EXTERIOR ORIENTATION IN PHOTOGRAMMETRY

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the line of intersection between the photographic and Survey planes. The positive
Y-axis points toward the North.

The rotation operations and matrix multiplications then take place as indicated
in Figure 6, and lead to the Church orientation matrix.

As stated earlier, the Church system is based directly upon the standard Eulerian
angles. Euler’s angles are three angles chosen to fix the directions of a new set of
rectangular space coordinate axes (photo) with reference to an old set (Survey). They
are the angles between the old and new Z-axis, θ, (corresponds to t), the angle be­
tween the new x-axis and the intersection of the new xy-plane with the old xy-plane,
ψ, (corresponds to s), and the angle between this intersection and the old x-axis, ϕ’
(corresponds to α−180°) (6) Figure 8 illustrates this relationship.

A major disadvantage of the Church system is that it breaks down for the ideal
aerial photograph, the true vertical, the primary purpose for which it was developed.
When there is no tilt, the two planes are either parallel or coincident, and there is no
line of intersection (or principal line); thus the angles of swing and azimuth are
undefined.

Use of Photogrammetry in Architecture and Other
Civil Engineering Construction

M. PLOTNICK,
Minerva Str. 29, Zurich 7, Switzerland

INTRODUCTION

Architectural Photogrammetry has de­
volved into a field of considerable tech­
nical and professional importance, during its
use since World War II. Sweden, France, The
Netherlands and Switzerland have found it
advisable to record their historical Architec­
ture by Photogrammetry. In the U.S.A. de­
vvelopments in this field have progressed at
Ohio State University’s School of Architec­
ture and Landscape Architecture, combined
with the Institute of Photogrammetry, Geodesy,
and Cartography. Professor Bertil Hallert, Director of the Photogrammetric
Institute of the Royal Swedish Institute of
Technology at Stockholm, published a paper
on the extensive work done there in determi­
ing accuracy and the theory of error, in archi­
tectural and other terrestrial photogram­
metry.

Photographic surveying of the historic
buildings has been carried out systematically
in Belgium during the years 1955/56 and a
publication of “Photographic Surveying of
Historic buildings in Belgium” was issued by
the director of the “Service de Topographie
de Photogrammetrie du Ministère des Travaux Publics,” Ing. F.J.G. Cattelain &
Ing. P. Vermeir.

A demonstration of the practical uses of the
method of stereophotogrammetry as applied
to Architecture/Archaeology is given below.
It has also proved its worth in city planning,
mining development and other civil engineer­
ing work, where the projects are investigated
and studied from elevation plans.

This paper is based on a project in archi­
tectural photogrammetry at the Swiss Fed­
eral Institute of Technology, in which the
WILD phototheodolite and WILD auto­
graph A-5 were used. However this procedure
does not need specific instruments but can be
carried out with any standard existing instru­
ments. Presented in this paper are some sim­
ple concepts of architectural photogram­
metry, which may serve as an aid in explain­
ing the relation of the subject to other
branches of civil engineering.

The project under review involved the
preparation of elevation plans at a scale of
1:100 of 50 ancient buildings within the old
city of Lucerne, Switzerland. Although mod­
ern buildings are being constructed in this
part of the town, too, the town-planning office
aims at retaining as much as is possible, the
character of the “old city.” The elevation
plans of the buildings to be demolished and
of the neighboring houses will be of great
assistance in this respect. The classical method involves far greater expenditure since scaffolding is necessary.

Equipment for taking photographs specially suitable for the task has been created and perfected by “Officine Galileo” Florence, under technical control of M. E. Santoni.

**General Principles**

As a first preliminary step one must decide on the scale to be used in the stereoplottter. This depends on the scale of plotting, the object distance, and the dimensions of the apparatus. In terrestrial photogrammetry the distance between the camera stations and the points to be plotted is introduced along the Z axis. In the Autograph A-5, the range of distance is from 114 mm. to 500 mm. The machine scale is so chosen that the minimum and maximum distances are within these limits, bearing in mind the transmission ratio from the stereoplottter to the drawing table.

At each of the camera stations $O'$ and $O''$, photographs usually are taken horizontally. The distance between the two stations, the so-called base $b$, is measured (Figure 1). It must be selected in relation to the object distance. (Base $b$ is about a 10th of the object distance.) Furthermore, the base must be parallel to the object plane, in order to get the true elevation plane.

Any point $P$ on the surface of this overlapping range will thus be represented by points $P'$ and $P''$ respectively. In the plotting instrument, the positions of each surface point $P$ can be mapped at a reduced scale from its images $P'$ and $P''$, provided that both the interior and exterior orientations of the cameras can be reproduced. The interior orientation includes the principal distance and the position of the photoplane relative to the projection-center. The exterior orientation corresponds to the absolute position of the cameras in space, in the given system.

In terrestrial photogrammetry, the outer orientation of the camera (phototheodolite) relative to the base and the horizontal, as well as the base length and the heights of the two stations, are known. (See Tables I, II.) Thus all orientation can be set in the stereoplottter right at the beginning of the operation.

Photographs are normally taken at right angles to the base: the computed base reduced to the machine scale is set as $bx$. Should there be any difference in height between the stations, it is introduced—reduced to scale as $by$. When the photographs are inclined relative to the horizontal, the angle of tilt, as set in the phototheodolite, is introduced to the

![Figure 1](https://via.placeholder.com/150)

**Table 1**

<table>
<thead>
<tr>
<th>MODEL N°</th>
<th>BASE</th>
<th>HL-HR</th>
<th>$bx$</th>
<th>$by'$</th>
<th>$by''$</th>
<th>$b z'$</th>
<th>$b z''$</th>
<th>$\phi'$</th>
<th>$\phi''$</th>
<th>$\omega'$</th>
<th>$\omega''$</th>
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<td>+1</td>
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<td>0.0000</td>
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<td>+8</td>
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<td>19</td>
<td>2.500 m</td>
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<td>0.0000</td>
<td>0.0000</td>
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<td>+2</td>
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<td>0.0000</td>
</tr>
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<td>1.80</td>
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</tr>
<tr>
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<td>0.0000</td>
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<td>0.0000</td>
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</tr>
</tbody>
</table>

**Table 1**

**Model Scale 1:50**

$F/L = 165.02$ mm.
stereoplotters on both cameras as an $\omega$ rotation. (In our project the phototheodolite was set up provisionally, and the maximum angle of tilt upwards used was about 18 g.) If the photographs are not taken at right-angles, but are swivelled to the base, then the two camera axes are parallel to one another, turned relative to the base by the horizontal angle $\phi$.

The base can be divided into two components $bx$ and $bz$. According to the formula:

$$ bx = b \cdot \cos \phi $$
$$ bz = b \cdot \sin \phi $$

where $\phi$ = angle of swivel (see Figure 2). These values are of course to be reduced on the plotting scale of machine.

The operation of plotting a model consists in setting the dimensionally-seen floating mark, by means of the two hand wheels, $X$, $Y$ and the foot disc $Z$ along surface lines to be mapped. In terrestrial photogrammetry, it is an advantage when the two stations of the base ($L$ and $R$) are at about the same height. Thus the maximum coverage of the model in the vertical direction by the two exposures is achieved.

To study the area surrounding the object to be measured by this method, it is necessary to first plan the number and location of the stations which will be fixed on the ground. However it is not necessary to establish the absolute position of the stations, since we are dealing with relative relationships.

### Table 2

<table>
<thead>
<tr>
<th>MODEL NO</th>
<th>BASE</th>
<th>ML-HR</th>
<th>$bx$</th>
<th>$by'$</th>
<th>$b''$</th>
<th>$b'x$</th>
<th>$b'y'$</th>
<th>$b''y'$</th>
<th>$\omega'$</th>
<th>$\omega''$</th>
<th>$\kappa'$</th>
<th>$\kappa''$</th>
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</thead>
<tbody>
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<td>09500</td>
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<td>100000</td>
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<td>082000</td>
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<td>000000</td>
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<td>06800</td>
<td>100000</td>
<td>100000</td>
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<td>000000</td>
</tr>
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<td>06500</td>
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</tr>
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<td>+1 m</td>
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<td>06800</td>
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<td>082000</td>
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<td>082100</td>
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</tbody>
</table>

**Characteristics of Instruments**

There are three common approaches to solving engineering problems by creating elevations: Analytical or mathematical, graphical and mechanical (i.e. with the use of an instrument).

For carrying out the concepts previously presented and in actual practice, the third approach has been found to be most feasible and useful for this method. We use a first-order plotting instrument of the universal type, which creates and measures a special model and records the results. The functional components of the instrument include different types of facilities available to the photo-

![Figure 2](image-url)
grammetric engineer and scientist. They are:

- Focal-length \( f \), swing \( \kappa' \kappa'' \), lateral-tilt \( \omega' \omega'' \),
- longitudinal \( \phi' \phi'' \).

base components \( b_x \)

base components \( b_y' \)

base components \( b_z' \) \( b_z'' \)

and \( X, Y, Z \). (Figure 3)

The values of all base-components can be set and read from measuring counters of which the smallest divisions correspond to \( 0, 0 \text{ in} \) \( mm \).

It is possible to shift from terrestrial to aerial work by a gear change. This is useful through allowing the drawing of elevations,—horizontal sections and partial ceiling plans,—without changing the orientation of the stereopair. By means of a knob, movement between \( Y \) and \( Z \) can be interchanged. The \( Z \) movement is then coupled to the hand wheel and the \( Y \) counter, the \( Y \) movement to the foot disc and the \( Z \) counter.

**CONTROL**

We have noted how this special model can be constructed with terrestrial photographs. To know the model-scale and its location in an established reference frame work, we have to achieve dimensional control in the model. The need for control can be illustrated by comparison with conventional aerial surveying. So we have to measure at least one distance in order to determine and to check the scale of the model. Another requirement is to give the heights of the two control points. The thus-mentioned requirements, scale and heights, can be satisfied by having two points in the model of a known horizontal position. In order to refer the elevation in our surface to a standard datum, such as mean sea level, we would need the mean sea level elevation of the two control points.

To this point an individual model formed by two exposures has been discussed. If we have oriented, scaled and leveled one of the individual models, it is possible to determine the special location and all points of the surface from other common overlapping models. The condition of this process can be tolerated until our error accumulation becomes too large. The important point in this method is that the outside control in every individual model is not necessary.

**FIG. 3**

**FIG. 4.** A stereopair taken at right angle to the base. (Not prepared for use with stereoscope.)
Every measurement system and its instruments contain sources of error. Of course we never completely eliminate all errors, but we strive to reduce the total residual errors until we obtain the desired precision and accuracy. However, the photogrammetric engineer can reduce and correct any measurements, as well as minute errors of the project. The accuracy of the method is therefore essentially concerned with our ability to construct the model. There are many factors such as the great importance of inner orientation in determining model errors. These errors affect the exterior orientation as well as the position errors of the model in space and its deformation. The accuracy of measurements from the photograph is largely dependent upon the care with which the work is done. Position errors should be ±0.2 mm. at all scales, provided the correct base-length is selected.

A description of errors is given by Prof. Dr. M. Zeller in his Text Book of Photogrammetry, where he investigates these errors and discusses their elimination in detail.

**Plotting of Elevation Plans**

The following actions apply to plotting elevation plans: selecting the model-scale, inserting negatives, setting the optical and mechanical equipment, introduction of the orientation elements and the base, connection with drawing table, compensation of minute errors at the time of exposure.

**Selection of model scale.** As in the plotting of aerial photographs, the selection of the model-scale depends on the scale of the elevation plan to be produced, as well as on the available gear ratios from the stereoplotter to the drawing table. The distance range of the model to be plotted must lie within the instrument’s ranges.

**Inserting the negatives.** When doing this the ranges of movements of the camera tilt, etc., must be considered together with the setting possibilities of the optical and mechanical parts. The model as seen in the stereoplotter is so presented to the observer that a rotation of the right hand-wheel to the right causes the floating mark to be moved to the right in the X direction.

**Orientation elements.** To reconstruct the interior orientation, the focal-lengths of the plotting cameras are set equal to that of the calibrated focal-length of the theodolite camera. The pictures themselves are so i-
For reconstructing the exterior orientation, the two plotting cameras are so placed that they have the same position relative to each other at the moment of exposure at the two ground-stations. The following orientation data or measured field survey must be set in the stereoplotter:

1. **Base length**: This is the measured horizontal distance between the exposure stations; it is set in the stereo-plotter as the reduced base in the model-scale \((bx)\).

2. **Height difference between the exposure stations** \((HL-HR)\): This is the measured height difference and is set as "by" in the model-scale.

3. **Tilt**: If the two photographic cameras were tilted the same amount, upwards or downwards, from the horizontal axis, the tilt angles are set in the instrument as \(\omega\) rotation. Finally, set on the plotting instrument the correct amount of \(\omega\) and eliminate any difference between \(\omega'\) and \(\omega''\) where \(\omega'\) and \(\omega''\) do not coincide exactly. The \(\omega\) rotation causes the images to appear rotated about the optical axes in the eyepieces. This effect can be compensated for by rotating the dove prisms.

In order to extend the surface surveyed from one base, additional pairs of swiveled exposures can be used. The two camera axes are then parallel to one another but swiveled relative to the base by horizontal angle \(\phi\). This swivel is introduced in the Autograph by movement of the base instead of by camera rotation. The measured base is divided into a component normal to the direction of exposure and one parallel to it and introduced as \(bx\) and \(bz\) (see Figure 2).

The swiveled exposures, however, have the disadvantage that the elevation has to be reconstructed in accord with the true horizontal plane—the house front not being in the true elevation position.

Thus additional drawing work had to be done, so as to reconstruct to a normal precise elevation plane.

**Compensations of minute errors in the orientation data.** We use the \(Y\), \(Z\) gears in the plotting of the elevation plane and we proceed by fixing two points along the true front plane. Connect the drawing table with the stereo-plotter and set the floating mark at point 1. The same point is set on the control sheet, which is inserted under the drawing table. Proceed with the two hand wheels and the foot disc along the surface line to point 2. If a scale-error is found here, it is best to
eliminate the latter by adjusting the base. This procedure should be repeated until both points are free from errors. In normal exposures the change of the base has an influence on by which corresponds to \( HL - HR \). By changing \( bx \) we have to change by proportionally.

In order to reconstruct the three-dimension situation to a two-dimension plane, it is necessary to have a set relationship between points on the photographs and model-scale. This is the case for the \( X, Y, Z \) axis in a normal exposure, provided the front is parallel to the base. It applies in the case of a swivel exposure only for the \( Y \) axis. In order to compensate for the \( X \) and \( Z \) differences in the swivel exposure which are caused by the angle-shift from normal, we use the following method:

The actual distance between the two control points is known. By rotating hand wheels \( X \) and \( Z \) we are able automatically to check and bring all surface points on the true horizontal plane. After this we draw our model in the elevation plane as if it were a normal exposure by using hand wheels \( X, Y \) and foot disc \( Z \) for distances. This plane is then reconstructed in accord with the distance between the control and all other surface points on the new true plane. A height check at control points 1 and 2 will insure us that the heights are correctly set. See Figure 6. (The point \( G \) in this figure is the photogrammetrical determined pass-point.)

**SUMMARY**

It has been shown that accurate economic and rapidly constructed models can be formed through using this method.

Our experience has included the construction of 54 models. This provided the opportunity to prove that the method is workable in a great variety of situations. Its accessibility to any object makes it far superior to the classical method.

The results indicate that the photographic method is sufficiently accurate for obtaining objects.

In closing we should point out that while our remarks apply only to Architecture, the method can also be used in Archaeology, Mining Development and other civil engineering constructions.

**ACKNOWLEDGMENTS**

I express gratitude to the Director of the Photogrammetric Institute of the Swiss Federal Institute of Technology, Prof. Dr. M. Zeller, for giving me the necessary facilities. Only through his help was it possible for me to complete this project in so short a time.

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**Forestry Applications of Aerial Color Photography**

**RUDOLF W. BECKING,**

*Dept. of Forestry, Agricultural Experiment Sta., The Alabama Polytechnic Institute, Auburn, Ala.*

*ERIAL photography has played an important role in developing forestry since first applied in 1920 and particularly after 1940 in the U. S. and Canada. It is highly valued by foresters, because it serves as an excellent record of contemporary forest conditions. In fact, aerial photographs comprise one of the best obtainable forest records, since they can be consulted and supplemented at any time. Foresters have made use of aerial

(Abstract is on next page.)

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