*Photograph and Map Transformations**

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ABSTRACT: *This paper presents a method and associated formulas for computing and accomplishing transformations of any type. This method goes beyond the normal and wide-angle lens photography applications and includes map transformations as well as narrow-angle photographs. All formulas have been derived as functions of the tilt and simplify the construction of a working guide for accomplishing the transformations in a photographic laboratory. The range of these formulas is,* $o < f \leq \infty$, $o \leq t \leq 90°$ *and any scale range encountered. The author recommends the possibility of designing an aerial camera lens to accomplish the affine transformation at the time of exposure. This article also gives a more detailed explanation of the material presented in the ACIC exhibit at the Annual ASP-ACSM Meeting, March 1956.*

1. Highly tilted wide and normal-angle photographs can usually be rectified in two steps where the affine transformation is introduced with negative offset or lens tilt. The purpose of this paper is to present a simplified laboratory procedure which will permit the rectification of long focal length, ultra narrow-angle photographs in only two steps. With the conventional methods outlined in the MANUAL OF PHOTOGRAMMETRY, seven or more steps would often be required.

2. The major difficulty encountered in rectifying long focal length, ultra narrow-angle photographs is in accomplishing the required affine transformation. As used here affine transformation means the lengthening of the principal line while maintaining the width perpendicular to the principal line at the principal point. Usually it is more convenient to speak of the ratio of this width to length as the affine transformation ratio. In the method presented herein, the affine transformation is accomplished while reducing the original photograph or map so that the negative can be accommodated in a conventional rectifier, such as the Bausch & Lomb which has a 9×9 inch negative carrier. The methods of rectification on the B&L, as described by the author in a paper in the September 1953 Issue of PHOTOGRAMMETRIC ENGINEERING, can then be applied.

3. The Variable Perspective Camera, shown in Figure 1, can satisfactorily perform an affine transformation ratio, $y = 0.30$. At the present time, a 70 inch process lens is used with a 30-inch diameter, 15-foot focal length spherical mirror. Figure 2 shows the relationships of the elements of the Variable Perspective Camera. Descriptions of other uses of this type equipment have been published previously.[1]

4. Theoretically, there is no limit to the affine transformation that can be performed with the Variable Perspective Camera. However, there is a practical limit in that the detail is compressed to such an extent that the resolution of the final product is only fair. Photographs requiring a severe compression are so highly tilted that the detail on the original photograph is barely adequate for mosaic compilation or photo-interpretation. A second limiting factor is the difficulty in keeping the copy material absolutely flat. A

[1] Merriam, Mylon, "The Control of Image Perspective as Applied to the Production and Photography of Terrain Models," PHOTOGRAMMETRIC ENGINEERING, XVII, 1951, p. 50–58; Merriam, Mylon, "A New Photo Method for Measuring Three Dimensional Objects," PHOTOGRAPHIC ENGINEERING, VII, 1956, p. 82–89.

* USAF Headquarters have cleared this paper and the Figures for publication.

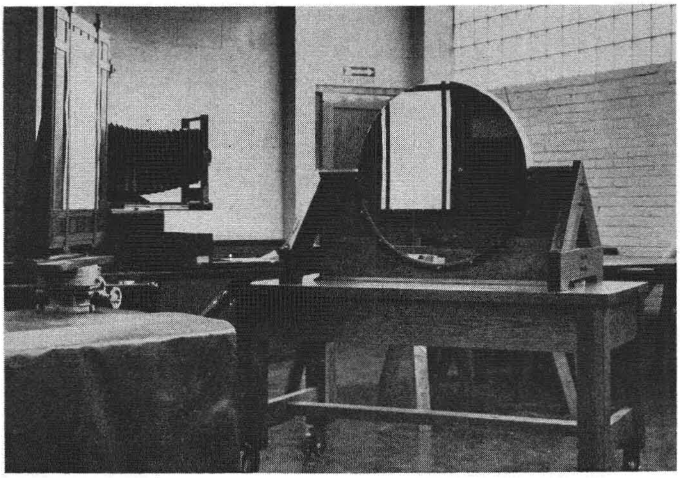

FIG. 1. Elements of a variable perspective camera.

third limitation is that a few degrees of convergence of the sides of the photograph is introduced into the negative when the Scheimpflug condition of focus is satisfied. A few degrees of convergence can usually be removed in the second step on the B&L Rectifier. Even with the limits enumerated above, the Variable Perspective Camera is still an acceptable arrangement for obtaining a reduced negative containing the required affine transformation with existing equipment.

5. The formulas and computations given below contain no limiting factor in the computations and the affine transformation diagram shown in Figure 3 graphically represents the requirements of any transformation. The accuracy of the formulas is dependent only on the number of significant figures used in computing. Usually, five-place trigonometry tables with all other measurements to four significant figures will suffice for any practical computations. The negative y values have no practical use.

The four formulas required for computing any type of transformed print pattern and the associated affine transformation diagram are as follows:

a. Angle of convergence, α

$$\tan \alpha = \frac{w}{f} \sin t$$

b. Affine transformation ratio of width to length of ground pattern at the principal point.

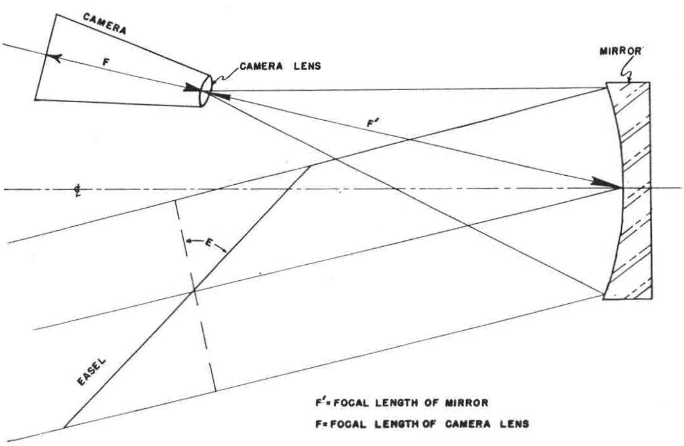

FIG. 2. Top view diagram of a variable perspective camera arrangement using a mirror.

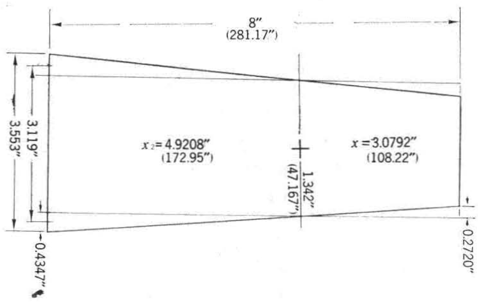

FIG. 3. Transformed print pattern with related data.

$$y = \frac{W'}{L'} = K \sec t(k_1 + k_2 \cos 2t)$$

$$K = \frac{w}{L} \quad k_1 = \frac{1}{2}\left(1 - \frac{L^2}{f^2}\right), \quad k_2 = \frac{1}{2}\left(1 + \frac{L^2}{f^2}\right)$$

$$k_1 + k_2 = 1$$

c. Distance of the principal point from one edge of the transformed print.

$$x_1 = \frac{C}{2}\left(1 - \frac{L}{f} \tan t\right)$$

$$x_2 = \frac{C}{2}\left(1 + \frac{L}{f} \tan t\right)$$

$$x_1 + x_2 = C$$

(A constant length of the diagram usually 8" for convenience)

d. Tilt formula.

$$\tan \beta = \tan t' = \frac{F(R-1)}{L(R+1)}$$

where

W' = Width of the ground pattern at the principal point
L' = Length of the ground pattern along the principal line
w = Half width of the photograph perpendicular to the principal line
L = Half length of the photograph in the direction of the principal line
f = Focal length of the taking camera
F = Focal length of the rectifier used in the second step
t = Air tilt of the photograph
$t' = \beta$ = Easel tilt of the rectifier used in the second step
R = Ratio of the larger width to the smaller width of the ground pattern
y = Ratio of the width to the length of the affine transformation diagram. (Also ratio of the width to the length of the ground pattern at the principal point.)

6. The diagram shown in Figure 3 was computed for an equivalent focal length of 100 inches, an air tilt of 79 degrees (11 degree depression angle), and the same data were used in preparing the materials shown in Figure 4. The computations for constructing the diagram and using the B&L Rectifier in the second step are assembled below.

a. Compute the affine transformation ratio in the following manner:

$$\frac{L}{f} = \frac{4.5}{100} = 0.045$$

$$\left(\frac{L}{f}\right)^2 = 0.002025$$

$$k_1 = \frac{1}{2}\left(1 - \frac{L^2}{f^2}\right) = \frac{0.9980}{2} = 0.4990$$

$$k_2 = 1 - k_1 = 0.5010$$

$$K = \frac{w}{L} = \frac{9}{4.5} = 2$$

$$y = 2 \sec 79° \left[0.4990 + 0.5010 \cos 158°\right]$$
$$= 2(5.2408)\left[0.4990 + 0.5010(-0.93358)\right]$$
$$= 10.4816\ (0.032) = 0.3355$$

b. Using the length of the diagram $C = 8$ inches, the half width of the affine transformation rectangle (w') in Figure 3 is

$$2w' = Cy = 8(0.3355) = 2.6840$$
$$w' = 1.342''$$

c. The angle of convergence of the ground pattern (α) is found is follows:

$$\tan \alpha = \frac{w}{f} \sin t = 0.09\ (0.98163) = 0.088347$$

and the remaining components are computed in the following manner:

$$x_1 = \frac{8}{2}\left(1 - \frac{L}{f} \tan t\right) = 4[1 - 0.045(5.1446)]$$
$$= 4(1 - 0.2302) = 4(0.7698) = 3.0792$$
$$x_2 = 4(1.2302) = 4.9208 \quad C = x_1 + x_2 = 8.000''$$
$$x_1 \tan \alpha = 3.0792(0.088347) = (0.2720''$$
$$x_2 \tan \alpha = 4.9208(0.088347) = 0.4347''$$

Only the four quantities, w', x_1, $x_1 \tan \alpha$, and $x_2 \tan \alpha$, are required to construct the diagram shown in Figure 3. This diagram is proportional to the ground pattern and the rectangular transformed negative required to accomplish the rectification on conventional rectifiers.

d. To find the dimensions of the transformed print at 1:1 when $h = f$, use the relationship

FIG. 4. Photograph of a panel in the ACIC exhibit at March 1956 ASP-ACSM meeting.

$$R_p = \frac{h}{f \cos t} = \sec t$$

for the ratio at the principal point. Then R_p times w (the half width of the original photograph) equals the half width of the transformed print at a scale of f/H. In Figure 3.

$$wR_p = 9 \sec 79° = 9.(5.2408) = 47.167''$$

and all other 1:1 distances are proportional to this ratio of $47.167/1.342 = 35.147$. These 1:1 distances are in parentheses in Figure 3. When other scales are required, merely multiply these distances by the appropriate ratio.

e. The easel tilt (β) of the B&L Rectifier is found by substituting into

$$\tan t' = \tan \beta = \frac{f(R-1)}{L(R+1)}$$

where

$$f = 5.5''$$

$$R = \frac{1.342 + 0.4347}{1.342 - 0.2720} = \frac{1.777}{1.070} = 1.660$$

$$L = 4''$$

$$\tan \beta = \frac{5.5(0.660)}{4(2.660)} = \frac{3.630}{10.640} = 0.3412$$

$$\beta = 18°50'$$

7. The computed y value is used for constructing the transformed print pattern. This value is also used as the minimum width at which the negative can be used on a rectifier such as the B&L and still maintain the correct x_1/x_2 ratio. The principal point of the rectified print will then be in the correct position when the proper convergence is attained. In most cases there is enough negative displacement in the B&L Rectifier for the image to move up the easel to its proper position. However, it is much easier for the operator to work nearer the bottom of the easel where the rate of change of scale is somewhat increased by the various movements of easel, enlargement and negative offset. For this reason, the width of the transformed negative is found by averaging the width at the principal point and the maximum width of the ground pattern. This width is computed for Figure 3 as follows:

$$2\left(1.342 + \frac{0.4347}{2}\right) = 2.684 + 0.4347 = 3.119''$$

or it can be measured on the diagram.

8. In practice, it is more economical to apply the formulas to one photograph of

average tilt of the flight for computing the affine transformation and to perform the rectification step to a templet containing map detail. Rectifying to a templet eliminates the computations required to place the principal line and to compensate for film shrinkage.

9. The tilt formula can be used for computations of highly tilted photographs if a nearly rectangular pattern of four scale check-lines using Anderson's scale-point method of placing the ratios is applied. The distance of these ratios from the principal point measured along the line connecting two opposite ratios is substituted into the following rearranged formula;

$$\tan t = \frac{f(R-1)}{L_s + L_L(R)}$$

where

$R = R_L/R_s$
R_L = Larger scale-check-line ratio
R_s = Smaller scale-check-line ratio
L_L = Distance from larger ratio to p along the principal line
L_s = Distance from smaller ratio to p along the principal line
f = Focal Length of taking camera, and
t = Air tilt

10. Figure 5 is a graph of affine transformation ratio y to air tilt for the conditions: $9'' \times 9''$ format photographs having focal lengths of 24 and 100 inches respectively. Most $9'' \times 18''$ format photographs also lie within the above range. (The ratio K in the formula need only be applied to $9'' \times 18''$ formats when constructing a transformed print diagram.) This graph shows that the affine transformation is a function of the focal length and tilt of the photograph. However, it is more directly related to the tilt than to the focal length. Therefore, a rule of thumb can be used to eliminate computations. For example, assume a 65 degree tilted camera installation. All photographs can then contain tilts ranging from about 60 to 70 degrees. One transformation ratio of about 0.4 would suffice for all of these tilts because any plus or minus correction can be introduced with negative offset (or lens tilt) on conventional rectifiers.

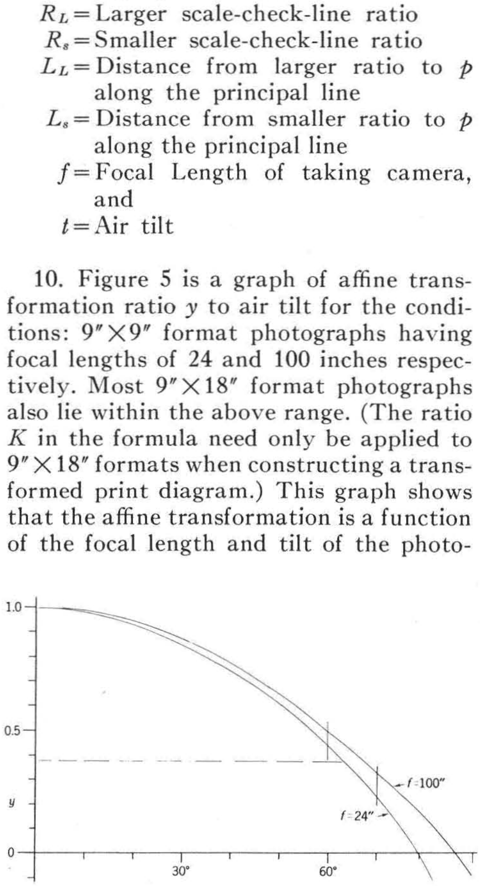

FIG. 5. Graph showing relation of the affine transformation ratio y to air tilt t.

11. The method outlined in this paper can be applied to photography of any focal length and is particularly useful for transforming photographs made by cameras of unusual design such as the K-30 or the Boston camera.[2]

12. An examination of Figure 5 suggests the possibility of designing an aerial camera lens which will accomplish affine transformation for negatives from narrow-angle oblique cameras installed at a constant depression angle, eliminate the need for special affine transformation equipment, and permit the photographs to be rectified on the B&L Rectifier in one step.

13. With these formulas the transformed print patterns can be precomputed using the simple functions of the tilt and the format size of the original photograph. In addition, these formulas can be applied to any changes in cartographic grid or projections with photo-mechanical means, thus handling a cartographic projection in the same manner as a photograph. Cartographic materials can be transformed from one geographic projection to any other in the following steps:

a. Establish a central point such as the intersections of the diagonals for nearly rectangular area and use this as the principal point.

b. Substitute the value of the focal length of the rectifier to be used in the final rectification step into the tilt formula given above.

c. Use the Anderson scale point method to position the ratios on the four scale check lines in a severe transformation such as from a Lambert Conformal to a Gnomonic Projection. (The areas transformed must be small enough so that the small difference in curvature of the latitude lines is insignificant.)

[2] Goddard, Col. C. W., "New Developments for Aerial Reconnaissance," PHOTOGRAMMETRIC ENGINEERING, Vol. XV, 1949, p. 51–72.

D. Determine the magnitude of the required affine transformation by dividing the two ratios found at the intersection of the lines connecting ratios on opposite sides of the principal point.

14. The transformation of one conformal projection to another conformal of a different band can usually be performed on a conventional recitifier in one step because the required affine transformation can usually be accomplished by the available negative offset or lens tilt.

15. Transformation data such as that discussed in this paper are a prerequisite to the construction of servomechanisms on electronic transformation equipment. These data are required because there is no rigidity of perspective in transformation in electronic equipment such as that obtained in optical projection.

16. This paper furnishes information about a technique which makes possible the rectification of long focal length, highly tilted photography which is well beyond the limits of the conventional rectifier from a single negative in which the appropriate amount of affine transformation has been introduced. It presents a method of computing the affine transformation pattern whereby the correct amount of easel tilt can be determined. The paper also provides an effective means of transforming one map projection into a radically different projection. And finally, it issues a challenge for the development of a camera system which would accomplish the affine transformation in the taking stage.

*The "Flying Carpet"–A Stereoscopic Grid Used in Photo Interpretation**

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THE "Flying Carpet" is an instrument that makes use of a stereoscopic grid in photo interpretation work. The principle of such a grid is familiar to many photogrammetrists and photo interpreters. The instrument provides for the superimposition of a stereoscopic grid on the stereoscopic model obtained by viewing two vertical aerial photographs through a stereoscope. The grid furnishes the viewer with a horizontal plane of reference that can be raised or lowered within the stereoscopic model. An attached vernier provides a means for measuring differential parallax.

The "Flying Carpet" consists of a metal framework on which are mounted two plastic plates (Figure 1). Thumb screws control the movement of the plates in the X and Y directions. Each plate is scribed with a $\frac{1}{2}$-inch square grid. The grid lines are oriented in the X and Y directions and are alternately red and blue. On the right frame a vernier is mounted that records directly to the nearest 0.1 millimeter. The vernier records only the X-direction movement of the right plate. By adjusting the movement of the right plate while the left plate remains at a fixed position, the vernier can be used for measuring differences of parallax, or for setting the stereoscopic grid at selected elevations within the stereoscopic model. When viewing near the margin of the stereoscopic model, small adjustments of Y parallax are sometimes necessary.

The grid can be used under a mirror or mirror-prism-type stereoscope that accommodates an image separation of 7 to 12 inches.

Photographs are fastened to the table at a proper separation for stereoscopic viewing, with the principal and conjugate principal points in alinement. The instrument is then positioned over the photographs so that the center X-direction line of each

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