*Conditions to Be Fulfilled by a Rational System of Analytical Aerial Triangulation**

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ABSTRACT: *It is emphasized that in a rational system of analytical aerial triangulation it is necessary to identify connecting points simultaneously in all three photographs pertaining to one triple overlap, to mark with full measuring accuracy the selected points, and so forth. A plea for definite theoretical investigations is made, with respect to the superior computational method ignoring relative orientation as a separate step. Desirability and means for using an electronic computer of the smallest size compatible with the nature of the problem are discussed. Formulae are given for the electronic computation.*

PRELIMINARY REMARK

The system of aerial triangulation summarily described by Randall D. Esten in PHOTOGRAMMETRIC ENGINEERING, Vol. XXI, No. 5, p. 697–698, December 1955, is an attempt at fulfilling the conditions specified hereunder. It is currently developed for the Engineer Research and Development Laboratories, Fort Belvoir, Virginia.

(1) *The stereoscopic identification of the connecting pass points must involve simultaneously all three photographs pertaining to a triple overlap.*—Failure to observe this point implies that one assumes the terrain around each pass point to be smooth, and provided with an identifiable "drawing" (lines, points) located in its surface. This means, in other words, that one assumes the point to be defined without ambiguity and with full measuring accuracy by its image on one photograph alone. This assumption is however quite erroneous over forest, rough country, etc., so that no method using a stereocomparator of classical design can claim to solve adequately the problem of identification.

(2) *The connecting points should be marked with utmost measuring accuracy on all photographs where they are used.*—Only thus can the expensive equipment triplication for measuring and recording the photographic coordinates be avoided. Only thus also can an easy and indisputable check of choice and identification of connecting points be effected during computation or later on, such a check being absolutely necessary for obtaining reliable results. Avoiding triplication of equipment is of especial concern if the correction of the photographic coordinates for systematic errors and film shrinkage is done (desirably so) at the measurement, and not at the computational stage.

(3) *For each connection between successive pairs of photographs, one should be able to use whenever necessary, a large number (20 at least) of connecting points.*—With terrain unfavorable to a good choice of connecting points, one should be able to replace one or more of the six classical connecting points thus affected by a group of three, four or more points. Only through such action is it possible to compensate for at least part of identification uncertainty. If no choices at all can be made in sections of the overlaps (due to lakes, sea, snow, etc.), one should be able to insert new groups of adequate strength. Furthermore, it should be possible, in mountainous terrain of unfavorable configuration, to use additional connecting points, located with respect to the general terrain surface at whatever contrasting levels one may discover within the overlap. These requirements dispose of the case from the outset, even if more gen-

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eral arguments about reliability (instant localization of identification mistakes) and accuracy were not in existence.... It should be noted that systems requiring only the theoretical minimum of connecting points, i.e. 4 in each overlap, are valueless in practice since any error on data goes undetected into the results of the whole triangulation. They are unacceptable also in that they leave aside the connecting points located near the nadir of each photograph, i.e. the ones deserving overwhelming weight in the transfer of azimuths.

(4) *The computing system must ignore relative orientation as a separate step.*—Only then is it possible to secure the maximum strength of connection for a given number of connecting points. Engaging in only relative orientation and scaling wastes the important supplementary determination of transverse tilt (ω) resulting from the resection on points on the edges of the strip in the triple overlap. Doing the scaling with only a portion of the available points also wastes valuable determination. If, on the contrary one were to proceed for example as described in *Photogrammetria*, 1951–2, p. 23–24 (and as described elsewhere long before this), combining in one operation space resection on previously determined points (in the triple overlap) and elimination of transverse parallax on new points (in the double overlap), all determining elements are taken into account simultaneously and the strength of connection is at once at maximum value. This is of still greater importance in mountainous terrain. If, for instance, the shape of the ground tends towards the classical "critical cylinder," as is frequently the case, pure relative orientation rapidly loses its determining power, while the combined resection-parallax method just mentioned is still effective. Only as above described, consequently, is it possible to operate with any certainty in mountainous terrain. Also only in this manner can a standard procedure be set up which makes the best of all shapes of terrain, dispensing with awkward decisions as to whether a given terrain is "flat" or "mountainous."

Of course, even the combined resection-parallax method can do nothing in the (rare) really critical cases of aerial triangulation. One such case seems to occur when the shape of the terrain is a certain cone corresponding to a very steep, wide open valley. But even in this case, there seems to be a way to proceed, if only one can get at the awkward overlap from the opposite direction (possibly with at first unknown scale and orientation).*

(5) *The computing system must allow a running check of the consistency of all partial results among themselves, and also permit immediate and unhurried critical examination in the pictures showing the choices and identifications of dubious connecting points, and finally permit an economical way of making the necessary corrections.*—If, for instance, one uses a large electronic computer, it must be programmed so as to stop computing after comparatively short intervals of time, the partial results al-

* NOTE: Photogrammetric literature contains many theoretical studies regarding pure relative orientation (orientation using only elimination of transverse parallaxes) and about the "critical cylinder." The practical consideration of such instances is seldom really justified except perhaps when tying an isolated pair of photographs to just three ground controls or groups of ground controls. On the other hand there exists practically no mention in the photogrammetric literature about the fundamental case of combined resection and elimination of parallaxes; this is the only rational method, and it is likely that it will be used as the standard method in all analytical photogrammetry, slated to form a large percentage of all future photogrammetric work. In the literature there is a great scarcity of information about the prevailing influence of the shape of terrain and still possible critical surfaces. The author attempted years ago to induce theoreticians to study these subjects—such a study appears to be of no great difficulty—simple geometrical considerations, while failing in the case of pure relative orientation, offer a good understanding of the subject. The author demonstrated this in a 1952 Technical Paper of Ohio State University. This is also discussed briefly by Ugo Bartorelli in *Bollettino Sifet* 1954/2–3, p. 17–18. Both attempts largely failed. Just as ineffective was the author's attempt at suggesting a comparison to be made between the accuracy of aerial triangulation with square pictures (single or quadruple camera) and the accuracy of aerial triangulation with rectangular pictures (twin camera), the photographs being "equivalent," according to a well-balanced key, from the combined viewpoints of coverage and obliquity of marginal rays. Rectangular pictures seem to give weaker transverse tilt and scale determinations, with hardly better longitudinal tilt determination. Essential decisions about the future of aerial triangulation depend on the results of such a study.

ready secured being delivered to a photogrammetric expert. At this point the machine must be turned over instantly to other problems, until one has checked the consistency of said results, carefully traced possible mistakes, stereoscopically checked (as is practically always necessary) the particulars of the connecting points involved, reasoned on the situation, and corrected the data for the computer, whenever mistakes have been found. If any success is to be achieved this should preferably be done under no pressure of time, and under full support of the documentation describing the connecting pass points.

There are variants, according to whether part of the consistency analysis can be done by the computer itself, whether data of the aerial triangulation and of the other tasks can remain stored internally or not during computing intermissions, and so forth. Also, special attention has to be given to the very efficient checks at the lowest level, that is, looking for contradictions between points of each group within the single overlap (the group replacing one of the classical six points).

All these requirements are essential, since it would be nonsense to allow a million dollar machine to turn out long sequences of results vitiated from the outset by wrong data. It would be particularly inefficient to stop such a computer during most of the time the aerial triangulation is supposed to be in progress. Nobody can entirely avoid mistakes in choice and identification. As soon as there is only one mistake in 200 given elements, the investigation time tends to grow towards a high multiple of the computing time of large computers. Even with small electronic computers (of the fifty thousand dollar size), these requirements should be heeded; although it may appear acceptable now, under certain circumstances, to keep the smaller machine idle while checking, investigating and correcting.

(6) *The system should lend itself for operation, whenever necessary, in small decentralized units, readily set up at the many places where needs for operations may arise.*—It may certainly be advantageous in certain cases to use time of a million dollar electronic computer in aerial triangulation; on the other hand, there is a definite need all over the world for many units using electronic computers of the fifty thousand dollar type, or using special purpose electronic computers, if such can be assembled for an appreciably lower price. Only in such a way can the work be done without dependency on long distance communications and on the availability of computers mainly devoted to other tasks, involving difficult problems of organization. A smooth work flow can be expected only exceptionally if the photographic data, the photogrammetric expert and the computer are not at the same location under a single management.

(7) *The system of computation should not imply strict adherence to the letter of the least squares adjustment method, but only acceptance of its general spirit.*—There is a definite danger in that the strict least squares adjustment requires a considerable amount of computation, and consequently a very expensive computer, or an unduly long time for carrying out with extreme accuracy a computation which one knows to be based on rough assumptions. The data are hardly ever worth this much care. The operation will be more satisfactorily handled if the same effort is put into more connecting points, into better correction and weighting, and into more critical analysis during computation. This is to say that concentration should be in those elements which tend to reduce the final contradictions, rather than indulging in their blind acceptance.

(8) *The electronic computation should be practically freed from calling in subroutines for trigonometrical functions.*—It makes a considerable difference in the computing economy whether the procedure requires only straightforward arithmetic operations, or on the contrary it uses numerous trigonometrical functions which load the programming with heavy sub-routines. In the latter case, the computing time can become a multiple of what it is in the first case. This may happen, for example, if in the connection of a new photograph in a strip (i.e. in the fundamental operation of aerial triangulation), one defines locations of rays or points by angles rather than by lengths, or if one uses angular rather than linear discrepancies and parallaxes for checking the progress of successive approximations. Or also if one tries to operate in some general system of reference, while ignoring the local cartesian coordinates bound to the strip or block. Trouble may already be generated by the use of trigonometrical functions of swing and tilts.

That the latter are avoidable, and that the computations actually required from the electronic computer can be easily mastered by a small unit, can be illustrated by the following set of formulae:*

$$x_a' = x' - y'q_k \quad x_b' = x_a'\phi(q_0) \quad x_r' = x_b' + z_b'q_f$$

$$v_a' = y' + y'q_k \quad y_b' = y_a' - z_a'q_0 \quad y_r' = y_b'\phi(q_f)$$

$$q_x' = \frac{x_r'}{z_r'}$$

$$z_a' = f\phi(q_k) \quad z_b' = z_a' + y_a'q_0 \quad z_r' = z_b' - x_b'q_f$$

$$q_y' = \frac{y_r'}{z_r'}$$

$$\left.\begin{aligned} D_X &= X - b_X - q_x'(Z - b_Z) \\ D_Y &= Y - b_Y - q_y'(Z - b_Z) \end{aligned}\right\}$$

for resection points only

$$\left.\begin{aligned} Z' &= \frac{b_X - q_x b_Z}{q_x - q_x'} \quad Y_a' = q_y'Z' \\ P_Y &= Y_b' - Y_a' \quad Y_b' = q_y(Z' + b_Z) - b_Y \\ X' &= q_x'Z' \\ Y' &= Y_a' + \frac{P_Y}{2} \end{aligned}\right\}$$

for parallactic points only

* These formulae simply translate into elementary algebra the operations described on p. 23–24 of *Photogrammetria*, 1951–2. Their derivation is immediate, the symbols having the following meaning:

$x'y'$	photographic coordinates of images of ground points
f	principal distance, in same (arbitrary) unit as x', y'
XYZ	coordinates of previously determined ground points
$X'Y'Z'$	coordinates of newly determined ground points
$q_x q_y$	slope components of previously determined rays
$q_x' q_y'$	slope components of newly determined rays
$b_X b_Y b_Z$	the usual basis components
$q_0 q_f q_k$	substitutes for the unhandy classical ω, ϕ, κ
$D_X D_Y$	resection discrepancies
P_Y	transverse parallax

One may find it convenient in particular cases also to use the electronic computer in connection with atmospheric refraction corrections, differential coefficients needed in further computations, "a posteriori" adjustments, etc., but there is no basic need for loading the electronic computer with such operations which generally are more efficiently done in analog devices since no high precision is required.

(REMARK: $\phi(q_{0,f,k})$ being $\sqrt{1+q^2_{0,k,f}}$ varies very slowly, so that it can either be introduced into the memory as a completed table, or generated with only one multiplication from a skeleton table at the expense of hardly noticeable computation time.)

The high precision part of the aerial triangulation computations, which is the only one where electronic speed is a basic requirement, needs no more arithmetic operations as implied by these formulae (adequately repeated according to the number of connecting points and approximations). There may exist some very special cases in which formulae with many trigonometric functions have their justification, but this is no reason for carrying such useless ballast in the bulk of the current work.

(9) *Other conditions.*—There are some rather obvious requirements to be fulfilled by a really utilizable system of aerial triangulation, i.e. one which does not fail as soon as there is some unfavorable circumstance in the actual working conditions. As examples of such requirements, one can mention the following: automatic settings and recordings; the use of stereoscopic fusion in transverse as well as in longitudinal identification; foolproof operation; correct introduction of photogrammetric-gravimetric data (atmospheric refraction); possible combination of certain purely geodetic operations with the photogrammetric operations (direct referencing of results on geoid, delivery of results already corrected for easy plotting in a definite map projection system); freedom from narrow geometrical limitations in acceptable orientation parameters or location of connecting points; possibility of introducing correctly weighted external data (from statoscope, profile recorder, radar, sun, horizon, etc., keeping however in mind that the aerial triangulation remains practically indispensable for securing continuity of final results, for getting higher accuracy, and especially to serve as a check for getting dependable results).

The scope of this paper is not to go into all these obvious points, but only to deal with conditions which are still being debated, or ignored, or misunderstood.