*The Spatial Model Concept of Photogrammetry**

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ABSTRACT: *Although photogrammetry is a well established field, photogrammetrists are constantly faced with the task of explaining the subject to other engineers and scientists. This paper presents some simple concepts of photogrammetry which the author has found useful in describing the subject to other engineers. A spatial model concept is developed and comparisons are made with conventional surveying.*

INTRODUCTION

DURING the past ten years, photogrammetry has developed into a field of considerable technical and professional importance. As a result of this growth, photogrammetrists are constantly faced with the task of explaining the subject to a growing audience of engineers and scientists. In many cases, the only explanation required is one sufficient to give the newcomer a general understanding and basic appreciation of the subject without creating an atmosphere of complexity and mystery. The purpose of this paper is to present some simple concepts of photogrammetry to serve as an aid in explaining the subject to others.

A BASIC DEFINITION

Photogrammetry is defined by the American Society of Photogrammetry as "the science or art of obtaining reliable measurements by means of photography." It is noted that this definition does not limit us to any one type of photography such as aerial photography nor to any one field of application, such as map making. *Photogrammetry is essentially a method of making spatial measurements.* By spatial we mean location, length, direction, size, shape, area, volume and similar dimensions, as contrasted with such dimensions as pressure, voltage and velocity. However, spatial measurements are often related to the determination of other variables, such as determining stress by measuring the spatial increment strain, or velocity by measuring time and spatial distance.

A BASIC CONCEPT

We may consider photogrammetry as a *means of creating a spatial model of the object photographed—a precision spatial model which can be measured.* This simple concept removes much of the mystery about photogrammetry. We merely create a model and measure it instead of the actual object of interest. Although our model may be mathematical, usually we create a three dimensional optical or optical-mechanical model of the object.

Essentially, photogrammetry is an analog process. By analog we mean the object being investigated is simulated by some other physical system. The use of electrical, mechanical, and hydraulic analogs is quite common in engineering practice for studying and measuring objects or conditions of interest.

APPLICATIONS

In general, we can say that photogrammetry can be used to measure any object or phenomenon which can be photographed. We usually find photogrammetry to be advantageous whenever it would be difficult or uneconomical to directly measure the actual object. For example, photogrammetry might be applicable if the

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object falls in one of the following categories:

(1) *Very large*—such as a portion of the surface of the earth
(2) *Very small*—such as a sand particle or metal surface roughness
(3) *Moving*—such as a structure under dynamic loading or a missile
(4) *Changing*—such as a hydraulic phenomenon or a glacier
(5) *Inaccessible*—such as an atomic particle in a cloud chamber
(6) *Complicated*—such as a deformed airplane wing or the human body

Our photogrammetric spatial model actually has four dimensions. In addition to the three linear dimensions of x, y, and z, the model has the important fourth dimension of *time*. The model is a true replica of the object photographed at a specific instance of time. If the object of interest is moving or changing, the time dimension is of paramount importance. Photogrammetry permits us to freeze the exact shape, size, and spatial location of the object at any selected time or time interval.

One of the important advantages of measuring with a model is that our work is conducted in the convenience of the laboratory instead of in the environment of the actual object. However, the photographs must be taken in the environment of the object under natural conditions. Therefore, the limitation on applying photogrammetric methods to a measurement problem often causes some difficulty in obtaining suitable photographs and not a breakdown in the photogrammetric method.

In summary, if we have an object which we would find difficult to measure directly but which can be photographed, we create a model of the object and measure the model. With this concept of where photogrammetry can be applied, the many novel applications of photogrammetry periodically reported are not so amazing, after all. In most cases, the object of investigation meets one or more of the six general object classifications listed above. As an application example, the problem of making spatial measurements of clouds in the high altitude jet stream was recently presented to D. R. Schurz of the M.I.T. Photogrammetry Laboratory by a meteorological research group. In this case, we have an object which is very large, moving, changing, inaccessible, and complicated all at the same time.

THE METRIC PHOTOGRAPH

In explaining the "how" of the photogrammetric method of making measurements, our first consideration is the nature of the photograph. Geometrically, the photograph is a perspective projection of the object photographed. In a perspective, all of the rays from the object converge to one point and the perspective projection is the trace of the intersection of the converging rays with a plane. The function of the lens of the camera is to cause all of the rays from the object to converge to one point. The plane of the photographic film or plate is the intersecting plane. The function of the photographic emulsion is to freeze the trace of the intersecting rays.

The metric photograph enables us to mathematically and physically recover the effective geometric point of convergence of the rays which formed the photograph. By recovering this point, we can recover and reproduce these rays, and hence the relative directions of and angles between all of the rays to every point on the object. We therefore say that the photograph is a *graphical record* of a set of directions or angles, and that the photogrammetric camera is an *angle recording* instrument.

A single photograph gives us the *direction* to each point in object space from the camera position, but not the *location* of points in space. A point in object space could be located by the intersection of two direction rays. This could be easily achieved by having two photographs of the same object from different camera positions.

It is observed that human vision geometrically operates in much the same manner. Each eye is similar to a camera, so our brain receives two separate and different "photographs" from two different camera positions. In the case of the two cameras and the two eyes, we are essentially triangulating distances. The distance between camera stations or eyes is the baseline, points different distances away subtending different angles opposite the baseline.

We can duplicate natural human vision by placing before each eye the corresponding photograph from two camera positions. This duplication is called stereovision and is an important part of the photogrammetric system since it enables us to *see* the model we have created. The average human eyebase is about 0.21 feet. This short baseline sets limits on our distance or depth perception ability. However, if we place photographs before the eyes which were taken 2,100 feet apart, we effectively expand our eyebase and extend our depth perception abilities by a factor of 2,100/0.21 or 10,000 times. This amplification of our human vision is an important factor in our ability to make very precise measurements with photogrammetry.

CREATING THE MODEL

When the photograph was formed, rays came from the object to the photograph. If we return to our camera position with a projector (which we may consider to be geometrically similar to the camera and which merely reverses the direction of the rays), the rays will return to their point of origin on the object. Similarly, if we have a second projector projecting the photograph from a second camera position, the rays from a given image point appearing in both photographs will intersect in space at their point of common origin. Under these conditions we will project a three dimensional model coinciding in space with the original object.

If we maintain the same relative orientation between the two projectors, but move them closer together along the baseline connecting the two camera stations, the rays from common image points will continue to intersect in space but will form a model smaller than the object. If we move the projectors apart, the model formed will be larger than the object. The ratio of the projector spacing to the camera spacing is the scale of the model. If the photographs were taken 3,000 feet apart and placed in projectors 1.5 feet apart, the projected model would have a scale of 1/2,000.

The spatial model formed by the projectors cannot be perceived by the unaided eyes. As mentioned before, each eye must be presented with a separate and different picture. Therefore, one eye is only allowed to see the picture from the first projector, and the other eye only the picture from the second projector. When this is done, the mind will fuse the two sets of images into a single spatial model.

THE NEED FOR CONTROL

We have seen how a spatial model of an object can be created with photographs. We cannot use the model for measuring the object unless we know the scale of the model and its location in an established reference framework. This is achieved by having dimensional control in the model. The need for control can be illustrated by comparison with conventional surveying. It was previously stated that a photograph is only a source of angular information. But angles alone do not define size. In triangulation, we know that we require the length of at least one side of a triangle (a baseline) in a network of triangles before we can compute the triangles. The same is true in photogrammetry. We have to measure at least one distance in our system in order to determine size or scale of the model.

A second basic requirement in any surveying system is horizontal orientation. What are the directions of our lines with respect to our spatial framework reference axes? Here again, as in ground triangulation, we need the horizontal orientation, or bearing, of at least one line in our model. These first two requirements, scale and orientation, can be satisfied by having two points in the model of known horizontal position. The distance between the two points establishes scale, and the bearing of the line between the two points establishes orientation.

A third consideration is the establishment of a datum plane in order that horizontal planes in the object will be horizontal in the model. Since it takes three points to determine a plane, we need the differences in elevation between three points in the model to define the datum plane. If we are compiling a topographic map and would like our elevations referenced to a standard datum such as mean sea level, we would need the mean sea level elevation of one of the three points. In summary, to locate our model within some established spatial framework, we would need at least two horizontal and three vertical control points within the model and referenced to the desired coordinate system.

We have spoken so far about an individual model formed by two photos, say *A* and *B*. However, photos *B* and *C*, *C* and *D*, *D* and *E*, etc, may form additional individual models which collectively form a continuous model of a large area. The individual models can be compared to individual triangles in a triangulation network. If we know everything about one triangle, we can compute the other triangles. Similarly, if we have oriented, scaled, and leveled one of the individual models, we can determine the spatial location and all points therein of the other models. Just as with ground triangulation, we can only continue this process of aerial or photogrammetric triangulation until our error accumulation becomes too large to tolerate. The important point is that we do not necessarily need "outside control" in every individual model.

MEASURING THE MODEL

The ultimate goal of most of our conventional surveying operations is the *xyz* coordinates of selected points, the graphical record of the *xy* position of selected lines, and the representation of relief by contour lines. Once we have created and controlled our photogrammetric model, we can achieve these goals directly without any intermediate measurements, computations, adjustments, or plotting. How this is done may be understood if you visualize that within the spatial framework of the model we insert a visible reference mark which can be freely moved in model space. The *xyz* position of the reference mark is so calibrated that the value of its position is always known. With this movable and calibrated reference mark, we have the perfect surveying tool. This may be compared to the surveyor having a little black box with three dials which always read the *xyz* spatial position of the box as it is carried from point to point.

When measuring the model, to locate a point on its surface, we merely place the reference mark thereon and directly read its value. To trace the plan position of a line, we merely move the reference mark along the line in the model. To run out a contour, we set the *z* value of our reference mark to equal the elevation of the desired contour. The mark is then moved until it touches the surface of the model. This is one point on the contour. Now if the mark is moved but constantly kept in contact with the surface of the model, we are tracing out the desired contour. A pencil point is coupled with the reference mark so that our point, plan line, or contour line is automatically plotted.

THE STEREOPLOTTER

There are three common approaches to solving engineering problems: (1) analytical or mathematical, (2) graphical, and (3) mechanical. By mechanical we mean with a machine, instrument, or analog. Most photogrammetric problems may be solved by any one or a combination of these approaches. For carrying out the concepts previously presented and in actual practice, we find the third approach to be the most feasible and use an instrument called the stereoplotter.

The stereoplotter is an instrument for *creating* and *measuring* the spatial model and *recording* the results. The functional components of the stereoplotter include (a) two or more projectors or other means of forming the spatial intersections of the reconstructed rays, (b) facility for moving these projectors along and around three mutually perpendicular axes to reconstruct relative and absolute orientation of the projectors, (c) means of separating the projections for the eyes, (d) a movable and calibrated reference mark, and (e) a plotting system. Although they all perform basicly the same function, there are a large number of different types and makes of stereoplotters available to the engineer and scientist.

PHOTOGRAMMETRIC ACCURACY

All measurement systems and their instrument components contain sources of error. We therefore design our measurement systems in such a way that we can (1) reduce the errors to a tolerable level, (2) correct for the errors by instrument design or measurement technique, or (3) make secondary measurements to enable us to compute the errors and correct the primary measurement. Of course we never completely eliminate all errors, but strive to reduce the total residual error until we obtain the desired precision and accuracy. In this respect, photogrammetry is no different from any other measurement system in that we are concerned with many sources of error. However, the photogrammetric engineer can reduce, correct, or compute these errors and so design any measurement project to obtain the desired accuracy.

Previously we presented a concept of photogrammetry as the creation of a spatial model of the object photographed and the measurement of the model. The accuracy of the method is therefore essentially concerned with our ability to create and measure the model. The accuracy with which we can create and measure the model depends on many factors such as the camera, lens, photographic materials, flight height, control, stereoplotter, human element, nature of the object, and the design of the over-all system. Errors originate from these and many other sources.

What is important is the value of all of the combined individual errors at the scale of the model. We therefore present a simple concept that the accuracy of our photogrammetric measurements is a function of the size of the model for a given system and set of conditions. As a hypothetical example, suppose at the scale of the model the total net residual error from all sources is 0.15 millimeters, 0.006 inches, or 0.0005 feet. If the object being investigated is 1,000 times larger (model scale 1/1,000), an error of 0.0005 feet in the model corresponds to an error a thousand times larger or 0.5 feet in terms of the object. For the case of our example, we could set up a table as follows:

If the Allowable Error Is	*We Need a Model With a Scale Of*
0.0001 feet	5/1
0.0005	1/1
0.001	1/2
0.01	1/20
0.1	1/200
1	1/2,000
2	1/4,000
5	1/10,000
10	1/20,000
20	1/40,000
100	1/200,000

If measurements of 1/10,000 of a foot are required, we would need a model five times the size of the object. If measurements to 10 feet are required, we could use a model 20,000 times smaller than the object photographed. The error in running a contour line is about twice that of a discreet point. Since the allowable error in a contour is one-half the contour interval, the model sizes given in the table must be multiplied by four if the numbers in the first column are contour intervals. Two foot contours would therefore require a model scale of approximately 1/1,000 for the hypothetical example.

It is obvious that there is no inherent limit to the accuracy of photogrammetric measurements in terms of object accuracy. In fact with microphotogrammetry, measurements to the order of microns at the scale of the object are quite possible if suitable photographs can be obtained. When people speak of limits for the accuracy of aerial photogrammetric mapping, they really mean there are minimum practical operating limits on the altitude of the airplane and associated problems in obtaining suitable photography.

PHOTO ANALYSIS AND INTERPRETATION

In this paper we have dealt entirely with basic concepts of the measurement or metric aspect of photogrammetry. In addition to recording angles, the photograph records qualitative information about the object photographed which is identified and interpreted by the photo analyst. Therefore, photo analysis and photogrammetry are closely allied but somewhat separate fields of endeavor, since the areas of knowledge involved are quite different. Mathematics and physics are basic to the photogrammetric engineer whereas geology, soils, and forestry are more important to the air photo analyst.

The nature and extent of the information and data which the highly trained professional photo analyst can obtain from aerial photographs borders on the fantastic. Soils, geology, drainage, land classification, vegetation, human activity are only a few of the many areas of data that can be obtained. It is not within the scope of this paper to discuss the qualitative aspects of photography, but it is important to note the distinction between the work of the photogrammetric engineer and the photo analyst even though there is some overlap between the two fields.

PROFESSIONAL PHOTOGRAMMETRY

Although the emphasis in this paper has been on simple concepts which give the newcomer an insight into photogrammetry, in theory and practice we are concerned with a subject which has extensive technical complexities. The photogrammetrist is called upon to have a working knowledge of many scientific and technical fields. Therefore, we classify photogrammetry as a narrow and highly specialized field but one involving broad professional considerations. A major step in the direction of professional recognition will occur when we remove some of the mystery surrounding the subject. It is hoped that the simple concepts presented in this paper will assist in obtaining a wider appreciation and understanding of the work of the photogrammetrists.