Determination and Correction of Systematic Errors in the Fundamental Operations of Aerial Triangulation

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ABSTRACT: *For the correct treatment of aerial triangulation procedures—instrumental or analytical—it is of fundamental importance that the systematic errors of the various operations be determined as correctly as possible. In this paper new methods for the determination of the inner orientation of the aerial camera, and the systematic disturbances in connection with the photography are demonstrated. All derivations are entirely founded upon the method of the least squares. Only in this way can reliable and unique information on the obtainable accuracy be determined. The sources of systematic errors obviously have to be determined in real photography and plotting conditions.*

INTRODUCTION

AERIAL triangulation consists of a series of operations, each of which necessarily introduces errors into the procedure. The discrepancies which always are found in superfluous control may be caused by large, systematic or accidental errors. The treatment of the discrepancies in order to obtain the most probable results of the triangulation procedure together with the reliability of the results is a very important task in all triangulation work.

First of all it is very important to remember that a correct treatment of the adjustment procedure necessarily requires that the nature of the sources of errors be clearly known. In the up-to-now usual adjustment procedures, *either* only systematic (constant) errors *or* only accidental errors are assumed to have caused the discrepancies. The adjustment procedures consequently are founded upon these assumptions. This, however, cannot be correct since both types of errors doubtless are always present; still worse, there certainly are different types of systematic errors present, not only constant errors but also systematic errors that can vary from model to model. Also the fact that the individual models are not strictly adjusted up to the same standard causes errors that will have systematic character, although they originally may be regarded as being caused by more or less accidental circumstances.

To obtain more reliable results of the aerial triangulation it is necessary to devote more research to the sources of the systematic errors and to the treatment of the individual models.

In other words, the fundamental operations of the photogrammetric procedure have to be carefully investigated. The fundamental operations are:

1. *The photography*
2. *The reconstruction of the bundles of of rays, optically, mechanically or numerically*
3. *The relative orientation*
4. *The absolute orientation and the coordinate measurements.*

Formally, we can distinguish between analytical and instrumental aerial triangulation. In reality, however, the two methods are very similar from a theoretical point of view. In both cases the same photographs are used. The difference is mainly concerned with the instrumental circumstances. There are doubtless larger systematic and accidental errors in the universal plotting machines than in the stereocomparator, and the systematic errors are doubtless more irregular in the former instrument. But from a theoretical point of view the conditions are very similar. The systematic errors of the universal plotting machines as well as of the stereocomparator have to be determined.

Below some new methods for the determination of systematic errors of the fundamental operations will briefly be described.

1. The Photography

The photographs are never mathematically-correct central-perspectives of the ground. The most important systematic disturbances are probably caused by the lens (radial distortion and asymmetries), the deformations of the negative material, and the influence of the atmospheric refraction. Furthermore, the earth's curvature causes an effect which in vertical photographs is very similar to the radial distortion.[1]

The combined effect of these factors is of systematic character, That, however, is dependant upon a number of circumstances, as for instance, the flying altitude, the temperature of the camera etc. Therefore it is necessary to determine the systematic disturbances of the photographs for each actual case of photography. First of all it is necessary to investigate which factors may cause systematic errors, particularly the temperature variations, but also other factors, for instance filters etc.

Such investigations must be performed under conditions which are as similar to the actual photography conditions as is possible, and which allow the influence of the actual factors to be distinguished, as well as possible, from other factors that may have similar effects upon the bundles of rays.

1.1. Camera tests from high towers

For such investigations a high radio tower has been used, see Figure 1.

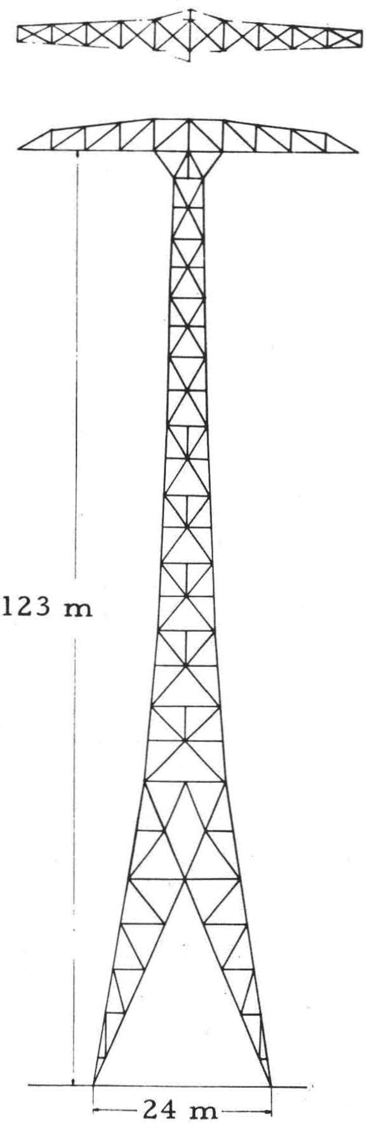

Fig. 1. The radio tower from which the camera tests were performed. Heavy antennas and stays make the tower very stable. Even at strong winds no deformations or tremblings could be determined. There is a lift and a small darkroom at the top.

The tower is about 125 meters high and from several points of view is very suitable for the camera tests. Under the tower a system of control points was constructed on the nearly flat ground, (see Figure 2a and 2b). The coordinates of the control points and of the external perspective center of the cameras were determined with high precision, by geodetic methods. The radial standard errors of the coordinates were found to be of the magnitude 2–3 mm. The negative plane of the camera was levelled with high precision, by spirit levels. For the tests glass plates were used, the flatness of which was measured with high precision in connection with the image coordinate measurements.

Since the camera was adjusted very accurately above the center of the control point figure on the ground, and the eleva-

[1] It must be emphasized that the atmospheric refraction and earth's curvature in *oblique* photographs have another effect than radial distortion.

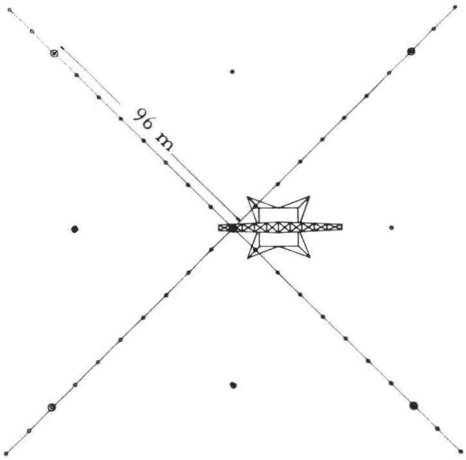

FIG. 2a. The control points on the ground. The five especially marked points served as first-order points from which other points were intersected.

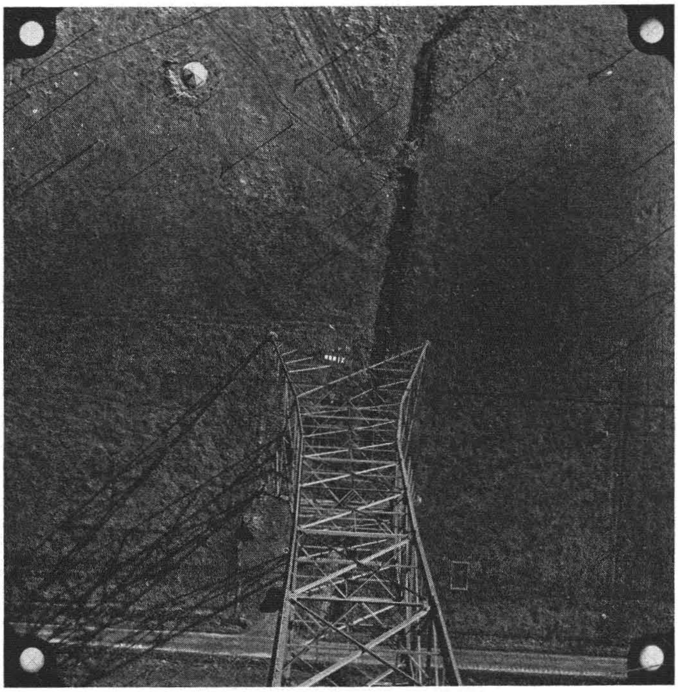

FIG. 2b. One of the test photographs from the tower. The resolving-power targets are clearly visible.

tion of the camera was known, the inner orientation obviously could be directly determined from simple image coordinate measurements and computations. The radial distortion was also easily determined from the scale variations of the circles into which the control points could be combined. Since all circles contain a great number of superfluous points, the final data of the inner orientation could be determined as results of adjustments according to the method of the least squares [1]. Simultaneously the standard errors of the observations and of the elements of the inner orientation could be determined. On the ground also a number of resolving power targets were located in the center and in the corners of the field.

With this arrangement it is very simple to study the influence upon the imaging procedure and the bundles of rays of, for instance, different filters, temperature influence, vibrations etc. Also asymmetries of various kinds can be determined in connection with the adjustment.

Up to now four different cameras have been preliminarily investigated in the tower.

Different filters have been investigated and some preliminary tests have been performed in different temperatures. Photographs were also taken with one of the cameras in normal temperature and after cooling it in a refrigerator. The most effective cooling can be performed with chemical means.

As an example of the performed tests, the results from a camera Eagle III with a Ross Xpress wide-angle lens will be described as follows:

Three exposures were made under the same conditions and the negatives were measured independently. For the image coordinate measurements an Autograph A7 was used. The projector of the Autograph was first checked with the aid of a high-precision glass-grid. The grid was checked with a special procedure, see [2].

For the computations of the measurements, the procedure in [3] was used. In this way a very convenient and quick adjustment in accordance with the method of the least squares is obtained. The results are demonstrated in Tables 1, 2 and 3.

TABLE 1

CAMERA CONSTANT c (CALIBRATED FOCAL LENGTH)

Experiment	c
	mm.
1	126.35
2	126.37
3	126.35
Average	126.36

Standard error of one determination: $m_c = 1\ \mu$

where μ is the standard error of the image coordinate measurements. This is demonstrated below.

TABLE 2

PRINCIPAL POINTS. DEVIATIONS FROM THE IMAGE CENTER

Experiment	dx_0	dy_0
	mm.	mm.
1	−0.40	+0.48
2	−0.41	+0.49
3	−0.41	+0.47
Average	−0.41	+0.48

Standard errors of one determination:
$mx_0 = 2{,}5\ \mu$
$my_0 = 2{,}5\ \mu$

TABLE 3

STANDARD ERRORS μ OF THE COORDINATE MEASUREMENTS FROM THE ADJUSTMENT PROCEDURE

Experiment	Circles					
	1	2	3	4	5	6
	Microns					
1	3.1	2.3	3.9	2.5	7.9	9.2
2	1.2	5.4	1.6	3.4	6.2	7.3
3	3.0	1.8	2.5	3.5	5.5	6.8
Averages	2.4	3.2	2.7	3.1	6.5	7.8

The average of all standard error determinations is $\mu = 4.3$ microns.

Probably due to asymmetries of the lens and small irregularities in the flatness of the plates, the standard errors of the observations increase with the radii of the circles.

The final results of the determination of the elements of the inner orientation are consequently

Camera constant (calibrated focal length):

$$c = 126.36 \text{ mm.} \pm 0.003 \text{ mm.}$$

Corrections to the image center:

$$dx_0 = -0.41 \text{ mm.} \pm 0.008 \text{ mm.}$$

$$dy_0 = -0.48 \text{ mm.} \pm 0.008 \text{ mm.}$$

The large corrections depend upon the fact that the supporting glass plate in the camera has been replaced but not correctly adjusted.

The radial distortion curve is demonstrated in Figure 3a. The three independent determinations of the curve are shown. (See also Figures 3b–3d.)

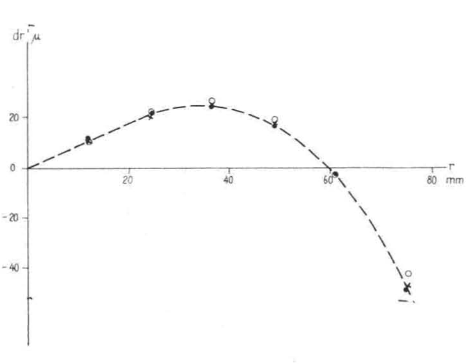

FIG. 3a. Radial-distortion curve, Ross Xpress. $c = 126.36$ mm. The agreement between the three independent determinations is excellent.

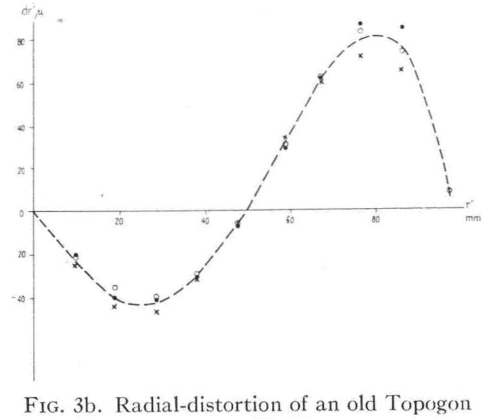

FIG. 3b. Radial-distortion of an old Topogon camera. $c = 99.33$ mm.

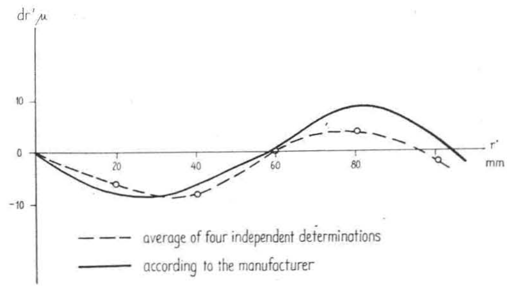

FIG. 3c. Radial-distortion of an Aviotar camera. Aviotar 27.

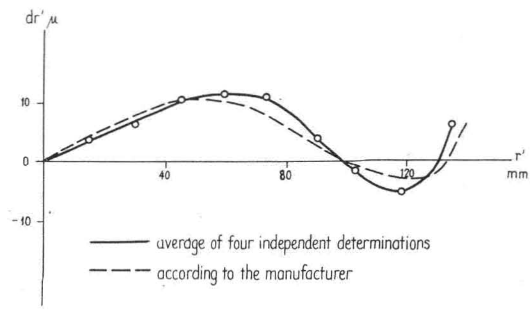

FIG. 3d. Radial-distortion of an Aviogon camera. Aviogon 29.

Up to now only preliminary tests have been performed in this way. It is quite obvious, however, that the method can be used for a very convenient and reliable, complete determination of the inner orientation of aerial cameras and in particular for the investigation of the influence of varying factors upon image quality from a photographic and geometric point of view. Due to the limited distance between the camera and the ground (125 meters), corrections can be applied to the obtained value of the calibrated focal-length in order to determine the value which corresponds to high flying altitudes (infinity).

1.2. CAMERA TESTS FROM THE AIR

In order to determine the systematic disturbances of the bundles of rays under real flying conditions, the method described in [1] has been applied.

A special test field was constructed in a

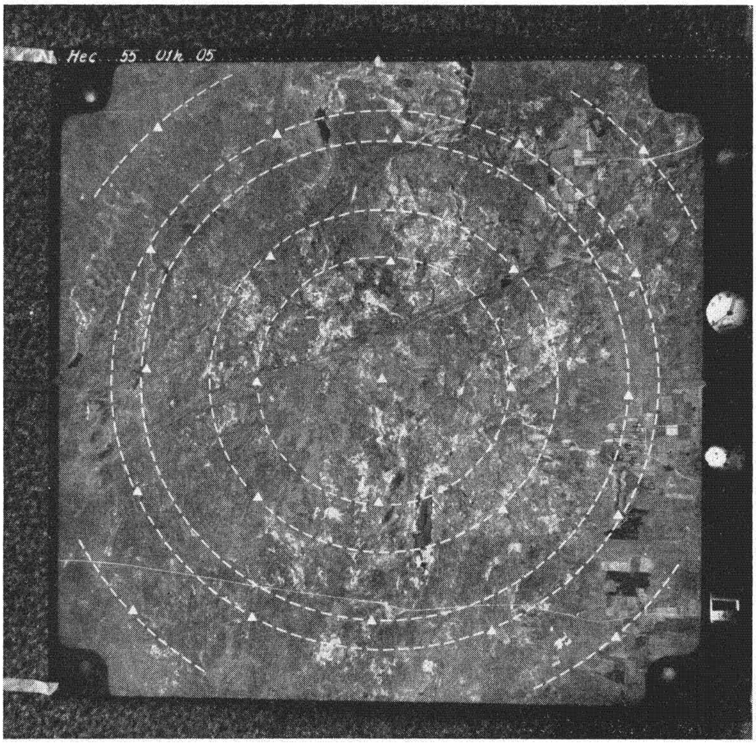

FIG. 4. Aerial test photograph (film) from the Öland test field. The control points and the circles are especially marked in the copy.

nearly flat bed-rock area on the island of Öland, see Figure 4.*

25 points forming a grid with the format 6×6 kilometers (about 4×4 miles) were marked in the rock and signalled with round, white, concrete plates. The coordinates of the points were determined with geodetic methods. The standard errors in planimetry and elevation are about 20 mm.

The test field was originally planned for flying altitudes up to 5,000 meters but will be extended and made more dense for larger and smaller flying altitudes.

A number of tests have been performed with the test field and an example will be demonstrated here.

The image coordinates of the diapositives of the photograph, Figure 4, were measured in one of the projectors of the Autograph Wild A7 nr 310, and independently also in another Autograph Wild A7 nr 362. The projectors were first calibrated with a high precision glass grid [1]. From the computations according to the formulae systems in [1] the radial distortion curves of Figure 5 were obtained. The standard error of the image coordinate measurements was of the magnitude of 0.01 mm. as an average, and the standard error of the radial distortion curves consequently was about 5 microns.

No absolute values of the elements of the external or interior orientation can be determined in this way, but systematic errors of the bundles of rays obviously can be determined under real photographic conditions. The residual errors after the correction of the radial distortion can be found as a vector diagram, see Figure 4 in [6].

From investigations of this kind, from photographs which have been taken under different atmospheric conditions, the influence of the atmosphere can be determined. For the continued investigations, special determination of the atmospheric conditions (temperature, humidity etc.) will be performed with the aid of the usual meteorological instruments, which are sent up in the atmosphere, simultaneously with

* The test field was constructed in cooperation between The Geographical Survey Office, The Board of Land Survey and The Division of Photogrammetry of the Royal Institute of Technology, Stockholm.

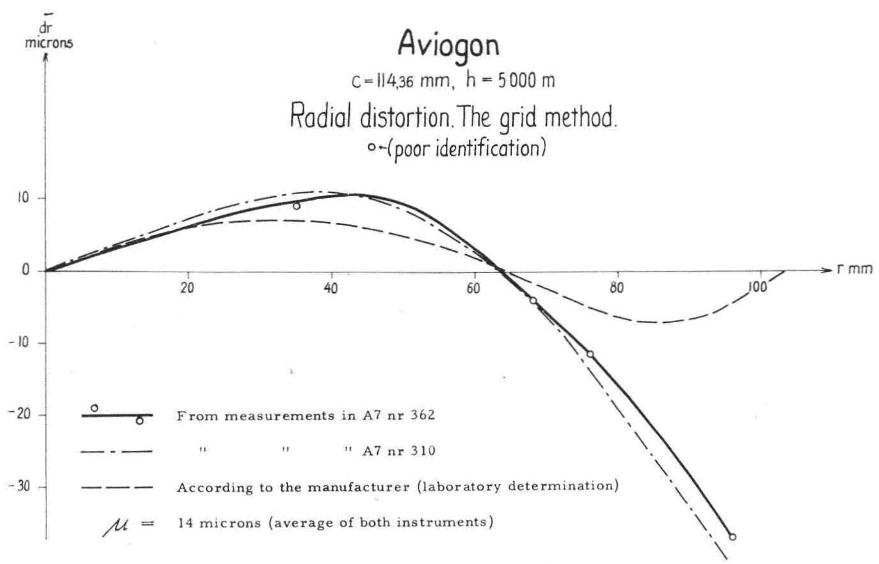

FIG. 5. Radial-distortion curves of the photograph Fig. 4. Two independent determinations. Standard error of the average curve about 5 microns.

the photography.

It is also possible to determine some types of systematic errors of the aerial photographs only from y-parallax measurements as described in [4]. It has to be noted, however, that the influence of the earth's curvature will not be determined from the y-parallaxes. This systematic influence can be computed from wellknown formulae and added to the radial distortion curve from the residual y-parallaxes, (See Figure 6).

We will here demonstrate the determination of the radial distortion curve of the aerial pictures from the Öland test area, one of which is demonstroted in Figure 4. See Figures 7 and 8. The y-parallax measurements were performed in the Autograph Wild A7 nr 310. The radial distortion curves therefore also include the errors of the instrument itself.

It should also be noted that the original y-parallaxes in the lateral orientation points may be systematically influenced by the unknown distortion. This may cause systematic influence upon the distortion curve via ϕ and bz. The curve can be corrected from iterations of the procedure and from a determined "ϕ-curve."

1.3. CORRECTION OF THE SYSTEMATIC ERRORS

From the determined systematic errors, automatic correction devices in the plotting instruments can be constructed in different well known ways. The accuracy of the correction devices have always to be checked, however; this can easily be done with the pictures that were used for the original determination of the systematic errors or with accurately measured grids. Simultaneously the systematic errors of the plotting instrument are obtained. Varying base- and elevation settings therefore must be used for the determination of residual systematic errors. The deformations of the instruments and the residual adjustment errors have proved to be of importance. The residual systematic errors normally have to be taken into account in a numerical way. See [5]. If direct image-coordinate measurements are to be used for the aerial triangulation (stereo-comparator measurements) the image-coordinates and coordinate-differences have to be numerically corrected for the known systematic errors. The standard errors of the corrected data are of fundamental importance for the determination of the accuracy of the aerial triangulation.

It is certainly not possible to determine and correct all systematic errors in all individual cases. First of all, the definition of the concept systematic error in comparison with accidental error is rather weak. For obvious reasons it is not possible to investigate the systematic errors in all individual pictures and therefore extrapolations from

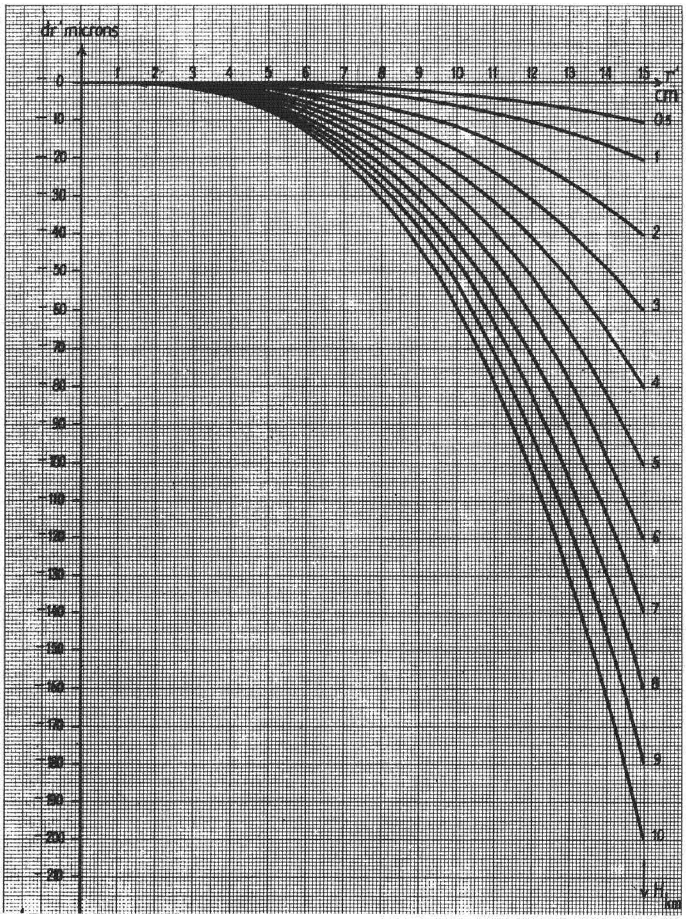

FIG. 6. Radial-distortion effect of earth's curvature, according to *Leijonhufvud*. $c = 115$ mm. Inst. f Fotogrammetri Stockholm 70. Jan. 1957.

rather few examined photographs have to be made. Doubtless there are varying systematic errors from photograph to photograph, for instance due to variations in the atmosphere, in the temperature, in the film base or in the glass plates which never can be determined in each individual case. The residual-errors, therefore, must be treated as accidental-errors although systematic-residuals always are present. The more careful the systematic-errors have been determined and corrected for, the more justified is the assumption that the residual-errors can be treated as accidental-errors. For this problem the wellknown *central limit theorem* from mathematical statistics is of the greatest importance. According to this theorem the resulting errors of a series of operations approximately form a normal distribution, even if the errors of the individual operations may have other distributions.

However, when the systematic-errors of the fundamental operations have been determined and correction made as carefully as possible, the residual-errors have to be determined in some way for the study of the error propagation. The best way is to use the principles of the method of the least squares also for this task.

2. The Treatment of the Relative Orientation

Relative orientation means that all corresponding rays of two overlapping bundles of rays shall be brought to intersect simultaneously. This condition is fundamental for instrumental as well as for analytical triangulation.

The lacking intersection means presence

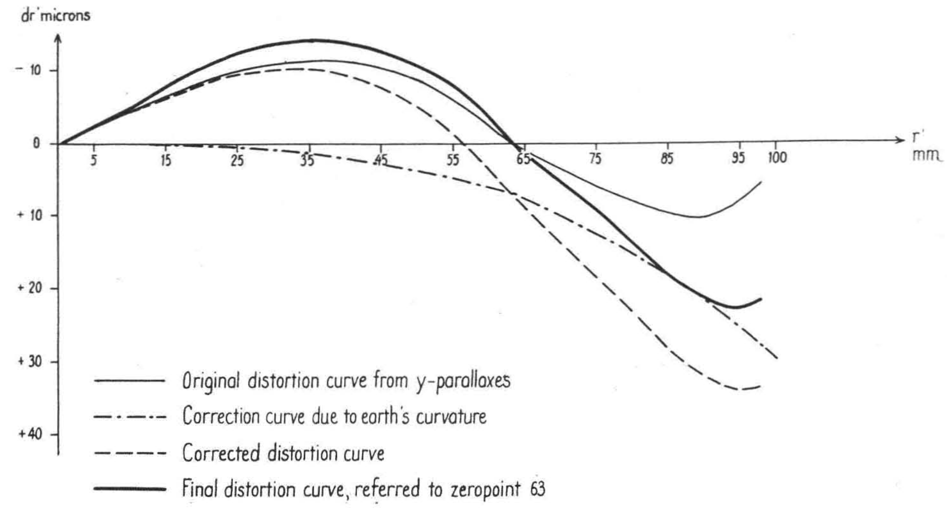

FIG. 7. Determination of the final radial-distortion curve according to the y-parallax method. The correction procedure. Aviogon. $c=114.36$ mm. $h=5{,}000$ m.

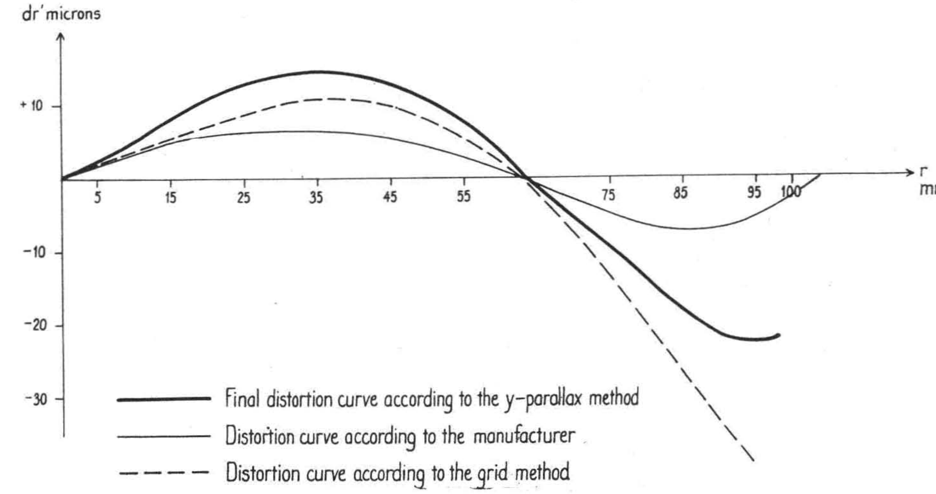

FIG. 8. Comparison between the results of the grid method and the y-parallax method. The agreement is rather good. The y-parallax method, as well as all parallax methods have to be regarded as approximate methods in comparison with the grid method. Aviogon. $c=114.36$ m. $h=5{,}000$ m.

of y-parallaxes in the plotter or in the figures of the analytical procedure.

Exact intersection can never be obtained since the procedure necessarily requires measurements, or at least estimations, of y-parallaxes or y-coordinate differences.

The actual quality of the relative orientation can most conveniently be determination of residual y-parallaxes by direct measurements (in the plotters) or by computation (in the analytical procedure). From residual y-parallaxes in suitably located points, corrections can be computed with convenient formulae to the preliminary data of the elements of the relative orientation, and (or) to the preliminary coordinates of the individual models and (or) of the entire triangulation strip [5]. Also here the method of the least squares is the most suitable and convenient solution.

Such corrections are of similar character as corrections for systematic errors, provided that the y-parallaxes themselves have been corrected for systematic errors of the reconstructed bundles of rays.

Finally, from the residual y-parallaxes in at least nine suitable points, the standard error of the y-parallax determination can be computed from convenient formulae according to the method of the least squares. *This standard error can be regarded as the statistical expression for the inevitable accidental errors of the orientation procedure*, and is of fundamental importance for the determination of the quality that can be expected from the entire triangulation procedure after an adjustment according to the method of the least squares. All expressions for the standard errors of the final coordinates of the triangulation procedure must contain this factor. Such expressions are always constructed as a product between the standard error of the fundamental observations and the square root of a weight number. In this weight number, among other things, the geometrical data of the photographs, the number of photographs etc. are included.

Summary

For the photogrammetric measuring procedure, as well as for all measuring procedures, the determination of the systematic sources of errors of the fundamental operations must be carefully performed if reliable results of high precision are wanted in the final results of the procedure.

The determination must be performed under conditions which are as similar as possible to those of the practical application of the measuring procedure. The method of least squares is the finest tool for the detection and determination of the systematic errors.

On the other hand the application of the method of least squares for the adjustment of the discrepancies of the triangulation in superfluous control points, and for the study of the error propagation, necessarily requires that at least the most important systematic errors and the discrepancies of the fundamental operations are corrected, optically, mechanically or numerically.

These facts must be remembered in all cases of aerial triangulation, instrumental or analytical, but are frequently neglected.

There is little to be gained in accuracy from the analytical aerial triangulation if those systematic errors which today are still present in the aerial photographs are neglected. Even if the accidental errors of the instrumental measurements can be decreased from 4–5 microns (the A7) to 1–2 microns (the stereocomparator) this means means very little in comparison with uncorrected systematic errors of the magnitude 10–20 microns or larger, depending upon the camera, flying altitude etc.

We certainly must start at the beginning in photogrammetry and devote much more research to the fundamental operations.

The methods which have been briefly described in this paper for the determination of the inner orientation of the aerial camera are doubtless of practical importance. Photographs from high towers can be used for absolute determinations of the elements of the inner orientation, more convenient, accurate and economical than with up to now usual methods.* The elements of the inner orientation are obviously defined in a statistical way in accordance with the method of the least squares. The entire bundle of rays is treated simultaneously.

The computations are very simple and give immediately unique solutions in accordance with the method of the least squares.

For special investigations of varying sources of systematic errors the tower method is very suitable.

The tower method in combination with the test field method will give the most reliable determination of the data of the inner orientation of the camera and the systematic errors under real flying conditions.

Finally, the y-parallax method will allow every photogrammetrist to check all individual models directly in connection with the plotting.

The latter method, however, has to be regarded as an approximate method in comparison with the grid methods.

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* According to information from Mrs. Norton, Fairchild Camera & Instrument Corp., New York, about 500 man hours normally are required for a complete determination of the inner orientation of an aerial camera using conventional methods.

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*Photogrammetric Methods in Reforestation Surveys**

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APPRECIABLE progress has been made during the last decade in the management of our forest lands in British Columbia. Artificial reforestation through planting and seeding is carried out by government and industry on an increasing scale.

Regeneration surveys are conducted on logged and burned lands to establish the state of reproduction and to determine the areas which need reforestation.

In the Douglas fir region of this province, the Forest Service requires that these surveys traverse the area to be examined with strips not more than 10 chains apart. At intervals of 1.25 chains along each strip a group of 4 milacres has to be tallied. Acceptable stocking is defined as 31 per cent or more of the milacres stocked, provided not more than four consecutive groups of 4 milacres are completely blank. Areas of 10 acres or more that are not satisfactorily stocked have to be delineated on a map of a scale of not less than 20 chains to the inch. These standards apply to forest lands where a period of eight years has elapsed since logging or burning.

These specifications are minimum requirements. It is apparent that the costs of such surveys will be appreciable when conducted over larger areas. The results may be statistically sound but it is obvious that a 10 chain spacing of examination strips can only provide a very rough estimate. The field-examiner's guess of what is in between the strips constitutes the sole basis for evaluation of about 90 per cent of the area covered. In other words, this type of survey answers one question only and that is: The number of trees established per acre. It does not give any description of the stand.

The use of air photographs and their detailed interpretation in the conduct of reforestation surveys can materially improve the results and, at the same time, cut down the costs.

The basic reason for this statement is simply the fact that an experienced interpreter can see on high-quality air-photos of approximately 20 chain scale, coniferous seedlings 2 to 3 feet high. If the photography is flown in late fall, these small trees can even be seen under dense herbaceous cover of Willow and Bracken fern.

This fact then enables the forester-interpreter to delineate on the photographs, without any field examination, three broad classifications:

1. Areas which are obviously fully stocked
2. Areas which appear partially stocked
3. Areas which appear not stocked.

What he cannot see, of course, are small seedlings of less than 2 feet height.

With this classification the costly field sampling with examination strips every 10

* Presented at meeting of the Puget Sound Section of the Society, Victoria, B. C., December 4, 1956.