formula:

$$dh = \frac{-h^2 d\theta}{b}$$

The exact expression can be derived as follows:

$$h = b \cot \theta$$

$$\frac{dh}{d\theta} = b(-\csc^2 \theta)$$

$$= -b(1 + \cot^2 \theta)$$

$$= -b\left[1 + \left(\frac{h}{b}\right)^2\right]$$

$$dh = -b d\theta - \frac{h^2 d\theta}{b}$$

A modification of this latter formula can be used to determine the limit of parallax perception on a stereoscopic pair of aerial photographs. Assuming the "normal" case, both photographs truly vertical and taken

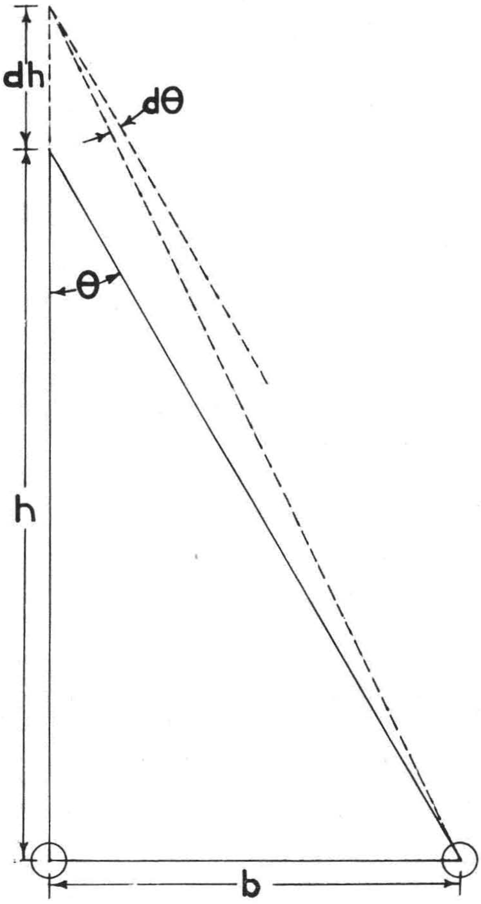

FIG. 1. The parallactic angle.

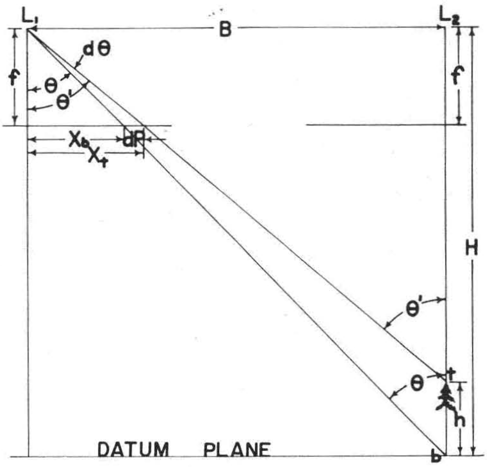

FIG. 2. The parallactic angle with vertical aerial photographs.

at precisely the same elevation above the datum plane[2], every plane parallel to the datum plane has its own unique absolute parallax. Thus absolute parallax can be measured at any point in the plane. Following this reasoning, it becomes possible to set up the condition shown in Figure 2 and accept the results as being general in application. Von Gruber's formula can be applied to this aerial photographic condition as follows:

$$\tan \theta = x_b$$

$$\frac{dx_b}{d\theta} = f(\sec^2 \theta)$$

$$= \frac{f^2 + x_b^2}{f}$$

$$dx_b = \left(\frac{f^2 + x_b^2}{f}\right) d\theta$$

$$= dP \text{ or differential parallax}$$

According to von Gruber (6) humans can detect changes in the parallactic angle (dJ) as small as 10 to 30 seconds of arc. Average individuals can detect changes of approximately 20 seconds. If this is the case, assuming a camera focal length of 8.25 inches, a format of 7×7 inches, and a standard endlap of 60 per cent, the average individual can detect differences in parallax (dP) of approximately 0.0010 inches. With 9×9 inch photographs, this is increased to 0.0011 inches. Thus, it can be seen that if a parallax measuring instrument is graduated in units finer than 0.001 inch it will not necessarily be more precise than one graduated in 0.001 inch units.

LITERATURE CITED

1. Nowicki, A. L. 1952. "Elements of Stereoscopy." *Manual of Photgrammetry,* 2nd Ed. Amer. Soc. Photogrammetry. Washington, D. C.
2. Sander, W. 1932. "The Development of Photogrammetry in the Light of Invention, with Special Reference to Plotting from Two Photographs." *Photogrammetry, Collected Lectures and Essays.* American Photographic Publishing Co. Boston, Mass.
3. Schlatter, E. J. 1946. "The Variable Elevation Wedge." PHOTOGRAMMETRIC ENGINEERING, 12: 354–359.
4. Spurr, S. H. 1948. *Aerial Photographs in Forestry.* Ronald Press Co., New York, N. Y.
5. Tewinkel, G. C. 1952. "Basic Mathematics of Photogrammetry". MANUAL OF PHOTOGRAMMETRY. 2nd Ed. Amer. Soc. Photogrammetry. Washington, D. C.
6. von Gruber, O. 1932. "Stereoscopic Vision and Measurement." *Photogrammetry, Collected Lectures and Essays.* American Photographic Publishing Co. Boston, Mass.