Cross-Bases Method in Aerial Triangulation

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ABSTRACT: *In this paper, the author gives the outlines of a relatively new method in aerial triangulation especially suitable for regions without geodetic foundations. The terrestial elements necessary for the triangulation and compensation of the aerial strip are kept to a minimum, and can easily be determined without being dangerously affected by eventual deviation of the vertical. Thus, this proposed method has a considerable advantage over the classical methods used in aerial triangulation in such regions.*

IN THE published doctoral thesis "Fehlerfortpflanzung und Ausgleichung von Aerotriangulationsstreifen mit Gemessenen Querstrecken" which appeared in Zurich, Switzerland in 1956, this relatively new method of aerial triangulation is thoroughly explained. Such a method may interest some American engineers, especially those concerned with regions where no geodetic surveying exists. Accordingly the author in this paper gives the outlines and features of the said method.

The Cross-Bases Method is especially recommended for regions without geodetic background. Because of the dangerous effect of the deviation of the vertical, which generally is not known in such regions, and which may result in extreme errors in the astronomically determined coordinates, (the errors can be more than one mile) it is obvious that any triangulation or compensation based only on such values cannot yield satisfactory results. On the other hand, this deviation of the vertical has a rather small effect on the astronomically determined azimuths of local lines (max. about several seconds). Due to this interesting and important fact an attempt was made to use local lengths and azimuths instead of coordinates in furnishing the terrestial material necessary for the triangulation of the strip and its compensation.

In the mentioned German publication of the author, the minimum requirements for the method are shown as:

A. TWO CROSS-BASES, one at the beginning and the other at the end of the strip, transverse to the flight direction (*B*1 and *B*2 in Figure 1). Each cross-base is to be determined by measuring: (1) Its length L, (2) Its azimuth A, (3) The height difference between its ends.

B. THREE ABSOLUTE HEIGHTS, one at the beginning, at the middle and at the end of the strip (Figure 1).

OR

TWO GROUPS OF HEIGHTS one of each at the beginning and at the end

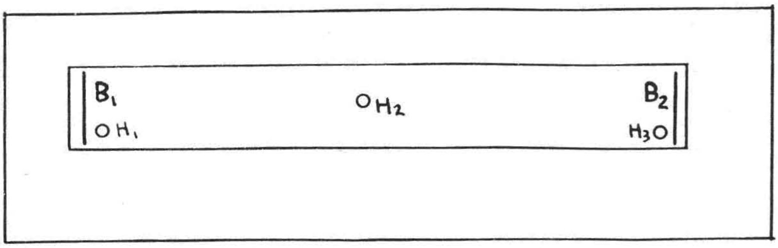

FIG. 1

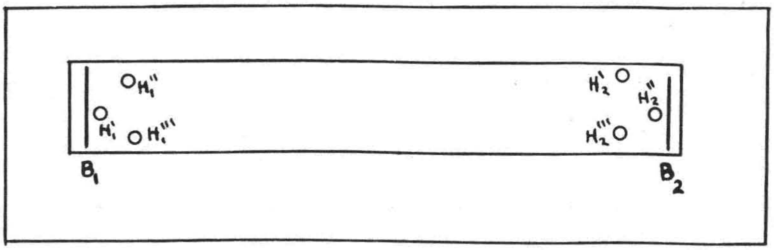

FIG. 2

of the strip, making it possible to determine the longitudinal tilt at the two ends of the strip (Figure 2).

The accuracy with which the above mentioned elements must be determined is explained in the following paragraph.

The heights of the control points could be determined easily using barometers. The Cross-Bases Method is applied mainly for small-scale problems, where relatively high flight heights are chosen, and where the accuracy of a stereoscopic determination of the heights corresponds with that of good barometers in determining the absolute heights.

For the *height differences* between the two ends of the cross base, an ordinary differential leveling should be sufficiently accurate.

As the error in determining the swing (K), for the relative orientation of the two pictures in order to have a plastic model, can be in the order of one minute to one and one-half minutes, the *azimuths* of the cross bases need not be determined with greater accuracy. For this purpose an ordinary transit can be used satisfactorily. The accuracy of the determinations of the *lengths* of the cross-bases need not be greater than that for determining these lengths stereoscopically. In general, an ordinary traverse should provide the necessary accuracy.

PRINCIPLES OF THE CROSS BASES METHOD

As in any triangulation method, there are two steps; namely the stereoscopic triangulation on the stereo-machine and the adjustment step. The stereotriangulation is carried on in the normal manner except that the *absolute orientation* of the *first pair* of the strip is *not necessary*. For instance the triangulation could be carried on with the first photograph horizontal and without any swing. The strip adjustment takes into account the effects of neglecting the absolute orientation of the first model. This is shown later.

From the stereoscopic coordinates and the heights of the critical points (the end points of the cross-bases and the height points), the following elements could be determined for each cross-base: its length Ls, its azimuth As, and its lateral tilt Ωs.

The errors in scale, azimuth and lateral tilt of each base is then deduced by comparing the terrestially determined values with those ascertained from the stereoscopic triangulation. These errors are defined as follows:

$$\Delta M = \frac{Ls - L}{L} = \text{Scale error}$$

$$\Delta K = As - A = \text{Azimuth error}$$

$$\Delta\Omega = \Omega s - \Omega = \text{Lateral tilt error}$$

If the determined heights permit determining the longitudinal tilt at both ends of the strip, the following error will also be available:

$$\Delta\Phi = \Phi_s - \Phi = \text{error in longitudinal tilt.}$$

If this is not the case, we just determine the errors in absolute heights:

$$\Delta H = Hs - H = \text{error in absolute heights.}$$

The theoretical and the practical investigations have shown clearly that the greatest part of the effect of the systematic as well as of the accidental errors in Photogrammetry seems to be systematic. This important fact permits drawing the diagrams in Figure 3 and to deduce the given relationships.*

Once the factors (dM_0, δM, $d\kappa_0$, $d\Delta\kappa$, $d\omega_0$, $d\Delta\omega$, $d\phi_0$, $d\gamma$) are determined, through use of these diagrams, the adjustment of the stereoscopic triangulation is carried out

* In all the diagrams and formulae: b is the mean base length, Z is the mean flight height above ground. The elements of orientation of the model are given in the symbols normally used. R is the earth radius.

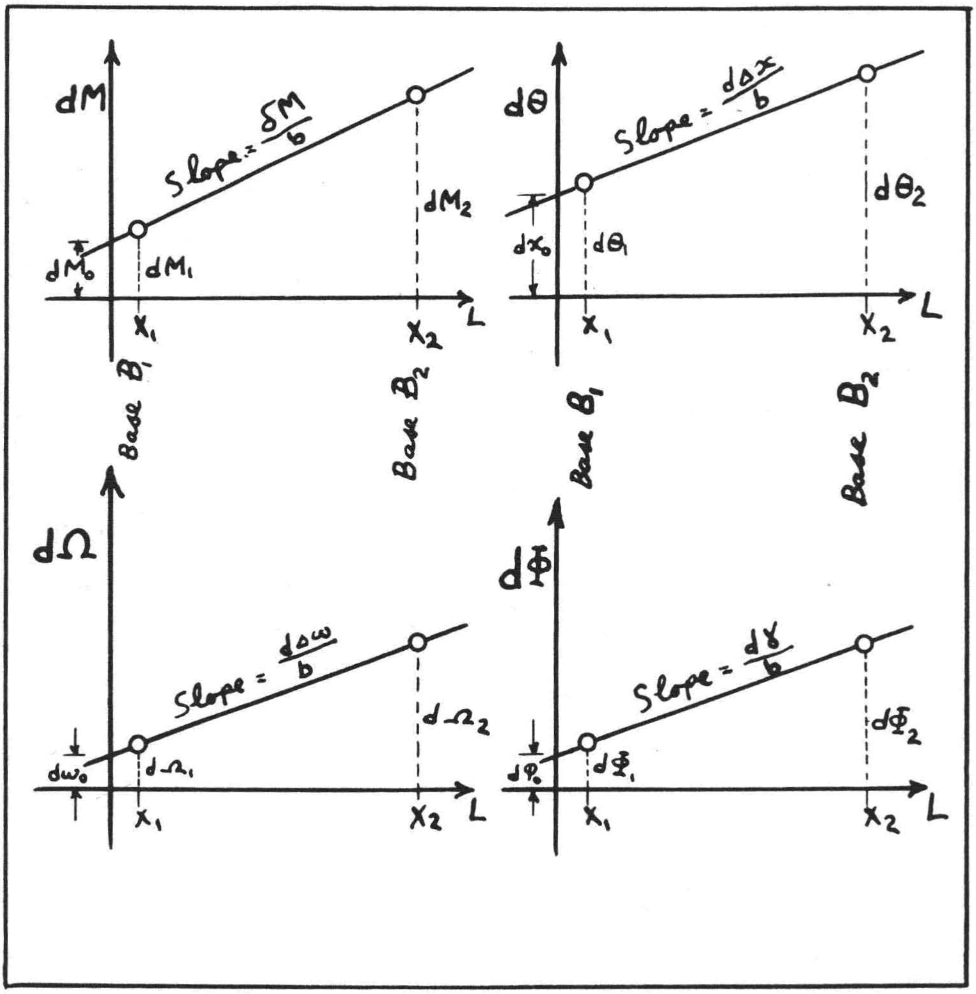

FIG. 3

very easily with the aid of the following formulae:

$$\Omega Xp = x\left(dM_0 - \frac{\delta M}{2} - \frac{d\kappa_0{}^2}{2} + \frac{d\kappa_0 \cdot d\Delta\kappa}{2} - \frac{d\Delta\kappa^2}{12} - \frac{d\phi_0{}^2}{2} + \frac{d\phi_0 \cdot d\gamma}{2} - \frac{d\gamma^2}{12}\right)$$

$$+ x^2\left(\frac{\delta M}{2b} - \frac{d\kappa_0 \cdot d\Delta\kappa}{2b} + \frac{d\Delta\kappa^2}{4b} - \frac{d\phi_0 \cdot d\gamma}{2b} + \frac{d\gamma^2}{2b}\right)$$

$$+ x^3\left(-\frac{d\Delta\kappa^2}{6b^2} - \frac{d\gamma^2}{6b^2} - \frac{1}{6R^2}\right) + x \cdot y\left(-\frac{d\Delta\kappa}{b}\right) + y(-d\kappa_0)$$

$$\Delta Y_p = x(d\kappa_0) + x^2\left(\frac{d\Delta\kappa}{2b}\right) + y(dM_0 - \delta M) + y^2\left(-\frac{d\Delta\kappa}{b} - \frac{d\Delta\omega}{2Z}\right) + x \cdot y\left(\frac{\delta M}{b}\right)$$

$$\Delta H_p = (-z[dM_0 - \delta M]) + x\left(\frac{-z}{b}\,\delta M - d\phi_0 + \frac{1}{2}\,d\gamma\right) + x^2\left(-\frac{1}{2R} - \frac{d\gamma}{2b}\right)$$

$$+ x \cdot y\left(\frac{d\Delta\omega}{b}\right) + y\left(z \cdot \frac{d\Delta\kappa}{b} + d\omega_0 - d\Delta\omega\right)$$

In the above equations, X_p, Y_p & H_p are the coordinates and heights of any observed point p. The adjusted coordinates of the point p will be $(X-\Delta X)$, $(Y-\Delta Y)$ & $(H-\Delta H)$.

If the heights measured in the field do not permit the direct determinations of $d\phi_0$ and $d\gamma$ from corresponding diagrams, the last mentioned values could very easily be computed by the aid of the formula for ΔH_p and three measured heights distributed along the whole strip.

Although the stereoscopic triangulation and the compensation could be accomplished with the minimum requirements as given, it is advisable and strongly recommended that more than these minimum requirements be measured in order to take into account, more or less, the possible irregularities in the propagation of errors.

Practical Results

A statement of the accuracy attained by the proposed method may be of some interest.

A strip about 45 miles long with a flight height 15,000 feet above the ground, using camera WILD RC 5a with plate adapter, f = 115 mms, photo 18/18 cms, with a photo-scale of about 1:43,000 and containing 22 models with a 60 per cent overlap, was triangulated by the author in 1956 using the WILD A7 of the Swiss Federal Institute of Technology in Zurich, Switzerland. The adjustment was carried out by using the proposed method. Forty two control points were terrestrially determined to check the values given by the said adjustment. The following results may give an idea about the accuracy attained by such a method:

Maximum error in position	= +52 feet
Mean error in position	= ±25 feet
Maximum error in height	= −30 feet
Mean error in height	= ±12 feet

In these results no mean square errors are mentioned. Most of the errors in Photogrammetry *are* or *seem* to be of systematic character or in other words having systematic effect. Accordingly in the opinion of the author mean square errors—built on the assumption that the errors are of accidental character and follow certain rules—should not be used as a criterium of accuracy.

The above mentioned errors remaining after the adjustment of the strip are within the permissible errors in small-scale mapping. For example, with scales of 1:50,000 or 1:100,000, the maximum error in position, remaining after the proposed treatment, is still within the drawing accuracy of maps, which can be given as +0.015 feet.

Advantages of the Cross Bases Method and Limits of its Use

Compared with the classical methods used in aerial triangulation and its compensation, the Cross-Bases Method has the enormous advantage that it is not dangerously affected by the eventually undetermined deviation of the vertical. In regions without geodetic background, this method is the only one—to the best of the author's knowledge—that can yield acceptable results.

In regions where the deviation of the vertical is determined, and where the terrestrial material necessary for adjusting the triangulation can be properly determined, the classical methods yield greater accuracy than the proposed ones. This should not be at all surprising as clearly any adjustment using direct coordinates is more accurate than computing the adjustment to these coordinates using the errors in scale, azimuth tilt etc.

A combination between the old and the proposed method, in regions where the vertical deviation is known, yields—without any doubt—an accuracy superior to that of any single method.

Obviously the Cross-Bases Method, as herein proposed, can deliver only free coordinates, i.e. the coordinates given for any point are not related to any terrestrial grid. In regions without geodetic background, one must be satisfied with the free coordinates because any astronomically determined coordinates could be dangerously falsified by the deviation of the vertical. The free coordinates in no way hinder full use of the triangulation and of the photos.

In regions with geodetic survey, obviously the usual grid coordinates could be deduced very simply out of the strip coordinates by means of simple transformation.

Reference

Karara, H. M., "Fehlerfortpflanzung und Ausgleichung von Aerotriangulationsstreifen mit gemessenen Querstrecken." Published by Dissertations Buchdruckerei Leemann AG, Zurich, Switzerland, 1956.