*Distortion Tolerance Specification for Mapping-Camera Lenses**

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ABSTRACT: *A very brief review is given of the status of lens distortion compensation. A method of analyzing the effect of lens distortion upon a stereomodel is presented. Data on distortion characteristics of Planigon lenses are presented, and conclusions are stated as to a suitable distortion tolerance for mapping camera lenses*

THE problem of "how much distortion we can tolerate in a mapping camera lens before it becomes a source of annoyance and loss of accuracy" has existed for a long time and no doubt will continue to be a source of uneasy concern for some time to come. Perhaps this is a desirable situation, for as many know, when a warped stereomodel that won't lie flat has been examined by a cluster of perplexed experts, and all logical reasons for the delinquent attitude of the model have been exhausted, we can always sadly shake our heads and blame it on uncompensated distortion in the aerial camera lens.

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But seriously, occasionally a new mapping instrument, technique or camera lens is introduced which requires re-evaluation of the significance of the phenomena of lens distortion on our proposed mapping system. The arrival of the so called "distortion-free" aerial mapping lens on the photogrammetric scene—specifically, the Planigon lens—is the principal reason for the evaluation discussed in this paper.

For a long time we have lived with aerial camera lenses which have considerable distortion, and we have met the problem by devising various optical, mechanical, and mathematical methods for annulling the distortion. However, as a rule these corrections have been based upon a characteristic "average" or "theoretical" curve representing a type of lens rather than individual lenses. Unfortunately, no two lenses of the same type have identical distortion characteristics and a certain degree of uncompensated residual distortion will exist when a specific lens is corrected by a device based upon a hypothetical lens. For example Figure 1 illustrates the distortion curves for two actual metrogon lenses along with the accepted theoretical distortion curve for the metrogon lens. If a device based upon the theoretical curve is used to remove the distortion from the individual metrogon lenses shown, the result will be a certain amount of residual distortion in our mapping system. This is illustrated in Figure 2.

* Presented at the 22nd Annual Meeting of the Society, Hotel Shoreham, Washington, D. C., March 21–23, 1956.

At the time of presentation the author was associated with ERDL. He is now Executive Vice President of O.M.I. Corporation of America.—*Ed.*

We have been accepting this residual distortion in our photogrammetric processes for some time—and those who have been using multiplex equipment or less accurate mapping methods, have not been unduly troubled—for a good reason. This amount of residual distortion did not cause sufficient error to hamper our mapping efforts seriously. But now (due to economic and military pressure) we find increasing emphasis on obtaining maximum performance from

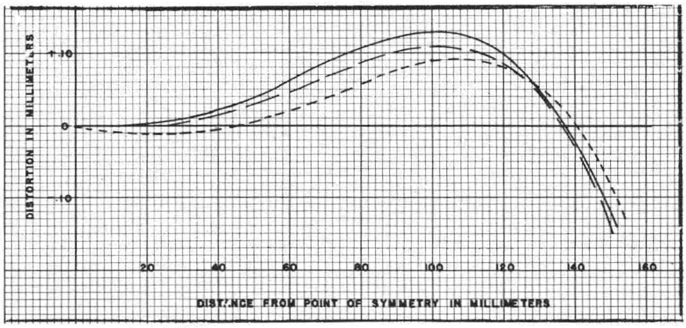

FIG. 1. Radial distortion of three metrogon lenses.

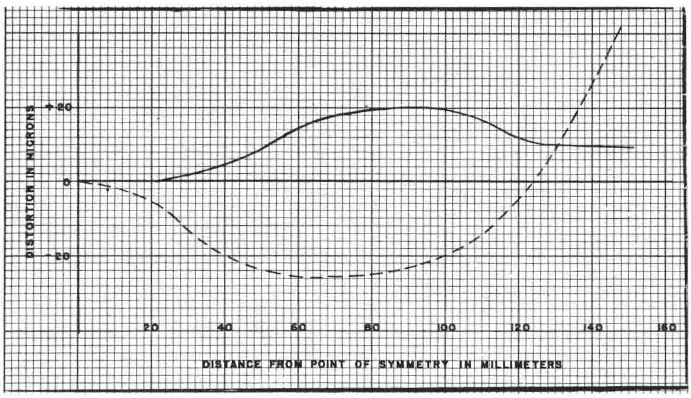

FIG. 2. Residual metrogon lens distortions.

photogrammetric equipment. The growing use of first-order European equipment in this country, the introduction of domestic precision instruments such as the ER55 and the Kelsh Plotter, and the development of increasingly stable film bases all require that we take another look at this uncompensated residual distortion that we are tolerating.

At the same time we can examine the new "distortion-free" Planigon lens and determine just how free of distortion it is. In this case our problem is simplified in that no correction devices are required. The theoretical curve is zero distortion—and any distortion displayed by the individual Planigon lens may be considered as residual uncompensated distortion. Figure 3 illustrates extremes in Planigon distortion, that is, what we consider a good, low distortion curve and a rather poor (for distortion-free classification) high distortion curve. The ideal curve of course is the X axis, where Y is zero for all values of X.

It may be noted that throughout this paper the reference is sometimes made to residual lens distortion, and at other times to just lens distortion. To clarify this, it should be made evident that we are actually concerned with the amount of distortion that is not compensated for. Where distortion-free photography is concerned, there is no correction by virtue of the definition, and in this case we are concerned with the total distortion of the lens. However in the case of Metrogon photography, we refer to the discrepancy between the actual distortion curve of the lens and the distortion values for the correction device, whether it be an aspheric plate, a ball cam, or another lens.

Large mapping organizations, for example the Corps of Engineers, must be prepared to map with photography taken with a rather large number of aerial cameras, even if of the same type. Designing a specific correction device for each lens is neither feasible nor practical. Obviously then attempts must be made to limit the variation in distortion characteristics of lenses of a single design, to a point where individual lens variation from a mean or theoretical value contributes insignificant error to the mapping system.

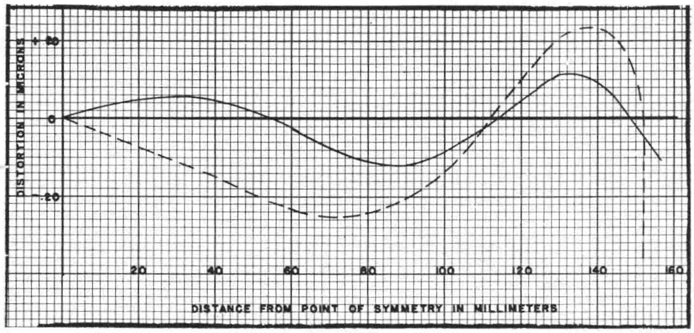

FIG. 3. Radial distortion of two planigon lenses.

A search in literature reveals very little published data on distortion tolerances for mapping camera lenses. There is considerable treatment of the effect of lens distortion on the stereomodel, but the problem of minimizing this source of error through the establishment of a suitable tolerance has been relatively neglected. The report of Commission I of The International Society of Photogrammetry to the Seventh International Congress[a] reveals that the French, in acceptance testing of new lenses, require that the average distortion should not depart by more than 16 seconds (sexagesimal) from that of a standard lens of the same type, and that there should not be more than 16 seconds variation in distortion at any given field angle. The English permit a distortion tolerance of plus or minus 0.02 mm. from zero for their Ross 16 cm W/A survey lens, a maximum tolerance of plus or minus 0.02 mm. from a standard curve is permitted for the Ross 6″ f:5.5 W/A survey lens. In addition the radial displacement for any particular point along either diagonal is permitted to depart from the mean curve for that lens by plus or minus 0.02 mm. This 0.02 mm. is roughly 25 seconds for a 6-inch lens. Neither of the above two cases cited specific reasons for selecting the quoted tolerances.

A suitable tolerance for lens distortion should preclude any lens from being accepted, of which the residual distortion, after theoretical compensation, permits excessive errors in the photogrammetric process. What constitutes excessive error is debatable. At Fort Belvoir, we feel that it is any error of a magnitude equal to the vertical accuracy of a spot reading. For example, we specify that our Kelsh Plotter stereomodels must be flat within plus or minus 1/5,000 of the

[a] Report of Commission I, Photography and Navigation, to the Seventh International Congress and Exposition of Photogrammetry, 1952, pages 23, 24.

projection distance. Therefore, we would designate as excessive any error due to residual lens distortion greater than 1/5,000 of the projection distance. This is a liberal view. Actually we would be happier with a much smaller distortion error, perhaps 1/7,500 to 1/10,000 of the projection distance, but at present we must compromise.

It would be most interesting if we could examine this problem quantitatively and arrive at some firm conclusions on what residual distortion is acceptable.

Uncompensated residual distortion may be resolved into two components where its effect upon the stereomodel is concerned. These are the direct x-parallax of a distorted image ray, and the tip factor. The direct x-parallax is simply the x vector, or the projection of the distortion displacement for any point, upon the x axis or base line of the stereomodel. This value, varying principally according to the residual distortion pattern, will effect a warpage of the model. The tip factor, which is a function of the y vector of the residual distortion, is caused by the requirement for clearing the y parallax, introduced by distortion, from the model; or to be to more correct, from designated points in the model, because complete elimination of y parallax is impossible where distortion is present. Using tip or phi to clear this y parallax introduces a certain degree of x parallax which also serves to warp the stereomodel. This warpage may tend to either compensate or further aggravate the warpage caused by the direct x parallax.

To investigate these conditions quantitatively, mathematical expressions can be developed. If we start with Von Gruber's basic equation for effects upon elevation in the stereomodel by differential motion of the various degrees of freedom we have:

$$dh = \frac{h}{b} ydK_1 + \frac{h}{b} ydK_{11} + \frac{(h^2 + x^2)}{b} d\phi_1 - \frac{(h^2 + (x - b)^2)}{b} d\phi_{11}$$
$$+ \frac{xy}{b} dW_1 - \frac{x - b}{b} ydW_{11} + \frac{h}{b} dx_1 - \frac{h}{b} dx_{11} + \frac{x}{b} dz_1 + \left(1 - \frac{x}{b}\right) dz_1 \quad (1)$$

where

h = projection distance
dh = elevation increment
dK = angle of swing displacement
$d\phi$ = angle of tip displacement
dW = angle of tilt displacement
x, y, z = coordinates of reference axes for stereomodel
dx, dy, dz = translation of perspective centers
b = steromodel base

The general equation (1) can be greatly simplified if recognized that we are concerned only with the terms of the equation containing elements of tip (ϕ) and x. All other terms are reduced to zero. We then have:

$$dh = \frac{h^2 + x^2}{b} d\phi_1 - \frac{(h^2 + (x - b)^2)}{b} d\phi_{11} + \frac{h}{b} dx_1 - \frac{h}{b} dx_{11} \quad (2)$$

where dx_1 and dx_{11} are assumed for the purpose of this treatment to be the direct x-parallaxes due to lens distortion rather than translation of the perspective centers as in the original equation. This assumption is a valid one under the conditions of this derivation.

Equation (2) contains all components due solely to lens distortion which will cause stereomodel distortion. However to render this equation practical for computational use it is necessary to express $d\phi$ in terms of measurable distortion values, i.e., x and y.

Figure 4 illustrates a projection of a cone of rays, angle ϕ, on any epipolar plane. If the cone is given a slight increase in tip ($d\phi$) the following relation is established between $d\phi$ and dx:

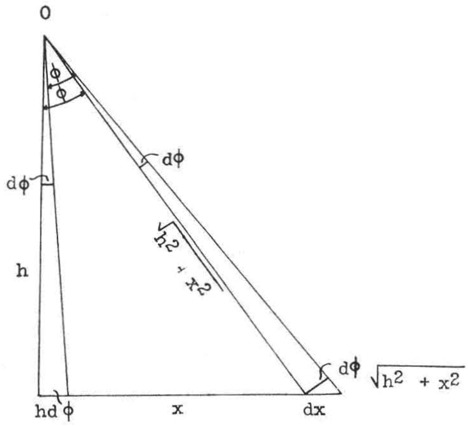

FIG. 4

$$\text{Cos } \phi = \frac{h}{\sqrt{h^2 + x^2}} = d\phi \frac{\sqrt{h^2 + x^2}}{dx}$$

(assumption of the value $d\phi\sqrt{h^2+x^2}$ for the small segment is not theoretically exact; however, the deviation from true value is insignificant in this treatment).

From the above

$$dx = \frac{h^2 + x^2}{h} d\phi. \tag{3}$$

After establishing a relation for $d\phi$ and dx, it is necessary to determine the amount of x-parallax, dx, introduced when the projector is tipped during relative orientation to clear dy parallax at a specific point. A relation between these terms is determined with reference to Figure 5.

$$y \cot w = \sqrt{h^2 + x^2}$$

squaring terms and differentiating

$$dx = \frac{y}{x} dy \cot^2 w = \frac{h^2 + x^2}{xy} dy. \tag{4}$$

We can state the amount of $d\phi$ required to clear a y-parallax by equating the two derived expressions for dx, equations (3) and (4).

$$dx = \frac{h^2 + x^2}{h} d\phi = \frac{h^2 + x^2}{xy} dy$$

$$d\phi = \frac{h}{xy} dy. \tag{5}$$

Substituting the value for $d\phi$ developed in equation (5) in equation (2) we have:

$$dh = \frac{h^2 + x^2}{b}\left(\frac{h}{xy} dy\right)_{\text{I}} - \frac{(h^2 + (x - b)^2)}{2}\left(\frac{h}{xy} dy\right)_{\text{II}} + \frac{h}{b} dx_{\text{I}} - \frac{h}{b} dx_{\text{II}}. \tag{6}$$

Equation (6) is the general equation for stereomodel warpage due to distortion of the bundle of rays emerging from each of the two projectors used to form the stereomodel. If we assume a given set of conditions, equation (6) can be reduced to a form permitting relatively simple computations. The first assumption is that the cone of rays emerging from each of the two projectors is

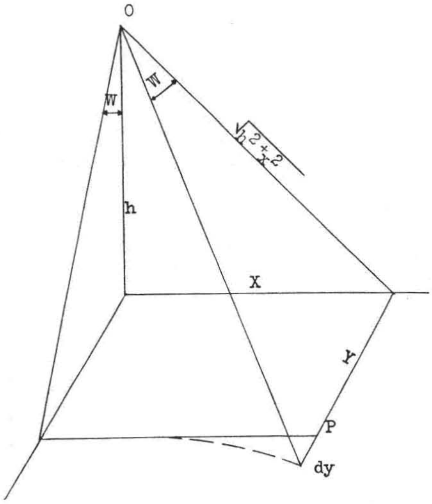

FIG. 5

symmetrically and equally distorted. That is, at equal angles subtended from the optical axes equal distortions will be present. This condition is particularly met when computing the theoretical stereomodel distortion caused solely by the aerial camera lens. In this case the two tip factors $d\phi_{I}$ and $d\phi_{II}$ are equal but opposite in sign, and equation (6) can be written

$$dh = \frac{2h^2 + x^2 + (x-b)^2}{b}\left(\frac{h}{xy}\,dy\right) + \frac{h}{b}(dx_{I} - dx_{II}). \tag{7}$$

After introducing the tip correction, to return the stereomodel to its original datum we subtract $2(h^2 d\phi)/b$ from equation (7). The quantity $hd\phi$ represents the amount of x displacement at the principal point (vertical photography) caused by tip correction $d\phi$ required to eliminate y parallax dy. Since by equation (5)

$$2\,\frac{(h^2 d\phi)}{b} = 2\left(\frac{h\,dy}{xy}\right)\left(\frac{h}{b}\right)$$

equation (7) becomes with addition of

$$-2\left(\frac{h^2 dy}{xy}\right)\left(\frac{h}{b}\right)$$

$$dh = \frac{h}{b}\left(\frac{dy}{xy}(x^2 + (x-b)^2 + dx_{I} - dx_{II}\right). \tag{8}$$

If we attempt to express dh as a function of lens distortion we find a certain degree of complication. The base-height ratio of the stereomodel affects our answer as evidenced by the reciprocal of this value, h/b in the last mentioned equation. The location x, of error dh in the model gives us a unique value, and the location of the point used for correction of the tip or phi relative orientation affects the value of dh through the term dy/x_1y_1. In addition it can be seen that with a dx_{I} and a dx_{II} in the equation unless they are both equal, or one is zero, dh is a function of not one but two variables in distortion. This is readily understood when we realize that a stereomodel is formed by two projections each with its own residual distortion.

It is obvious with all the above mentioned parameters to be considered that a tolerance limitation on lens distortion which is general enough to cover a wide range of stereoscopic situations, cannot be simply formulated on a strictly theoretical basis. It appears that the only solution is to compute, for each distortion curve in question, a stereomodel warpage pattern and to determine whether this pattern is within tolerable limits. This is not as impractical as may appear at first glance.

If sufficient conditions are fixed, a simple computation form based upon the above derived equation can be devised which will permit conversion of a lens distortion curve into a stereomodel warpage pattern within several minutes involving no more than simple arithmetic operations. Figure 6 illustrates such form as developed at Fort Belvoir. This computation form was prepared based upon a stereogrid model, Figure 7, formed with a base-height ratio of 0.67 and where tip parallax is cleared at point $x_1 = 100$ mm., and $y_1 = 100$ mm., measured at the aerial negative scale. This represents a stereomodel allowing maximum exploitation for vertical photography. Small deviations in base height ratio or location of tip solution points will not affect the computation significantly. If considerable deviation is anticipated or planned, it is relatively a simple matter to revise the computation form.

At the Engineer Research and Development Laboratories we have on file a calibration certificate prepared by the Fairchild Camera and Instrument Corporation for each of 112 KC-1 cameras which, as most of you know, mount the Planigon lens. As a matter of record we compute a stereomodel warpage diagram for each of these cameras and attach the computation to the certificate. In our efforts to simply correlate the residual distortion of the aerial camera lens and the vertical warpage error in the stereomodel, we plotted these values against each other. To be more specific, for each of the 112 Planigon lenses we plotted the maximum value from the distortion curve, when balanced for equal positive and negative distortion out to 42.5 degrees, against the maximum vertical error in the computed stereo-grid model for that lens distortion curve. We were very pleased to find a relatively well defined empirical relationship.

With some least squares curve fitting we evolved the curve shown in Figure 8. One must be very careful about using this curve as it would be easy to accept this relation as generally valid. (Too many are always impressed and awed by non-linear curves plotted neatly on cross-section paper). However, it must be stressed that this curve was developed under very special conditions; although it applies consistently to Planigon lenses forming stereo models with base height ratios of approximately 0.67, it may not predict, with any degree of certainty, error due to residual distortion of other lens systems. In fact we had to prove to ourselves that the curve had any validity even with Planigon lenses.

We accomplished this test in a straightforward manner. Two KC-1 cameras were taken on a flight over a well controlled area and identical simultaneous photography was obtained. One camera had what we considered a very acceptable distortion curve while the other had a very poor curve. They are shown in Figure 3. The same stereopair was selected from each of the two rolls of photography referred to now as "good" and "poor." These stereopairs were absolutely oriented in an identical manner on the same first-order precision stereoplotting instrument which had been specially calibrated before the test.

On the basis of our empirical curve we could predict, allowing for no other sources of error, that the maximum error for the good model would be 1/6,000 of the projection distance. Our experimental value was 1/5,500. For the "poor" model the prediction called for 1/2,000, the experimental value was 1/2,500.

These data are presented only on a qualitative basis. We cannot pretend that this was a complete scientific experiment. Too many variables exist in a limited test of this nature to exclude coincidence. Nevertheless we were pleased to note that the data tie in so neatly on the first attempt, a far too rare occurrence in research work.

What does all this mean insofar as determining a tolerance for lens distortion? Based upon our theoretical studies of the 112 Planigon lenses and our limited practical experience with KC-1 photography, we feel that it is a reasonable generalization to state that a residual lens distortion tolerance of plus or minus 10 microns for any half-angle from zero to 42.5 degrees would in great probability

STEREOMODEL WARPAGE COMPUTATION FORM

Base-Height Ratio, 0.67

Point	Distance from 1 D_1	Distortion d_1	X Component dx_1	Distance from 2 D_2	Distortion d_2	X Component dx_2	Δx_1	Δx_2	$\Sigma dx = dx_1 + dx_2$ $\Delta x_1 + \Delta x_2$	$dz = 1.5(\Sigma dx)$	dz' adjusted datum	Instrument error DZ'
C	141		$(.709)d_1$	100	—	0	$(1.00)\Delta y$	0			0	
B	112		$(.446)d_1$	112		$(.446)d_2$	$0.25)\Delta y$	$(0.25)\Delta y$				
E	106		$(.708)d_1$	79.1		$(.316)d_2$	$(0.56)\Delta y$	$(0.06)\Delta y$				
H	112		$(.893)d_1$	50.0	—	0	$(1.00)\Delta y$	0				
G	70.7		$(.707)d_1$	70.7		$(.707)d_2$	$(0.25)\Delta y$	$(0.25)\Delta y$				
J	79.1		$(.948)d_1$	35.4		$(.706)d_2$	$(0.56)\Delta y$	$(0.06)\Delta y$				
M	100		$(1.000)d_1$	0	—	0	$(1.00)\Delta y$	0				
L	50.0		$(1.000)d_1$	50.0		$(1.000)d_2$	$(0.25)\Delta y$	$(0.25)\Delta y$				

$\Delta y = dy_2 - dy_1 =$

dy_1 is equal to 0.709 times distortion for radial distance of 141 mm.

dy_2 is equal to the distortion for radial distance of 100 mm.

FIG. 6

eliminate any vertical error in the stereomodel, due to lens distortion, greater than 1/7,500 of the projection distance—or flight height if you prefer.

The basis for the above generalization is limited to stereomodels with a base-height ratio of approximately 0.67. However, if we refer to the general equation developed earlier, we find that as the base-height ratio is increased, the term h/b, its reciprocal, tends to decrease. So we can anticipate that dh, the vertical error will not exceed the limiting value set for b/h of 0.67, to any significant degree. As the base-height ratio becomes less than 0.67 the probability that dh will exceed $h/7{,}500$ increases until at b/h of 0.4 it may approach $h/5{,}000$, or

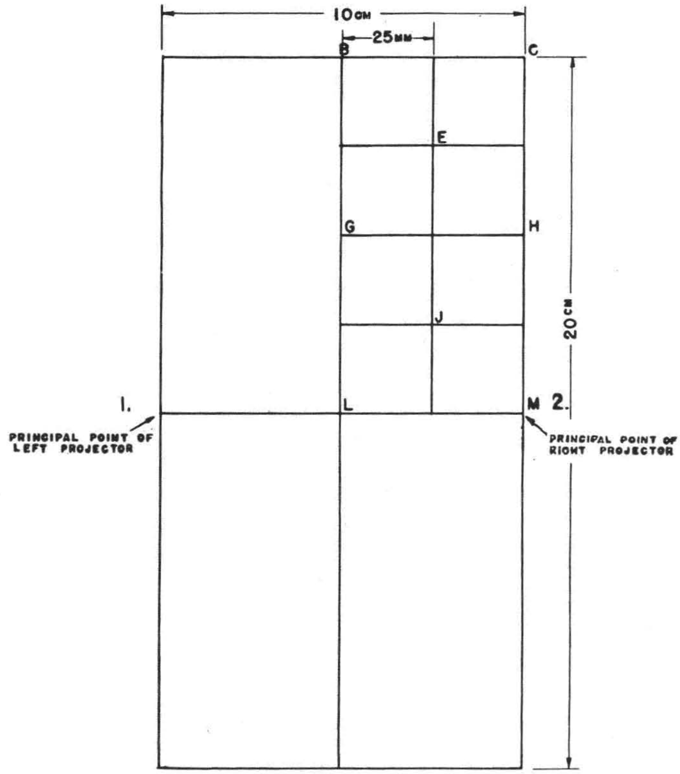

FIG. 7. Stereo-grid-model format.

under unusually adverse circumstances even greater error may result. However, the advantages of the greater base-height ratio are well established and it is an exception to the rule for mapping photography destined to be used in high precision work to be accomplished with a base-height ratio of less than 0.5.

A tolerance limitation of 10 microns for a 6-inch focal-length lens is of the order of 10 seconds of arc. This figure is very close to the mechanical limitations of the best stereo-photogrammetric instruments in use at present, and also approximates the order of magnitude of error due to dimensional stability of aerial film base. To establish a tolerance of less than 10 microns for a 6-inch lens, although desirable, could be unnecessarily severe. To permit a tolerance of greater than 10 microns will permit errors in the stereomodel, caused by lens distortion, which are measurable in high precision plotting instruments.

To conclude, arriving at a general tolerance for lens distortion based solely upon theoretical considerations is not feasible. Too many variable conditions

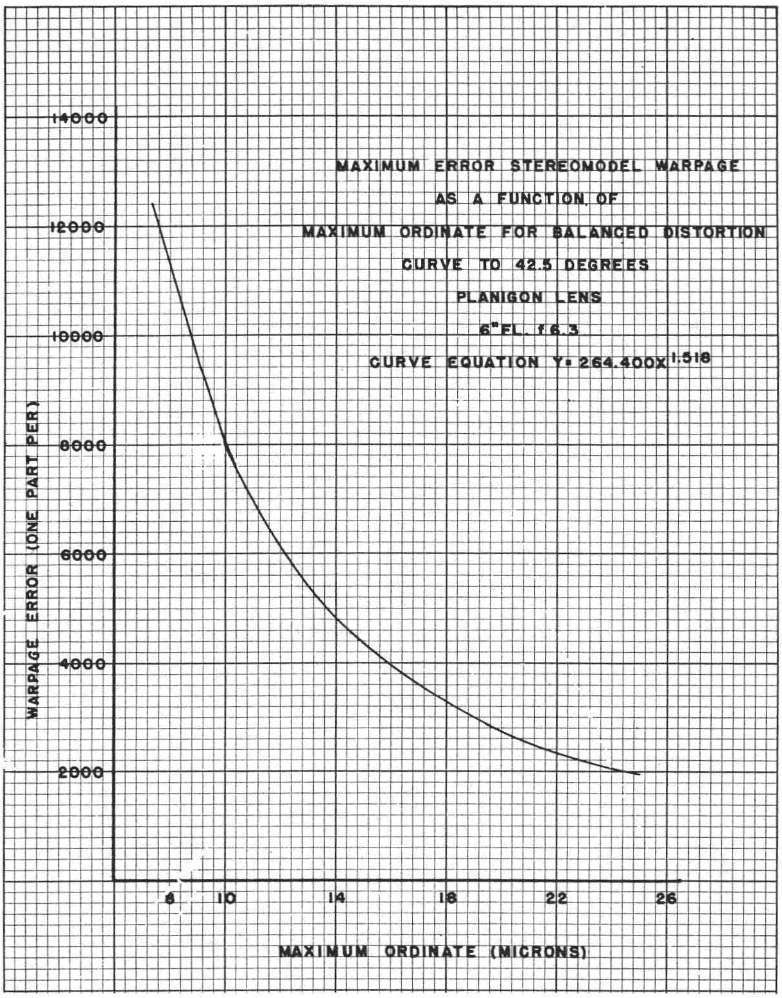

FIG. 8. Empirical stereomodel warpage—lens distortion relation.

exist, under which the photography taken with the lens may be used. However, if some of these variables are limited to conditions of general practice we can, on the combined basis of theory and empirical observation, establish a working tolerance consistent with accuracies of current photogrammetric procedures.

Our thinking at the Engineer Research and Development Laboratories is that the present goal for distortion free lenses should require that no distortion value exceed plus or minus 10 microns between zero and 42.5 degrees half-angle. We are actually now accepting plus or minus 15 microns in the distortion-free category, that is, requiring no correction; however, this is a concession to the transitional period, during which our major lens manufacturers are becoming familiar with the extremely difficult procedures called for in producing a truly distortion-free wide-angle aerial-mapping lens.