*The Application of the Balplex Plotter to Trimetrogon Obliques**

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ABSTRACT: *The development of the Balplex Plotter gives rise to several new techniques of map compilation. A more complete investigation of the methods and accuracy of plotting from trimetrogon oblique photography is warranted, by virtue of the Plotter's unique facilities for improved model illumination, attainment of the Scheimpflug condition, and suspension of the projector.*

Empirical procedures of relative orientation for both the methods of independent and dependent pairs of high-obliques are shown. In addition, formulas for the computation of the orientation element errors have been evolved, based on the observed parallaxes in the model. These formulas may be applied to a direct realization of relative orientation in a first-order plotter, or to a numerical relative orientation procedure with any plotter equipped to indicate the by-setting of one projector. The standard errors of the elements of orientation have been developed so that the horizontal and vertical accuracies of the model may be evaluated in future applications.

TRIMETROGON oblique-photography has generally been regarded as satisfactory only for reconnaissance mapping and superficial general plotting. Consequently, an analytical determination of a relative orientation procedure for trimetrogon-obliques and the accuracy of such an orientation do not appear to have been performed. The recent advent of the Balplex Plotter now makes such a determination desirable. Several unique features are incorporated in the design of the Plotter which make it well suited to plotting from trimetrogon obliques.

The purpose of this paper is to establish the procedure for relative orientation of high-obliques and to evaluate the accuracy of the elements of such an orientation. Since similar data have been determined for vertical and convergent photography by the application of the method of least squares, that method is again employed, so that all related data are expressed in a comparable manner. Because the accuracy of the model is primarily dependent upon the accuracy with which the relative orientation is accomplished, expressions for the accuracy of each element of relative orientation are also shown.

Relative orientation of two projectors of a photogrammetric instrument may be accomplished by the recognition and removal of y-parallaxes at six standard locations on the model (Figure 1). Both projectors of the Balplex Plotter are equipped with three translational and three rotational motion screws, thus making a total of twelve adjustment motions available. It can be shown that five of these motions are theoretically sufficient to remove the parallax at the first five points, after which no parallax will be present at any other point on the model. In practice, parallax is frequently found at a sixth point, in contradiction to this theory. However, any such parallax is due to unavoidable factors which cannot be treated directly in theoretical data. These factors may be

* This is a rewrite and condensation of prize-winning paper for B & L award in 1956.

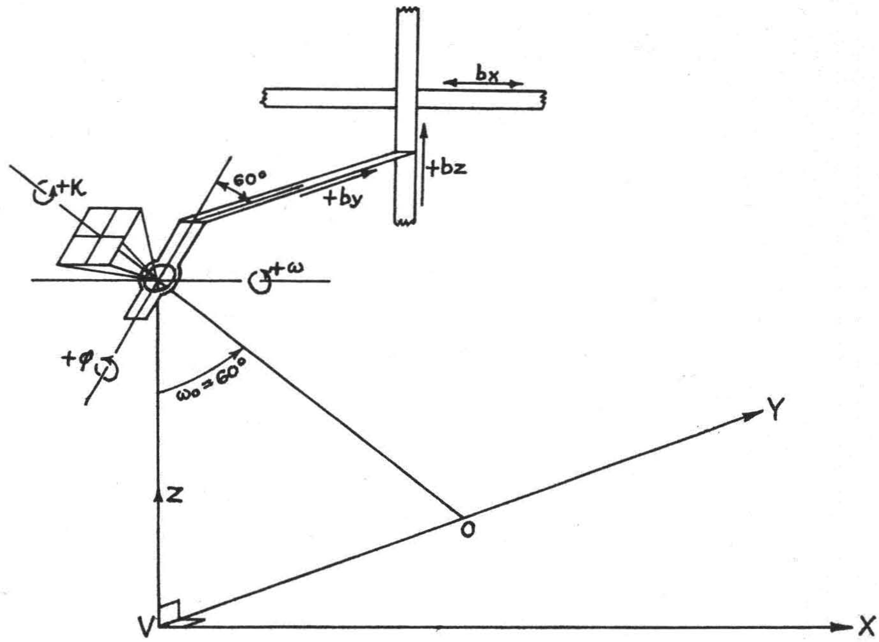

FIG. 1. The 60° oblique position of the left projector, showing the six orientation motions, their sign convention for this report, and their relations to the mapping axes.
ϕ = angle of rotation about the primary axis.
ω = angle of rotation about the secondary axis.
κ = angle of rotation about the tertiary, or camera, axis.
ω_0 = initial 60° transverse tilt at exposure.

such as lack of symmetry between the projector and aerial camera lenses, differential film shrinkage, or other known and unknown factors.

To obtain a picture of the geometric conditions required for relative orientation, consider that a rough orientation has been performed to reduce the y-parallaxes to small values over the entire model. This may be done in projection-type plotters, such as the Balplex, without using the anaglyphic spectacles. Then, by removing the x-parallax at a chosen point A, the geometric conditions shown in Figure 2 are obtained.

From Figure 2, the following identity can be established:

$$py = Y_2' - Y_1 = Y_2 - Y_1 + by. \tag{1}$$

Relative orientation is accomplished when $py=0$ at five points. In order to make the y-parallax equal zero, corrections must be made to all terms in formula (1). Thus

$$py + dpy = (Y_2 + dY_2) - (Y_1 + dY_1) + (by + dby) = 0. \tag{2}$$

Subtracting (1) from (2):

$$dpy = dY_2 - dY_1 + dby = - py. \tag{3}$$

Formula (3) shows that the parallax correction at any point is accomplished by a change in the Y_1 and Y_2 coordinates, and by the by difference between the projectors. In addition, it is obvious that the parallax correction will be the negative value of the existing parallax.

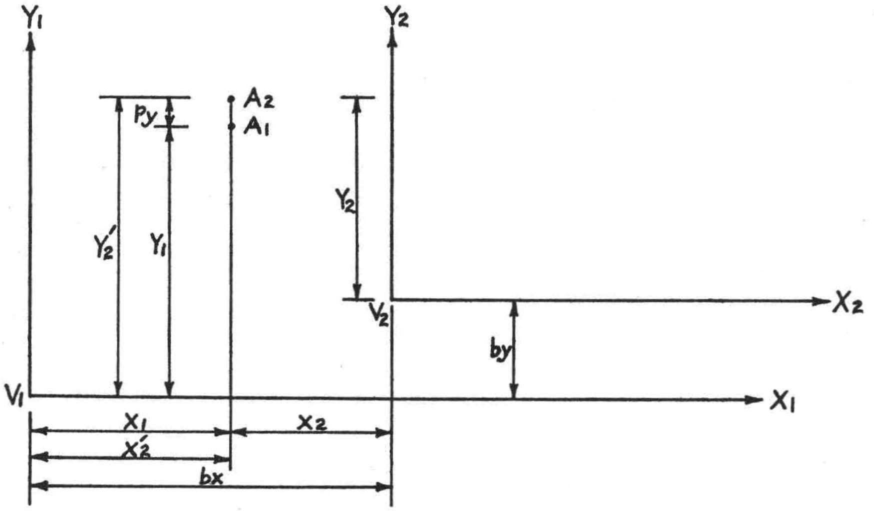

FIG. 2. Coordinates of a projected point related to the plotting plane axes of the left and right photos.

A_1 and A_2 represent the projected images of homologous picture points a_1 and a_2.

V_1 and V_2 are the nadir points of photos 1 and 2.

X_1, Y_1 and X_2, Y_2 are the plotting plane axes for photos 1 and 2.

py denotes the existing y-parallax.

The dY_1 and dY_2 corrections of formula (3) are made by changes in the elements of orientation, as is shown in the well-known general expression relating the model Y-errors to the differential errors of relative and inner orientation:

$$dY = \frac{\delta Y}{\delta\phi} d\phi + \frac{\delta Y}{\delta\omega} d\omega + \frac{\delta Y}{\delta\kappa} d\kappa + \frac{\delta Y}{\delta bz} dbz + dby + \frac{\delta Y}{\delta x} dx + \frac{\delta Y}{\delta y} dy + \frac{\delta Y}{\delta f} df. \quad (4)$$

In order to evaluate the dY terms in formula (3) and so derive parallax equations for six chosen points on the model, each partial derivative in equation (4) must be evaluated for the high oblique set-up. This may be done in the following manner:

(a) First, expressions must be found relating the projection plane X and Y coordinates of a point P to the x and y image coordinates of p on the multi-tilted high oblique. This is accomplished in progressive steps, by expressing the X and Y coordinates of the point in terms of the photographic x and y coordinates in the initial $\omega_0 = 60°$ tilt situation, then writing expressions relating the photographic coordinates of the point through each stage of successive ϕ, ω and κ rotations. Thus the photo has been given all possible rotations that could occur in practice. After rather lengthy reductions, the desired expressions relating the coordinates of a point on the projection plane to its image coordinates on the high oblique are found to be:

$$X_P = \frac{(x\cos\kappa - y\sin\kappa)\cos\phi + f\sin\phi\cos\omega - (x\sin\kappa + y\cos\kappa)\sin\phi\sin\omega}{\begin{array}{l} f\cos\phi\cos\omega\cos\omega_0 - (x\sin\kappa + y\cos\kappa)\cos\phi\sin\omega\cos\omega_0 \\ \quad - (x\cos\kappa - y\sin\kappa)\sin\phi\cos\omega_0 - (x\sin\kappa + y\cos\kappa)\cos\omega\sin\omega_0 - f\sin\omega\sin\omega_0 \end{array}} \quad (5)$$

$$Y_P = \frac{\begin{array}{l} f\cos\phi\cos\omega\sin\omega_0 - (x\sin\kappa + y\cos\kappa)\cos\phi\sin\omega\sin\omega_0 \\ \quad - (x\cos\kappa - y\sin\kappa)\sin\phi\sin\omega_0 + (x\sin\kappa + y\cos\kappa)\cos\omega\cos\omega_0 + f\sin\omega\cos\omega_0 \end{array}}{\begin{array}{l} f\cos\phi\cos\omega\cos\omega_0 - (x\sin\kappa + y\cos\kappa)\cos\phi\sin\omega\cos\omega_0 \\ \quad - (x\cos\kappa - y\sin\kappa)\sin\phi\cos\omega_0 - (x\sin\kappa + y\cos\kappa)\cos\omega\sin\omega_0 - f\sin\omega\sin\omega_0 \end{array}} \cdot \quad (6)$$

(b) Expressions for each of the partial derivatives in (4) can now be obtained from (6). It is evident that such a direct evaluation would be extremely tedious. However, by employing symbolization, formulas (5) and (6) can be combined in more convenient form, and the partials evaluated more easily.

(c) The resulting general formulas for the partials are lengthy for the most part, and are not shown. These formulas may be reduced considerably by evaluating them for high oblique photography in which:

$$
\begin{array}{lll}
\omega_0 = \omega_0{}^\circ & \text{Sin } \omega_0 = \text{Sin } \omega_0 & \text{Cos } \omega_0 = \text{Cos } \omega_0 \\
\phi = 0^\circ & \text{Sin } \phi = 0 & \text{Cos } \phi = 1 \\
\omega = 0^\circ & \text{Sin } \omega = 0 & \text{Cos } \omega = 1 \\
\kappa = 0^\circ & \text{Sin } \kappa = 0 & \text{Cos } \kappa = 1.
\end{array}
$$

After substitution of these values in the general formulas, the final expressions for the partial derivatives in formula (4) are:

$$\frac{\delta Y}{\delta \phi} = X\left(-\sin \omega_0 + \frac{Y}{h} \text{Cos } \omega_0\right) \tag{7}$$

$$\frac{\delta Y}{\delta \omega} = h\left(1 + \frac{Y^2}{h^2}\right) \tag{8}$$

$$\frac{\delta Y}{\delta \kappa} = X\left(\cos \omega_0 + \frac{Y}{h} \sin \omega_0\right) \tag{9}$$

$$\frac{\delta Y}{\delta bz} = \frac{Y}{h} \tag{10}$$

$$\frac{\delta Y}{\delta x} = 0 \tag{11}$$

$$\frac{\delta Y}{\delta y} = \frac{1}{f}(Y \sin \omega_0 + h \cos \omega_0)\left(\cos \omega_0 + \frac{Y}{h} \sin \omega_0\right) \tag{12}$$

$$\frac{\delta Y}{\delta f} = \frac{1}{f}(Y \sin \omega_0 + h \cos \omega_0)\left(\sin \omega_0 - \frac{Y}{h} \cos \omega_0\right). \tag{13}$$

The derivations of many of the formulas in this paper are of considerable length and are not shown in the interests of brevity and conciseness of presentation. Interested readers may obtain the derivations and diagrams from the author.

By substituting in formula (4) the values for the partials given in formulas (7) to (13), the following general Y-error formula for high obliques is obtained:

$$
\begin{aligned}
dY = {} & X\left(-\sin \omega_0 + \frac{Y}{h} \cos \omega_0\right) d\phi + h\left(1 + \frac{Y^2}{h^2}\right) d\omega + x\left(\cos \omega_0 + \frac{Y}{h} \sin \omega_0\right) d\kappa \\
& + \frac{Y}{h} dbz + dby + \frac{1}{f}(Y \sin \omega_0 + h \cos \omega_0)\left(\cos \omega_0 + \frac{Y}{h} \sin \omega_0\right) dy \\
& + \frac{1}{f}(Y \sin \omega_0 + h \cos \omega_0)\left(\sin \omega_0 - \frac{Y}{h} \cos \omega_0\right) df.
\end{aligned} \tag{14}
$$

Assuming that the errors of inner orientation dx, dy and df are zero, as is the usual procedure in the determination of relative orientation procedure, then five elements remain with which to establish the relative orientation.

For the method of independent pairs, with reference to formula (3), dY_1 is accomplished by employing orientation elements ϕ_1 and κ_1; dY_2 is accomplished by using elements ϕ_2, κ_2 and ω_2. With $by = bz = 0$, the following substitutions may be made in order to express all coordinates in the system of the left hand nadir point:

$$X_1 = X \qquad Y_1 = Y_2 = Y$$
$$X_2 = X - b \qquad h_1 = h_2 = h.$$

Then, by substituting in equation (3) the appropriate terms from equation (14), the parallax equation for independent pairs becomes

$$\begin{aligned} -py = dpy &= (X - b)\left(\cos\omega_{02} + \frac{Y}{h}\sin\omega_{02}\right)d\kappa_2 \\ &+ (X - b)\left(-\sin\omega_{02} + \frac{Y}{h}\cos\omega_{02}\right)d\phi_2 \\ &+ h\left(1 + \frac{Y^2}{h^2}\right)d\omega_2 - X\left(\cos\omega_{01} + \frac{Y}{h}\sin\omega_{01}\right)d\kappa_1 \\ &- X\left(-\sin\omega_{01} + \frac{Y}{h}\cos\omega_{01}\right)d\phi_1. \end{aligned} \tag{15}$$

With the same substitutions, the general parallax equation for the method of dependent pairs becomes

$$\begin{aligned} -py = dpy &= (X - b)\left(\cos\omega_{02} + \frac{Y}{h}\sin\omega_{02}\right)d\kappa_2 \\ &+ (X - b)\left(-\sin\omega_{02} + \frac{Y}{h}\cos\omega_{02}\right)d\phi_2 \\ &+ h\left(1 + \frac{Y^2}{h^2}\right)d\omega_2 + \frac{Y}{h}dbz_2 + dby_2. \end{aligned} \tag{16}$$

It should be noted that in order for these equations to be applicable, the orientation elements $d\kappa$, $d\phi$, etc. must be small; that is, the photography must not show wide deviations from the assumed conditions.

In an attempt to obtain a larger b/h ratio, the possibility of plotting from alternate obliques was investigated. It was found that this procedure cannot be used with standard high-obliques taken in conjunction with the usual 60 per cent overlap vertical photography. An attempt at mapping from alternate obliques would leave gaps in the plot, consisting of the lower central portion of each alternate oblique. These gaps cannot be entirely covered by the corresponding vertical photographs. Therefore, models must be formed from successive obliques. Then the best portion of each photograph is utilized, and six points can be chosen so that their projected images are symmetrically located on the projection plane. These conditions are convenient for the determination of the five elements of relative orientation, and the least squares determination of the accuracy of these elements.

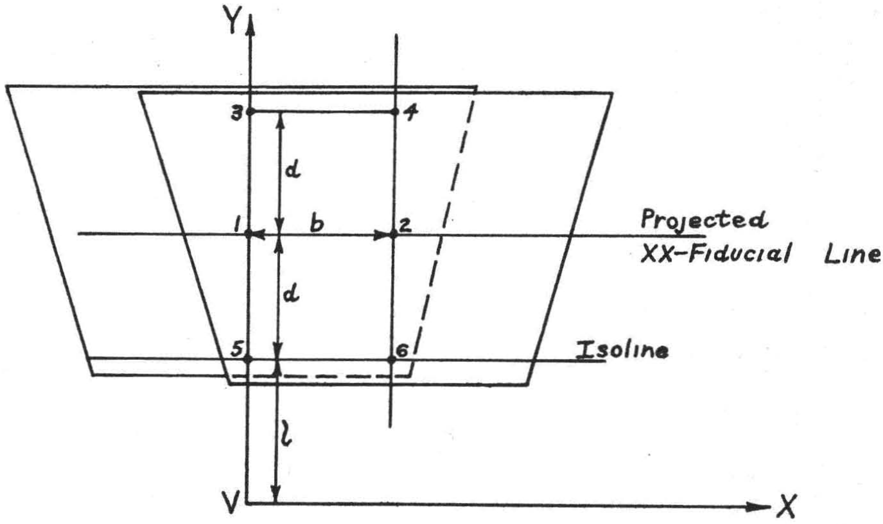

FIG. 3. Orientation points on the model plane.

Depending on the conditions under which the photography was obtained such as the flying height, air-base, air-camera focal-length, tilts, etc, the six points can be chosen on the oblique photos in the form of a simple trapezium, so that their projected images will form a rectangle on the projection plane, as shown in Figure 3.

By choosing the nadir point V of the left projector as the origin for the machine coordinates, the coordinates of the six points are:

$$\begin{array}{lll} X_1 = 0 & Y_1 = l + d & h_1 = \\ X_2 = b & Y_2 = l + d & h_2 = \\ X_3 = 0 & Y_3 = l + 2d & h_3 = \\ X_4 = b & Y_4 = l + 2d & h_4 = \\ X_5 = 0 & Y_5 = l & h_5 = \\ X_6 = b & Y_6 = l & h_6 = \end{array} \left.\vphantom{\begin{array}{l} a\\a\\a\\a\\a\\a \end{array}}\right\} \text{(Constant) } h$$

where l is the distance from the X axis to the isoline. The assumption of h = constant is justified only on the grounds of convenience. The effect of non-flat terrain has not been investigated for this type of photography.

To facilitate the evaluation of the parallax equations for the six orientation points, the following values are substituted as applicable to trimetrogon obliques:

$$\omega_{01} = \omega_{02} = \omega_0 = 60° \text{ and } \sin \omega_0 = .866, \cos \omega_0 = .500$$

In addition, model lengths b, l, and d (Figure 3) can be expressed in terms of the vertical projection distance h:

$$l = h \tan 30° = .577\ h$$

$$l + d = h \tan 60° = 1.732\ h$$

$$\therefore d = 1.155\ h$$

and

$$l + 2d = 2.887\ h.$$

Since the corresponding vertical photography will also be used for plotting, it should have the conventional 60 per cent longitudinal overlap. Then, for the standard conditions for vertical photography of $f=6$ inches, 9×9 inch format photos, and the 60 per cent overlap, the base-height ratio for both vertical and high oblique photography will be .600. Therefore, $b=.600\ h$.

Now, by substituting the foregoing identities into the general parallax equations (15) and (16), parallax equations for each of the six points are determined.

For the method of Independent Pairs:

$$p_1 = 1.200\ h\ d\kappa_2 \qquad -4.000\ h\ d\omega_2 \tag{17}$$

$$p_2 = 1.200\ h\ d\kappa_1 \qquad -4.000\ h\ d\omega_2 \tag{18}$$

$$p_3 = 1.800\ h\ d\kappa_2 + .346\ h\ d\phi_2 \qquad -9.333\ h\ d\omega_2 \tag{19}$$

$$p_4 = 1.800\ h\ d\kappa_1 + .346\ h\ d\phi_1 \qquad -9.333\ h\ d\omega_2 \tag{20}$$

$$p_5 = .600\ h\ d\kappa_2 - .346\ h\ d\phi_2 \qquad -1.333\ h\ d\omega_2 \tag{21}$$

$$p_6 = .600\ h\ d\kappa_2 - .346\ h\ d\phi_1 \qquad -1.333\ h\ d\omega_2. \tag{22}$$

For the method of Dependent Pairs:

$$p_1 = 1.200\ h\ d\kappa_2 \qquad -4.000\ h\ d\omega_2 - 1.732\ dbz_2 - dby_2 \tag{23}$$

$$p_2 = \qquad -4.000\ h\ d\omega_2 - 1.732\ dbz_2 - dby_2 \tag{24}$$

$$p_3 = 1.800\ h\ d\kappa_2 + .346\ h\ d\phi_2 \qquad -9.333\ h\ d\omega_2 - 2.887\ dbz_2 - dby_2 \tag{25}$$

$$p_4 = \qquad -9.333\ h\ d\omega_2 - 2.887\ dbz_2 - dby_2 \tag{26}$$

$$p_5 = .600\ h\ d\kappa_2 - .346\ h\ d\phi_2 \qquad -1.333\ h\ d\omega_2 - .577\ dbz_2 - dby_2 \tag{27}$$

$$p_6 = \qquad -1.333\ h\ d\omega_2 - .577\ dbz_2 - dby_2. \tag{28}$$

Consider that the relative orientation is to be started using the first steps of the conventional independent pairs procedure. This means that κ_2 (the κ motion of the right-hand projector) is used to clear y-parallax at point 1 (see Figure 4), then κ_1 is used to clear point 2.

Next, the two ϕ motions can be used to clear parallax at either points 3 and 4 or points 5 and 6. Considering that the ϕ motion is very sensitive in stereotriangulation, and that points 5 and 6 are nearer the projectors than points 3 and 4 and thus will be more clearly defined, it seems preferable to apply the ϕ corrections at points 5 and 6. So the next two steps consist of clearing point 5 with ϕ_2 and point 6 with ϕ_1.

Thus the parallax has been removed from points 1, 2, 5 and 6 in turn by κ_2, κ_1, ϕ_2 and ϕ_1. However, equations (17) to (22) show that the parallax at each point is due not only to the element applied in the above procedure, but to additional elements as well. For example, equation (17) shows that the parallax at point 1 is due to an ω error in addition to the applied κ_2. Therefore, false values of κ_2, κ_1, ϕ_2 and ϕ_1 have been used to clear the four points. In the process of clearing each point, then, parallaxes will actually be introduced at all other points on the model. Consequently, a method must be determined in which the one remaining element, ω, will compensate the errors of this procedure.

Equations (17) to (22) state that the parallax due to an ω error is 4 $h\ d\omega$ at model points 1 and 2, 9.333 $h\ d\omega$ at points 3 and 4, and 1.333 $h\ d\omega$ at points 5 and 6. This condition can be represented graphically as in Figure 4.

Following the independent pairs procedure, parallax is removed at points 1 and 2 by κ_2 and κ_1 respectively. This will result in the parallax distribution shown in Figure 5.

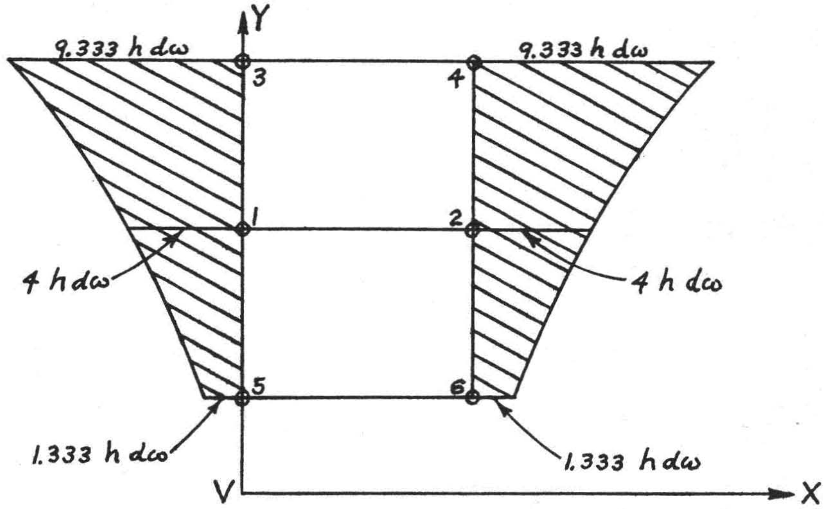

FIG. 4. Graphical diagram of initial ω parallax in model.

Next, after the parallax has been removed at points 5 and 6 by ϕ_2 and ϕ_1 respectively, the distribution of ω parallax will be as indicated in Figure 6.

Thus, the resultant parallax at points 3 and 4 is 2.666 $h\ d\omega$. If the first four steps in clearing the model had been without error, this final parallax could be corrected directly by the ω motion, a parallax-free model would be obtained, and relative orientation accomplished. However, it is known that the first four steps are in error, so an overcorrection factor for the final ω motion is required.

From Figure 3 it may be seen that 9.333 $h\ d\omega$ is the amount actually required to be removed from points 3 and 4. The correction factor n for the ω motion then is

$$2.666\ h\ d\omega\ n = 9.333\ h\ d\omega$$

$$n = 3\tfrac{1}{2}.$$

Therefore, the required overcorrection factor is

$$\kappa = n - 1 = 2\tfrac{1}{2}.$$

This means then that after the first four steps have been completed, the resultant parallax at points 3 and 4 should be overcorrected by $2\frac{1}{2}$ times. This will reintroduce parallax over the whole model, but the proper amount of ω parallax will have been introduced so that after a repetition of the first four steps, there will theoretically be no parallax at the final points 3 and 4.

In practice, a parallax-free model may not be obtained after the second operation. The development of this procedure is based on certain necessary assumptions which are usually not applicable in actual plotting work. For example, the parallax equations were derived for ideal high oblique photographs which were assumed to have no ϕ, ω, κ rotations at exposure. Also, the terrain was assumed to be perfectly flat. If the overcorrection is applied by eye, as is the normal procedure, then it will probably not be exact, and the model will not be parallax-free after the second operation. However, these detrimental factors can be overcome by repeating the orientation procedure until a satisfactory model is obtained.

Summarizing, relative orientation of independent pairs can be accomplished as follows:

1. Clear parallax at point 1 with κ_2.
2. Clear parallax at point 2 with κ_1.
3. Clear parallax at point 5 with ϕ_2.
4. Clear parallax at point 6 with ϕ_1.
5. Note the resultant parallax at points 3 and 4. It should be equal at both points and of the same sign. Overcorrect this parallax $2\frac{1}{2}$ times with ω_2.
6. Repeat the entire procedure until a parallax-free model is obtained.

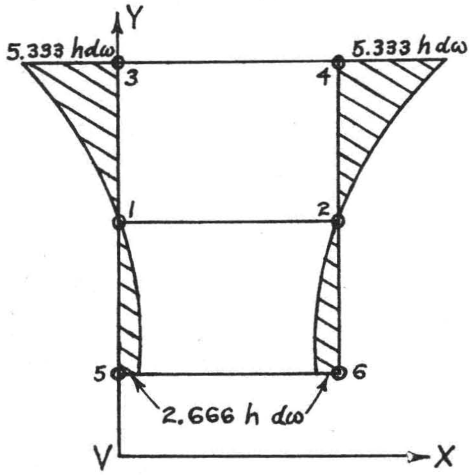

FIG. 5. ω parallax distribution after clearing points 1 and 2.

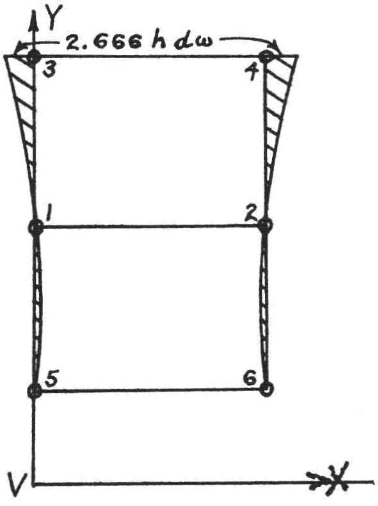

FIG. 6. ω parallax distribution after points 1, 2, 5 and 6 have been cleared.

In the parallax equations for the method of dependent pairs, (23) to (28), the $d\omega_2$ terms have the same coefficients as they have in the independent pairs equations (17) to (22). It is analogous then that the overcorrection derived for the method of independent pairs will be the same for the method of dependent pairs.

Thus, the relative orientation procedure for dependent pairs can be summarized as:

1. Clear parallax at point 2 with by_2.
2. Clear parallax at point 1 with κ_2.
3. Clear parallax at point 6 with bz_2.
4. Clear parallax at point 5 with ϕ_2.
5. Note the resultant parallax at points 3 and 4. It should be equal at both points and of the same sign. Overcorrect the parallax by $2\frac{1}{2}$ times with ω_2.
6. Repeat the entire procedure until a parallax-free model is obtained.

Formulas (17) to (22) provide a set of 6 equations with 5 unknowns, in terms of the observed parallaxes, for the method of Independent Pairs. Similarly, formulas (23) to (28) provide the same data for Dependent Pairs. Therefore, having one extra observation equation in each set, it is possible to solve each of the two sets for the orientation corrections by the Least Squares Adjustment method.

From the solution of the normal equations for independent pairs, the expressions for the corrections to the elements of relative orientation, in terms of the observed y-parallaxes, are found to be:

$$d\omega_2 = \frac{.375\,(p_1 + p_2) - .187\,(p_3 + p_4 + p_5 + p_6)}{h} \tag{29}$$

$$d\phi_2 = \frac{1.202\,p_1 + 1.683\,p_2 + .121\,p_3 - .841\,(p_4 + p_6) - 2.766\,p_5}{h} \tag{30}$$

$$d\phi_1 = \frac{1.683\,p_1 + 1.202\,p_2 - .841\,(p_3 + p_5) + .121\,p_4 - 2.766\,p_6}{h} \tag{31}$$

$$d\kappa_2 = \frac{1.804\,p_1 + 1.527\,p_2 - .485\,(p_3 + p_5) - .763\,(p_4 + p_6)}{h} \tag{32}$$

$$d\kappa_1 = \frac{1.527\,p_1 + 1.804\,p_2 - .763\,(p_3 + p_5) - .485\,(p_4 + p_6)}{h}. \tag{33}$$

In the same manner, the expressions for the corrections to the orientation elements for dependent pairs are:

$$d\omega_2 = \frac{.577\,(p_1 + p_2) - .289\,(p_3 + p_4 + p_5 + p_6)}{h} \tag{34}$$

$$dby_2 = .642\,(p_1 + p_2) - .321\,p_3 + .096\,p_4 - .654\,p_5 - 1.404\,p_6 \tag{35}$$

$$dbz_2 = 2.000\,(p_1 + p_2) + 1.000\,(p_3 + p_5) + .567\,p_4 + 1.433\,p_6 \tag{36}$$

$$d\kappa_2 = \frac{.278\,(p_1 + p_2 + p_3 - p_4 - p_5 - p_6)}{h} \tag{37}$$

$$d\phi_2 = \frac{-.481\,(p_1 + p_2) + .962\,(p_3 - p_4 - p_5) + 1.924\,p_6}{h}. \tag{38}$$

It should be noted here that relative orientation of high obliques can be accomplished directly in a first-order plotter by using formulas (29) to (33), or formulas (34) to (38). When the plotting instrument is equipped with counters for all orientation elements, the measured parallaxes can be inserted in these formulas, the required orientation corrections computed, and the corrections applied directly to the instrument.

The least squares adjustment of the observation equations also provides means of evaluating the accuracy with which the relative orientation can be accomplished. The weight of each element and the correlations of all elements can be computed from the results of the solution of the normal equations. The standard error of each orientation element is given by

$$m_i = \mu\sqrt{Q_{ii}} \tag{39}$$

where

m_i = the standard error of the element i.

μ = the standard error of the observation of y-parallaxes for the particular instrument and set-up.

Q_{ii} = the weight number of the element.

Before numerical values can be obtained for the standard errors of the orientation elements, the standard error μ for the high-oblique set-up of the Balplex Plotter must be determined from practical work.

Substituting each weight number into formula (39), the standard error of the correction to each element of orientation is found to be:

For Independent Pairs:

$$md\omega_2 = \mu \frac{.649}{h}$$

$$md\phi_2 = \mu \frac{3.655}{h}$$

$$md\phi_1 = \mu \frac{3.655}{h}$$

$$md\kappa_2 = \mu \frac{2.689}{h}$$

$$md\kappa_1 = \mu \frac{2.689}{h}$$

For Dependent Pairs

$$md\omega_2 = \mu \frac{.649}{h}$$

$$md\kappa_2 = \mu \frac{.680}{h}$$

$$md\phi_2 = \mu \frac{3.118}{h}$$

$$mdby_2 = 1.407\mu$$

$$mdbz_2 = 2.332\mu.$$

A simplified method of numerical relative orientation for vertical photography is described in detail in the December 1953 issue of PHOTOGRAMMETRIC ENGINEERING. An adaptation of this same procedure may also be used for high-oblique photography. By substituting these correction equations (29) to (33) for independent pairs, and equations (34) to (38) for dependent pairs, in place of those given in Mr. Tewinkel's paper, simple computation forms can be drawn up for high-oblique photography.

Following the application of one of these procedures for relative orientation, absolute orientation of high obliques may be accomplished by the established methods for vertical photography, after which the model will be ready for compilation.

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