*Methods of Computing Cantilever Control Extension**

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ABSTRACT: *Two methods are described for performing cantilever extension of ground-control analytically. One method is similar to the familiar instrumental method of control-extension employing a procedure to perform relative orientation of a model first and then to perform the absolute orientation or sealing of the model. The other method combines the relative and absolute orientation into a simultaneous solution for each stereomodel. The second method is shown to be more favorable. For cantilever extension ground-survey control is given only in the initial photograph of the strip. Investigations have been conducted using both a minimum and a redundant number of pass-points. Each method employs an iterative process with the electronic-calculator performing the tedious computations.*

INTRODUCTION

ANYONE who has had the experience of solving a photogrammetric problem, such as space resection and orientation by the analytical method using a desk calculator, understands how tedious and time-consuming the work is. Yet, compared to this simple problem, the computation of control-extension in a strip of photographs is indeed a monstrous task.

Up to now, control-extension in photogrammetry has been performed by either radial triangulation or stereo triangulation. Radial triangulation usually utilizes slotted templets as a means of horizontally controlling maps, charts, and mosaics. Stereo triangulation as performed on the precise stereoscopic mapping instruments provides horizontal and vertical control for topographic mapping. These two methods have dominated the arena of control-extension in photogrammetric mapping for approximately 30 years. It was not until the introduction of the high-speed electronic-computer that the concept of applying analytical methods to photogrammetric control-extension received warm attention.

Under the sponsorship of the U. S. Army Engineer Research and Development Laboratory, Cornell University has engaged in the study of a number of methods for solving the problems of space resection and orientation, extension of control, and bridging. These studies have been concentrated largely upon existing methods with some necessary modifications.

COORDINATE SYSTEMS AND EARTH CURVATURE

To eliminate the problem of earth curvature and to obtain the true azimuth of the principal plane of each photograph, three rectangular coordinate-systems are introduced into the computations. These are: (1) geocentric, (2) local, and (3) photographic.

The geocentric coordinate-system† has its origin at the center of the reference ellipsoid of the earth. The positive Z-axis is toward the north pole. The X and Y axes lie in the plane of the equator with the positive X-axis in the meridian plane through Greenwich.

The local coordinate-system, which varies for each photograph, is considered to have its M and N axes in a plane tangent to the reference ellipsoid with the positive N axis toward true North. The

† Shu, C. S., and McNair, A. J., "Determination of Geographic Coordinates, Flight Heights, and True Orientations for Extensions of Strips of Aerial Photographs," PHOTOGRAMMETRIC ENGINEERING, September 1956, p. 637.

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positive K axis is toward the Zenith and contains the exposure station. The photographic coordinate-system has its origin at the perspective center with the negative z axis toward the principal point. The x and y axes lie in a plane parallel to the plane of the photograph with the positive x axis, as projected on the plane of the photograph, pointed toward that fiducial mark which allows this axis to best describe the direction of flight.

The conversion from geographic coordinates of latitude, longitude, and elevation above the reference ellipsoid of the earth to geocentric coordinates of X, Y, and Z or vice versa is straightforward. With an electronic-computer the conversion requires only a second or two. In this computation, for a limited area, the surface of the reference ellipsoid of the earth can be regarded as coinciding with sea level. A matrix multiplication makes it possible to convert between the photographic coordinate-system and the local coordinate-system. Thus, for any photograph in a strip the true azimuth of the principal plane is readily established independent of convergence of the meridians.

The method of computing a cantilever control-extension is to solve for the geocentric coordinates of the exposure stations and of the pass-points, and also to solve for the relations between the photographic coordinates and the local coordinate-system of each photograph.

The relations between the photographic and the local coordinate-system provide tilt, swing, and azimuth, or pitch, roll, and crab of each photograph.

Computational Procedures

Two methods, each of which is representative of a general approach to the problems of computing cantilever control-extension will now be described: (1) A method which is very similar to the instrumental approach; (2) A method which solves relative and absolute orientation simultaneously.

(1) THE MODIFIED BRITISH ORDNANCE SURVEY METHOD

The first method consists of first computing the relative orientation for each pair of overlapping photographs in the strip. Each is called a stereogram. Convenient values are first assumed for the relative orientation elements. Then corrections to these assumed values are sought. The formulas applied to this computation are a set of first-order linear equations. These equations are derived from the concept that, if a pair of overlapping photographs are in relative orientation, the rays emitted from both exposure stations to any common image point in their overlap area must intersect. This is accomplished by reducing the volume of a four-sided figure to zero by an iterative process. The edges of this figure are the stereo base, the two rays from the exposure stations to the common image point, and the line representing the shortest distance or discrepancy between those two rays.

For cantilever control-extension, known ground-controls are provided only in the area covered by the initial aerial photograph of the strip. A minimum of there ground-control points is required to establish the correct space resection and orientation of this photograph with respect to the geocentric coordinate-system. This is computed by some other method such as the Church or Herget method. Since the relative rotational elements for each model have been obtained by the previous computation, the correct space orientation for the second photograph can be computed by means of a simple transformation process or matrix multiplication. The space resection for the forward photograph in any stereogram can be obtained if at least two ground control points appear in it to fix the scale of the model.

The control-extension can now proceed by selecting a minimum number of two pass-points which are in the triple overlap of the first three photographs. The geocentric coordinates of these pass points can now be computed. At this point the second photograph is completely oriented and resected, and two points, the geocentric coordinates of which are known, appear in the second model. These forward points are now treated as ground-control and the resection and orientation of photo three is accomplished in the same manner as photo two. Thus each exposure station in turn is located out to the end of the strip.

(2) THE MODIFIED HERGET METHOD

The second method differs from the first in that the relative and absolute orientations for each model are completed in a single step. The space-resection and space-orientation of the initial photograph are accomplished as in the first method. Then for the second photograph, six differential error equations with six unknowns can be established. These are composed of equations for the two ground-control points which are in the overlap area of the initial and the second photograph and of equations for the two pass-points, which are in the common area of the first and second models. Three of these six unknowns represent the three corrections to the approximate geocentric coordinates of the exposure station of the second photograph. The other three unknowns represent the rotational corrections for the camera axis of the same photograph with respect to the geocentric coordinate-system.

Convenient values are assumed for these elements and then corrected by an iterative process. As soon as the computations for the space resection and orientation of the second photograph are completed, the geocentric coordinates of the two pass-points can be computed by established formulae. Again these two pass-points are used as ground-control points for the orientation and resection of the third photograph in the strip. By repeating the process, the computation of the control-extension can be continued to the last photograph in the strip.

Tests and Error Studies

To test these two methods, data from a set of fictitious photographs with an average scale of 1:80,000 were processed. The computations were programmed to suit two types of high-speed electronic computers.

After the methods had been successfully tested using a minimum amount of data, tests were run for redundant data using six, ten, and fifteen pass-points per model. These tests correspond to an actual case in which every systematic error had been eliminated for each pass-point. Each pass-point selected for computation was displaced from its true position on the photograph. This was done in a random manner to simulate accidental errors in measurement. The standard error for the measurement was assumed to be 0.050 mm.

Seven sets of fictitious photographic strips, ranging from four to nine photographs per strip were tested with various random photo-image measurement errors and with different numbers of pass-points per model. The results indicate that there is no assurance that fifteen pass-points would give better results than six pass-points per model. It can also be concluded that the modified Herget method gives better accuracy than the modified British method.

The results of these sample sets fail to provide a strong confidence limit for any prediction of the range of the accumulated errors from model to model. It is apparent that after three models, the position of the ground points does not maintain an accuracy of 1:5,000. Thus the results of cantilever extension will probably not meet the required accuracy for controlling topographic mapping so long as the standard errors in photographic image measurement are 0.050 mm. or a maximum error of 0.150 mm. However, it should be remembered that similar accidental errors of this magnitude are present in any instrumental extension and would not permit the achievement of National map accuracy standards by any cantilever method.

Analytical Advantages

Although the methods mentioned were restricted to the cantilever extension, some advantages by analytical control-extension versus the instrumental control-extension can be ascertained from this study:

(1) The analytical method can introduce any number of pass-points or any existing control-points into the computation without complicating the problem, except that a redundant number of equations requires a least squares solution. As we all understand, since the advance of the electronic computer, the least squares solution by the electronic-computer has become the least troublesome routine. Six points is the minimum number of pass-points required in the control-extension to solve the space-resection and space-orientation of each new photograph in the strip. Our study made evident that using 10 or 15 random selected pass-points may not always give a better result than by using 6 points. However, it can be expected that when a redundant number of random selected pass-points are used in the solution, the resulted model will have its error more uniformly distributed over the entire area of the model, which will be the basis for mapping. In other words the model determined by a

minimum number of pass-points may result in a uneven distribution of errors; some locations in the model may tend to exaggerate the errors while in other locations the degree of errors may be less. The flexibility of the analytical method for adopting any number of pass-points or ground-control points in the control-extension was lacking in the instrumental method.

(2) When the minimum number of pass-points are used in the analytical control-extension, the pass-point distribution will be confined to certain patterns as in the conventional instrument methods. This will give the well defined pass-points on the photograph a chance to be selected in the control-extension, hence better accuracy can be expected than when the poorly defined pass-points must be used as in the instrumental method, when there is restriction in choosing the pass-points in certain restricted areas on the photograph.

(3) Due to the limitation of the plotting instruments, it is not natural that the orientations of a strip of photographs have to be divided into two separate steps—relative-orientation and absolute-orientation. By means of the analytical method, the space-resection and orientation of each photograph can be solved simultaneously. Furthermore such methods offer not only the solution of the resection and orientation of each photograph in the strip one by one, but also the simultaneous solution of the entire strip, that is, in a strip with known controls at the two ends of the strip or with known controls scattered over the entire strip. Theoretically a simultaneous solution of a strip or several adjacent strips, pooling all the available control data over the strip or strips in a least square solution, will give the best fit of the models in the strip or strips and the most probable solution for the entire scheme of the photo strip or strips. However, the amount of computations involved in solving the simultaneous equations is in proportion to the cube of the number of unknowns in the simultaneous equations. Whether the accuracy achieved by the simultaneous solution of the strip or strips will warrant the great many computations should be a subject for further study, since the quality of photographic image-points and the accuracy of the measuring devices have their own limitations.

(4) Since the measured photographic-coordinates are the initial data in the computation of analytical aerotriangulation, any known systematic errors such as lens-distortion, film-shrinkage, refraction of light rays, etc., which can be made as corrections to the photographic-coordinates, can be eliminated. In addition, the plotting instrument errors, optical or mechanical, will not exist in the analytical method, because no plotting instruments are used at this stage.

(5) If the analytical method can take the place of instrumental method in aerotriangulation, only simple stereo-plotting instruments will be needed to serve the purpose of making the contour or topographic maps.

(6) It has been proved that the analytical method of control-extension by means of the electronic-computer is faster than the instrumental method. It could be expected in the future improvements of computation speed in the electronic-computer and by the simplification of the formula and process of the analytical methods, the aero-triangulation for a strip or strips of photographs can be completed in a small fraction of the time required by the instrumental method. Also in analytical control-extension no skilled and highly trained personnel are necessary as are highly demanded in the instrumental method. Theoretically, the accuracy achieved by the analytical method should be higher than by the instrumental method, because of the certain advantages of the former that have been mentioned in this paper.

Therefore, it can be concluded that in the future, the control extension by means of the analytical methods will be much more widely accepted in the field of photogrammetry.