tional photogrammetric activity, and it was agreed that this participation is not in proportion to the extremely dynamic development of photogrammetry in this country. We also felt that during the next few years, prior to the next Congress, this situation ought to be changed.

In accordance with the decision of the International Society of Photogrammetry, experimental work along the lines described in my short talk will be continued. As a member of the Canadian and American Photogrammetry Societies, we should decide upon our participation in this activity now. Also it would be worthwhile to consider the use of our own test area on this continent in order to experiment with techniques which are in use here.

The Measurement of Elevation Differences by Photogrammetry Where No Elevation Data Exist

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ABSTRACT: *This paper presents a rapid and simple method for determining approximate ground elevation differences by photogrammetry. This method has been used in Creole for about four years as an aid to the geologist and engineer, but to the author's knowledge has never been published. It is not unlike other methods in principle, in that it measures parallax differences as an expression of differences of ground elevation. However it ignores the flying height; reduces the photographic scale of the spatial model to a given datum; uses a transparent templet for parallax and other photographic measurements; limits itself to the geometry of the lens-photographic plane, except for one final step; then utilizes one measured horizontal distance to determine the absolute height difference required, within a ten per cent precision range; and it requires the use of no special devices.*

THE Creole field geologist has been trained in the use of air photographs as an aid in planning, orientation and measurement. Through the medium of photogrammetry, he can measure or check the thickness of a stratum; or make a topographic profile for a geological cross-section.

He has been taught to make these measurements simply and rapidly with a few simple tools. These tools consist of a stereoscopic pair of vertical photographis, photogrammetric map, stereoscope, millimeter scale, triangle, needle, pencil, grease-pencil, scratch pad and transparent templet approximately of the size of a photograph.[1] It takes him about ten minutes to determine the height of a hill with an accuracy of within ten per cent. He does not have to know nor determine the photo scale, nor airplane height; nor does he worry about lack of good reference points, either vertical or horizontal. He does need the approximate focal length of the air camera. He makes one measurement on the map; the rest is done on the photographs with the aid of the templet.

[1] If the operator wishes, he may also use a conventional parallax bar instead of the templet.

For simplicity, speed and accuracy this method is considered unrivaled. Also the elimination of the camera (airplane) height and average photo scale in computing elevation differences not only avoids additional steps in the procedure, but also increases the resulting accuracy, inasmuch as these factors are at best only approximate and may, in rough terrain, cause large errors. It is in this type of terrain that elevation measurements are often made.

The method described below is not only useful for the geologist in measuring thicknesses of strata and in preparing cross-sections, but may also be applied by the engineer as a convenient means of finding stream, highway and pipeline gradients in location planning, or wherever the usefulness of air photographs can be exploited.

The formula which is used differs from the accepted text book formulas in the following points:

a) The geometry on which it is based (see Figure A) does not depart from the lens-focal plane of the air camera, until the final step. Then the absolute difference in height is found by simple ratio and proportion, using one horizontal ground distance measured on the map.

b) The photographic base (b_1) is accurately adjusted to the horizontal plane which passes through the lower point, and is not a theoretical measurement, nor an average of the center-point distances measured on the two photographs of the stereo-pair, as in most height measurement formulas.

FORMULA:

$$\Delta H = \frac{dp \cdot f \cdot B_m}{b_1(b_1 + dp)};$$

where:

ΔH = absolute difference in height in meters between upper and lower points.

dp = difference of parallax in millimeters between upper and lower points (to 1/10 mm.).

f = focal distance of air camera in millimeters (to two decimals).

b_1 = photographic base adjusted to datum passing through lower point (to 1/10 millimeter).

B_m = ground distance in meters between photo-centers (can be measured on map).

PRECISION AND SOURCES OF ERROR

The precision obtainable with this method is ± 10 per cent of the true height difference as determined by accurate ground survey methods, provided all scale measurements are taken to 1/10 millimeter and stereoscopic point transfers are accurate.[2]

Photographic tilt, if not previously compensated, will contribute error to any photogrammetric computation based upon truly vertical photographs. However, if the tilt is not excessive, or if the points being considered are not spaced too far apart on the photographs, the result will probably fall within the precision range stated above.[5] See bottom of page 777.

DEFINITIONS OF TERMS USED IN THIS ARTICLE

1. *Stereo-model:* The stereo-model represents the image area which is common to the two overlapping pictures of a stereo pair.
2. *Stereogram:* same as stereo-model.
3. *Optical center* (*c*): The optical center of an air photograph is found at the intersection of the straight lines that join the pairs of opposite fiducial marks at the edges of the photo, and is the only point around which true horizontal angles can be measured.
4. *Photographic base* (*b*): The photographic base is the distance, measured in millimeters, between the original center points of a stereo pair, when the common images of points lying at a selected datum plane are in coincidence, and with the center point images in alignment.
5. *Adjusted photographic base* (b_1): This is the photographic base adjusted to the horizontal datum plane passing through the lower point and is found in step VI of the procedure outlined below.
6. *Absolute parallax:* When two vertical overlapping air photographs are superimposed with their bases (lines connecting their centers) in alignment and with the images of a point lying at a given elevation in coincidence (see Figure A), all other points in the

[2] Needless to say, the calculations must be correctly done, with special care in carrying the decimal point.

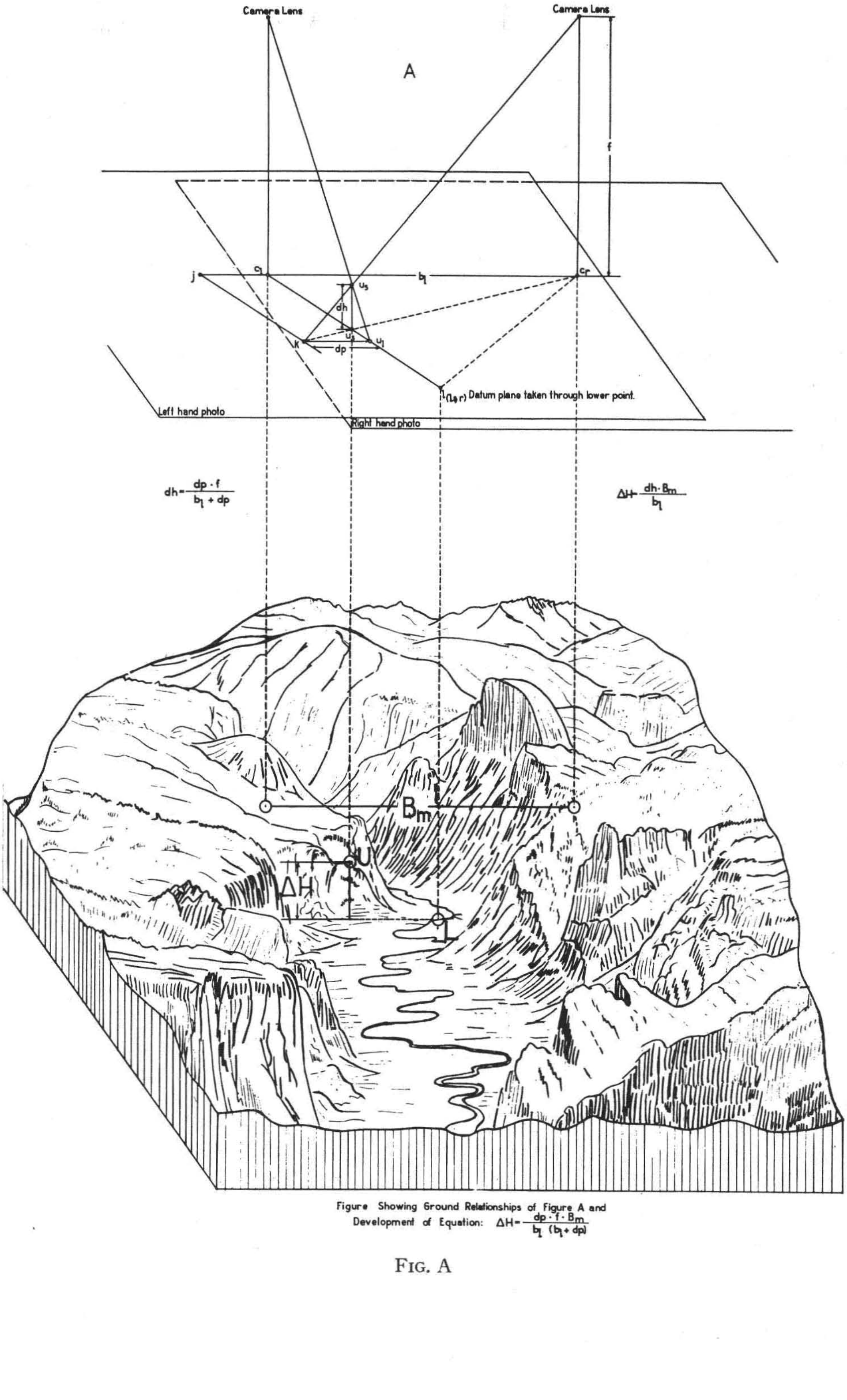

Figure Showing Ground Relationships of Figure A and Development of Equation: $\Delta H = \frac{dp \cdot f \cdot B_m}{b_l\ (b_l + dp)}$

FIG. A

stereo-model which lie at the same elevation will also coincide. These points are said to have the same "absolute parallax."

7. *Difference of parallax* (dp): Difference of parallax is the measurement in a stereo-model of the difference in absolute parallax between two points which lie at different elevations. Parallax differences are always measured parallel to the photographic base. Difference in parallax can be measured by using a parallax bar, or a transparent templet. With the parallax bar (containing floating dots and a micrometer) a stereoscopic view is indispensable; with the transparent templet method, the measurement of parallax differences can be made directly on the photographs with a scale.

PROCEDURE BY STEPS, USING TEMPLET

I. Locate on each photograph of a stereoscopic pair its optical center and the upper and lower points to be measured.[3]

II. Transfer the center points carefully, using a stereoscope.

III. Draw a fine pencil line from the center point of each photograph through the transferred center. This line represents the direction or bearing of the photographic base line (b).

IV. Prepare the templet for measuring b_1 and dp by placing a sheet of transparent material over the right hand photograph. Punch through the templet, at the center, upper and lower points of the right photograph, and also transfer the base line (b).

V. Now, discarding the right hand photograph, place the templet over the left photo, carefully placing the lower point of the templet in coincidence with the lower point of the left photograph. Then, pivoting on the lower point, swing the templet into adjustment, so that the base line of the templet and that of the left picture are either superimposed or parallel.[4]

VI. With the lower points in coincidence and base lines superimposed or parallel, mark on the templet the position (on the base line) of the center point of the left photograph. Now measure on the templet to 1/10 millimeter the distance between the two centers. This is the distance b_1 of the formula, which is the photographic base adjusted to the datum passing through the lower point.

VII. Without changing the relative positions of photograph and templet, mark on the templet the position of the upper point of the left photograph.

VIII. Repeat operations, V, VI and VII, with the *upper* points in coincidence instead of the lower. That will give a new center point position of the left photograph along the base line, and also another position of the lower point taken from the left photograph.

IX. From an examination of the templet, it will be seen that it contains two upper and two lower points, as well as two positions of the left center point along the base line. The parallax difference (dp) can now be measured (to 1/10 millimeter) in *three different places*, all three of which should be equal, provided there is no relative tilt nor human error.[5] The parallax difference (dp) is measured on the templet between the two center point positions of the left photograph along the base line. Also the two upper points and two lower points should be the same distance apart from each other and equivalent to the paral-

[3] Transfers should be made stereoscopically, preferably by perforating the photograph at each point of detail with a small round needlehole. Special care and accuracy in this operation will greatly improve the result.

[4] If there is a difference in scale between the two photographs, the base lines will not coincide, in which case they should be swung into parallel alignment. This difference in scale will be the result of either camera tilt, or of a change in height of the airplane between exposures.

[5] When tilt exists, it can usually be perceived in the stereoscopic view and its direction approximately ascertained, in which case the three dp measurements can be weighted and the one (or two) least affected by tilt chosen.

lax difference (dp) measured along the base line.

X. We now have all the elements of the formula necessary to calculate the difference in height (ΔH), except the value of B_m, which is the ground distance in meters between the two photo-centers. This distance can be scaled off on the map. Preferably the map should have been compiled by the radial-line slotted-templet method or its equivalent, and should already contain the photographic center points.

With these values, we can now solve the equation given above and repeated here:

$$\Delta H = \frac{dp \cdot f \cdot B_m}{b_1(b_1 + dp)}.$$

We have now found the ground height difference between two selected points on a stereo pair of vertical air photographs.[6] The same method may be applied to calculate the relative height differences of a number of points which fall in the same stereo-model. In this case, the lowest point, or a point lower than all the points to be calculated, should be used as a reference or datum. Thus, a single value for b_1 will serve for all points in the model and the height difference of each point will be referred to that low point or datum.

In the event it may be preferred to use a parallax bar to measure the parallax difference (dp), the two photographs of the stereogram must be placed with their base lines in longitudinal alignment, inasmuch as parallax differences are always measured parallel to the photographic base.[7] The distance of separation of the photographs will, of course, correspond to that most convenient for the operator and to the type of stereoscope used. With a parallax bar, parallax differences are usually measured to 1/100 millimeter.

[6] Where height differences (ΔH) are desired in terms of feet, or any other unit of measurement, the term B_m should be expressed in the unit of measurement desired. Likewise the other terms, which are given herein in millimeters, may be expressed in inches (to 1/100) if so desired.

[7] Even when the parallax bar is used to determine parallax differences, the templet will be needed to obtain the value of b_1.

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