*Some Aspects of HIRAN-Photogrammetry**

DR. SIMO H. LAURILA,
Research Associate, Mapping and Charting Research Laboratory, Ohio State University

RECENT developments of SHORAN (Short Range Navigation) have proved its feasibility for application to medium and small-scale photogrammetric mapping. Shoran equipment consists of an airborne set and two ground sets, weighing about 100 kg. each. The airborne set transmits 0.8 microsecond duration pulses on the frequencies of 210 and 260 megacycles per second. Each ground station receives, amplifies and retransmits these pulses, after certain delay, back to the airborne set on a common frequency of 320 mc.-sec.

The signals arriving at the airborne set are indicated in an oscilloscope whose circular time base has the so-called "natural repetition frequency," viz. 93,109 c-sec; this is one half of the average propagation velocity of electromagnetic waves in the lower atmosphere—V = 186,219 miles-sec. Thus with repetition cycles of 93,109; 9,310.9, and 931.09 c-sec one complete revolution of time base corresponds to measured distances of 1, 10 and 100 miles respectively. HIRAN (High Accurate Shoran) is a modified version of Shoran specially adapted to accurate surveying purposes.

In photogrammetry Hiran can be used in two ways: a) to establish a primary network of control points for large block triangulations; b) to introduce control point extension in areas where dense network is needed. The sides of a large block triangulation may be tens or even hundreds of kilometers. To establish a complete adjustment for such a block only nine primary control points are needed as pointed out by Brandenberger (1). In Figure 1 an ideal network is represented in which all possible distances are measured by Hiran. The number of conditions for the adjustment of the network is given by the formula

$$C = L - 2P + 3 \qquad (1)$$

where C is the number of conditions, L is the number of measured lines and P is the number of points. In this case there would be 21 conditions and 36 measured lines. To make the work as economical as possible, an optimum number of lines to be measured must be selected to give the best conditions for the adjustment.

Each line is measured by the so-called "line-crossing method." An airplane at a known altitude H crosses the line several times following a figure eight path, which eliminates back lash errors in the Hiran dials, Figure 2. The arc distances, D_{R_A} and D_{D_A}, to both ground stations are measured simultaneously and recorded.

The sum of the measured distances as a function of the time of observation is a second order parabola, Figure 3

$$\Delta Y = AX^2 - BX + C \qquad (2)$$

where ΔY is the difference between a measured and an approximate sum distance, X is the time when measurements are recorded and A, B and C are constants to be determined by a least squares curve fitting method. After A, B and C have been calculated, the minimum ΔY is obtained by

$$\Delta Y = -\frac{B^2}{4A} + C \qquad (3)$$

Electromagnetic waves radiated by the antenna are bent towards the earth by atmospheric refraction Figure 2. The path of radiowaves is therefore not a straight line, but approximately the arc of a circle. At the same time, the propagation velocity of radio waves also changes as a function of this refraction. The radius of curvature

* This paper is a contribution to Commission I of the International Society of Photogrammetry. Prepared for presentation by Prof. Fred Doyle at The International Congress.

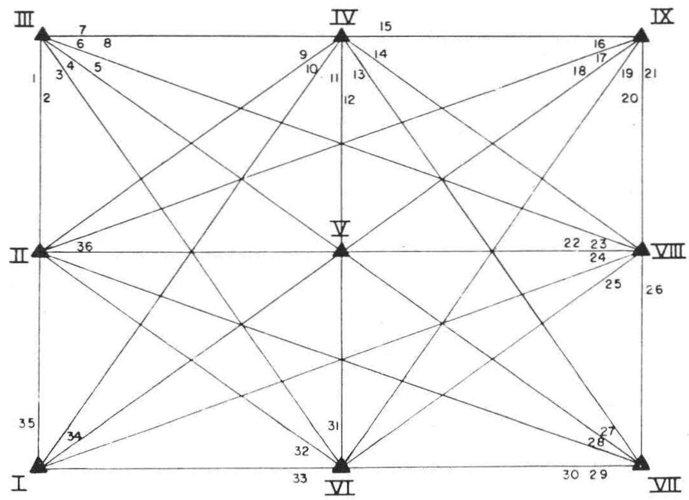

FIG. 1

r of the ray path is determined by the following formula

$$-\frac{1}{r} = B + C(H + Z) - \frac{CD_A^2}{6}\left(\frac{1}{R} + B + C(H + Z)\right) \quad (4)$$

where Z is the elevation of the ground antenna, H is the elevation of the airborne antenna, D_A is the arc distance between the ground and air stations, R is the curvature radius of the international sphere, B and C are empirically determined meteorological constants. Using the well known arc-chord equation

$$D_C = D_A - \frac{D_A^3}{24r^2} \quad (5)$$

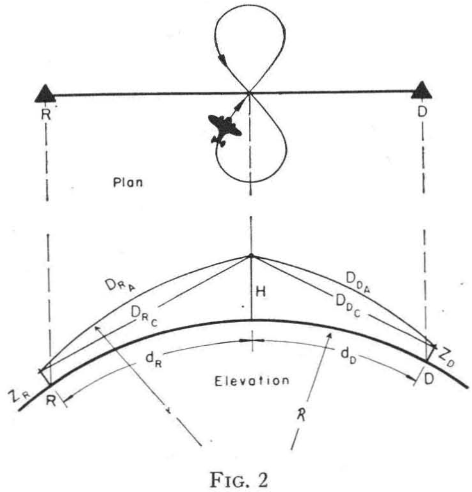

FIG. 2

the chord distances D_{R_C} and D_{D_C} are obtained.

As already mentioned the Hiran equipment has been constructed to correspond to an assumed velocity of electromagnetic waves $V = 186{,}219$ miles-sec. The actual velocity V_m for the measuring conditions is obtained from

$$V_m = \frac{V_0}{n} \quad (6)$$

where V_0 is the velocity in vacuo ($V_0 = 186{,}282$ miles-sec) and n is a refractive index along the ray path. This can be obtained by a formula derived by Essen (1951).

$$n = 1 + \left[\frac{77.62}{T}\cdot P - \left(\frac{12.92}{T} - \frac{37.19}{(T/100)^2}\right) e\right] 10^{-6} \quad (7)$$

where T is the temperature in absolute units, P is the pressure in millibars and e

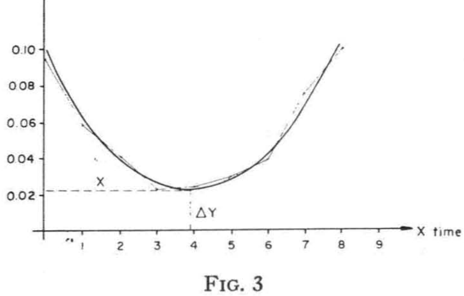

FIG. 3

is the partial pressure of water vapor in millibars. To obtain these elements in practice, the measuring airplane starts from one ground station, flies along the chord D_C to the measuring elevation H and makes meteorological observations at certain altitude intervals. After the mission the same process is repeated towards the other ground station.

The geodetic distances d are then calculated by the following formula

$$d=R_m \cdot \cos^{-1}\left[\frac{(R_m+H)^2+(R_m+Z)^2-D_C{}^2}{2(R_m+H)(R_m+Z)}\right] \tag{8}$$

where R_m is the curvature radius of the reference ellipsoid for the latitude and azimuth concerned.

The accuracy of line crossing measurements has been tested by extensive trials made in 1950–51 in Florida. [2]. The total error in Hiran distance measurements can be divided into two classes, namely errors m_i independent of distances (instrumental and observation errors); and errors m_d dependent on the distances (propagation anomalies). These errors are mutually independent and thus the total standard error will be

$$M = \pm\sqrt{m_i{}^2 + m_d{}^2}. \tag{9}$$

The magnitude of these error components have proved to be

$$\begin{aligned} m_i &= \pm\ 0.0015 \text{ mile} = (\pm 2.4 \text{ meters}) \\ m_d &= \pm\ 1/70{,}000. \end{aligned} \tag{10}$$

At a distance of 100 kilometers the standard error of one line crossing thus will be about $M=\pm 3$ meters or ± 1 /33,000. At longer distances this proportional error will be correspondingly smaller.

For control point extension by photogrammetry the situation is essentially different. In this case there are two stationary ground stations from which the arc distances D_{R_A} and D_{D_A}, Figure 4, are measured and recorded at each exposure of the surveying camera. By this process the location of the aerial camera at the moment of exposure can be obtained.

The curvature radius of the ray path is obtained similarly to the line-crossing method. The velocity correction, however, is not obtained by direct meteorological observations, but is based on a determination from a standard atmosphere. The basic idea is to assume that the refractive

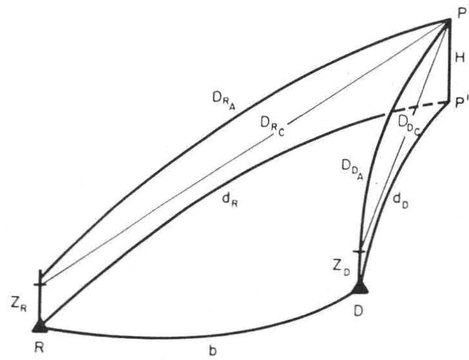

FIG. 4

index n changes with the elevation H according to a second order parabola,

$$n = 1 + A + BH + CH^2. \tag{11}$$

By using values obtained from Essen's formula (7) and by applying the least squares curve fitting adjustment, the meteorological constants A, B and C can be determined for any area in question. After these constants have been obtained, the following equation for a refractive index can be derived

$$\begin{aligned} n = 1 + A + \frac{B(H+Z)}{2} &+ \left[\frac{C}{3}(H+Z)^2 - HZ\right] \\ &- \frac{D_A{}^2}{12}\left[\frac{1}{R} - \frac{1}{r}\right]\Bigg[B + C(H+Z) \\ &- \frac{CD_A{}^2}{10}\left[\frac{1}{R} - \frac{1}{r}\right]\Bigg] \end{aligned} \tag{12}$$

From the spatial chord distances D_{R_C} and D_{D_C} the location of an exposure station can be obtained by three different ways, namely

a) by making calculations in space based on the Space Rectangular Coordinate System;
b) by reducing spatial elements first to the ellipsoid and then to a plane, on which the calculations may be made;
c) by constructing a circular Hiran coordinate system as a function of spatial chord distances and by superimposing this system on an existing rectangular grid.

Of these methods the first two are accurate, but if hundreds of points are to be located, the last method gives an economical solution of sufficient accuracy.

To investigate the accuracy of the Hiran

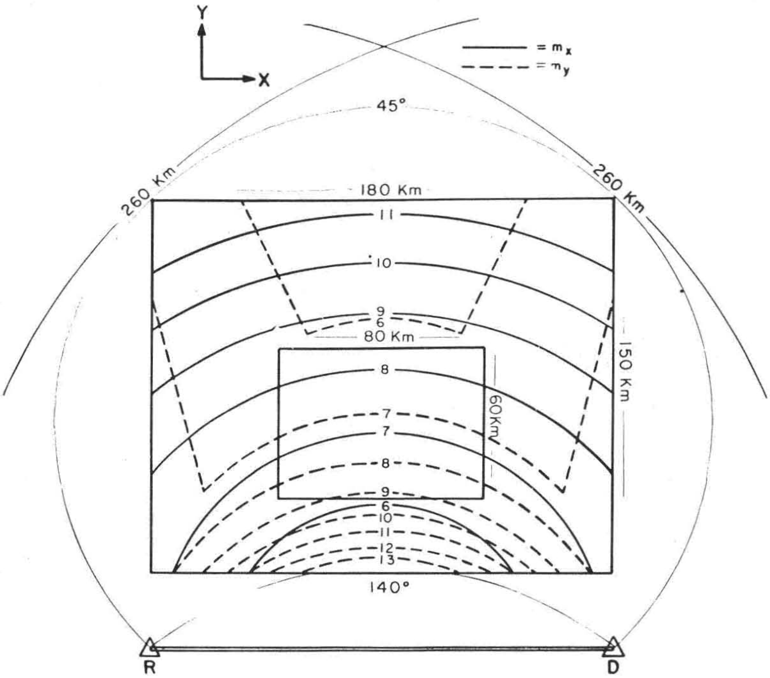

FIG. 5

control point extension a test flight was made in Arizona in 1952 at an altitude of 6,000 meters. The camera used was a Fairchild wide angle Metrogon camera with focal length f = 152. 53 mm. Every photograph contained about 8–16 well defined control points measured on the ground. Using these points a least squares adjustment was made, and the plane coordinates of ground nadir points were determined. The same coordinates were also obtained by the Hiran observations. Comparing these values in 17 photographs the standard error of one Hiran location was computed. Both meteorological observations and standard atmosphere assumptions were used, but since the test area was small compared to the measuring distances—about 150 km.—essential differences did not exist in the random distribution of errors. The standard error in one distance measurement at a distance of 150 km. [3]. proved to be

$$m_D = \pm 7.1 \text{ meters.} \quad (13)$$

In Figure 5 an analysis is made to establish the error propagation for X- and Y-components separately. The solid line gives the error component along the X- axis and the dotted line along the Y- axis. Two different areas are considered, 60×80 km. and 150×180 km. Figure 5 shows that the average standard error components all over both areas are about

$$m_X = m_Y = \pm 7.5 \text{ meters.} \quad (14)$$

Of course, a successful Hiran control point extension presupposes a method for the exact location of nadir points on the photographs. For this purpose the use of horizon-pictures or gyroscopic stabilization of the camera may be employed.

The use of Hiran for establishing the basic network for extensive photogrammetric mapping programs, especially in the so-called underdeveloped countries, should be carefully evaluated.

REFERENCES

[1] *Brandenberger, Dr. A.* "The Practice of Spatial Aerial Triangulation," Commissioned by the Photogrammetric Institute of the Federal Institute of Technology, Zurich, 1955.

[2] *Jacobsen, Carl E.*, "High-Precision Shoran Test Phase I," AF Techn. Report No. 6611, USAF, Wright Air Development Center, Wright-Patterson Air Force Base, Dayton, Ohio, July, 1951.

[3] *Laurila, Simo*, "An Investigation of Shoran-Photogrammetric Position Fixing," Techn. Paper No. 188, Mapping and Charting Research Laboratory, Ohio State University, November, 1954.