Structural Engineering Application of the Stereometric Camera*

A camera system developed for traffic accident analysis finds an application in the materials testing laboratory.

(Abstract on page 100)

INTRODUCTION

Non-topographical use of photogrammetry has led to a variety of methods and special instruments; [6], pp. 97–102. The Wild C12 stereometric camera was designed specifically for the survey of traffic accidents (Figure 1). It consists of a fixed base of 1.20 m. length onto which two identical cameras are mounted. Their principal distance is 90 mm. and their glass plate format is 90×65 mm. The lens shutters are synchronized for exposures varying from 1 to 1/300 sec. The f-stop can be varied from 12 to 36. The camera unit can be rotated for azimuth and for ω-inclination at fixed settings of +15, 0, −15 and −25 grads. The unit can be mounted on a tripod; it can be leveled and vertically raised.

The stereophotographs taken of traffic accidents can be evaluated in the so-called "police-autograph" Wild A4 which was specifically designed for the plotting of stereometric camera photos; or they can be plotted on an ordinary first order stereoplotter, in which a multiple of the camera principal distance has to be introduced, resulting in a z-affinity of the stereomodel. For the Wild A-5 stereoautograph this ratio is usually chosen as 2:1.

It suggests itself that the stereometric camera can be applied in a similar manner to other non-topographical uses of photogrammetry: [4]; Figure 90 in [6].

Structural engineering makes use of various model tests to determine material properties and design criteria. Many of these tests restrict themselves to the measurement of strains. As long as the loads are kept small, the tested material (concrete, or steel) can be considered to have elastic properties. Elastic range tests lead to so-called "elastic design criteria" [1]. Elastic design, however, has its shortcomings, inasmuch as it is not valid for greater loads under which plastic properties of the tested material become noticeable. This is always the case near failure. Strain gauges fail to register in the plastic range so that deflection measurements of the structural model can lead to a much more accurate indirect determination of design parameters [2, 5].

The measurement of deflections should be fast, so that changes due to settling of the structure during the measurement are ineffective. Photogrammetric techniques are ideally suited for this purpose. They also give the advantage of not having to disturb the

The Wild C-12 stereometric camera was applied to two separate structural tests.

**COMPOSITE CONCRETE-STEEL BEAM TEST**

In the first test a composite concrete-steel beam (Figure 2) was subjected to loads varying from 0, 450 psi., 600 psi., 800 psi. to 1,200 psi., at which intervals photographic stereo-exposures were made. The load was applied by five hydraulic pressure pumps subtended on a steel structure on top of the beam, simulating a uniform load. The top part of the composite beam consisted of a concrete slab, which was connected to the bottom part composed of steel by shear connectors. Failure of the composite beam occurred at 700 psi. when one of the shear connectors broke, freeing the concrete part from the steel.

The objective of the test was to measure the deflections of the beam at 21 points which were marked by chalk. Secondly, an attempt was made to measure horizontal differences between points 1 and 2 etc. situated alternately on concrete and on steel (Figure 3.)

A dial indicator was used as a check measurement of the deflection at midspan each time photographs were taken.

The camera was set in horizontal position at a distance of 7 m. from the beam.

**REINFORCED CONCRETE SLAB TEST**

The second test was directed toward determining the deflections of a reinforced concrete slab under varying loads (Figure 4). The slab, 4.5 by 4.5 feet square, rested along the edges on a 3-inch diameter steel frame, leaving a net span between supports of 4 by 4 feet. The bottom of the slab contained 64 regularly distributed hooks connected to the reinforcement (Figure 5). Sixty-four cylindrical metal containers were suspended on the hooks. These were filled with sand and hung freely in a water tank 36 inches deep. By draining the water tank at intervals of 6 inches, varying “uniform” load stages could be achieved, increasing the load up to failure of the slab.

The 4 by 4-foot net span was marked by lines of white chalk on the slab which, for this purpose, had been previously painted black (Figure 6). A 6-inch grid further subdivided the net span into a total of 81 points. However only every second point was intended for measurement of the deflection, thus leaving 25 points to be measured (Figure 7). For the determination of absolute orientation, four control points, A to D, were determined using a level.

Photography of this arrangement was made for varying load stages. The slab (4 by 4 feet) was tested using oblique photography. The stereometric camera was set up on the tripod, 5 m. to one side of the test structure and at a
Another slab (4 by 6 feet) was tested using vertical photography. A metal frame was erected outdoors directly above the test structure. The stereometric camera was directly supported by the frame, 3.5 m. above the slab. Again four level points, A to D, were used to determine absolute orientation of the photographs.

**ANALYTICAL EVALUATION OF THE PHOTOGRAPHS**

Since all measurements needed referred to discreet points rather than areas, consideration was given to analytical procedures. Measurements of photo co-ordinates of the left photograph and of the x-parallax between right and left photograph were made on the Jena 1818 stereocomparator for each point. From double measurement of all plates the following standard errors were obtained for the mean of both determinations:

- $m_x' = \pm 3.0 \mu$
- $m_z' = \pm 4.9 \mu$
- $m_{pz} = \pm 3.7 \mu$

$m_z'$ for oblique photography is equivalent to $m_y'$ for vertical photography.

Ground co-ordinates were computed on the LGP-30 electronic computer according to the formulas:

- $y = b \times f / (p_x - p_{x_0})$
- $x = (y/f) (x' - x_{0'})$
- $z = (y/f) (z' - z_{0'})$
- $Z = z \cos \omega + y \sin \omega$
- $Y = y \cos \omega - z \sin \omega$
- $X = x$

**Fig. 2.** A composite concrete-steel beam being tested.

**Fig. 3.** Arrangement of the points measured on the composite concrete-steel beam.

Co-ordinates $x' z', p_x$ refer to photo-coordinates of points measured in the stereocomparator, $x_{0'} z_{0'}, p_{x_0}$ to photo-coordinates of the principal point, determined as the mean between the photo-coordinates of the two fiducial marks of the C-12 camera. $b$ and $f$ represent base and principal distance of the camera, $x, y, z$ are inclined ground co-ordinates, while $X, Y, Z$ are horizontal ground co-ordinates. $\omega$ can be determined from two con-
control points, at which level readings were taken (Figure 8); the formulae valid for oblique photographs become:

$$\tan \xi = (Z_B - Z_A)/(Y_B - Y_A)$$
$$\sin \eta = (Z_A - Z_B)/(Y_B - Y_A) \cos \xi$$
$$\xi + \eta = -\omega$$

In the case of four control points A, B, C, D (Figure 7), \(\omega\) can be obtained as the mean from two determinations via A, B and C, D.

The co-ordinates \(X, Y, Z\) for each of the points should then be compared with the corresponding co-ordinates \(X, Y, Z\) at various loading intervals. The differences in \(Z\) co-ordinates will represent the deflections. These are the most significant deformations.

According to measurement accuracy, it should be possible for the values \(m_x', m_y', m_pz\) (as quoted previously) (in the absence of other influencing factors such as distortion or interior orientation errors) to reach the following co-ordinate accuracies for the conditions of the experiment:

- \(m_x = \pm 0.23\) mm.
- \(m_y = \pm 1.7\) mm.
- \(m_z = \pm 0.38\) mm.

The deflection \(\Delta Z\) should be determinable with an accuracy of \(m_z\sqrt{2}\) or \(\pm 0.54\) mm. This presumes, however, that the orientation of the photography remains constant, or that an eventual change in exterior or interior orientation can be taken into account. This is al-
Abstract: The paper describes the application of the Wild C12 stereometric camera for the study of conditions of failure for a concrete slab and a composite concrete-steel beam. Experiments were designed so that successive stereoexposures could be made at varying loads up to the moment of failure. Both the slab and the composite beam carried a painted grid system, which allowed point measurements on a Jena 1818 stereocomparator. The concrete slab was photographed both as a vertical as well as a high oblique picture to allow a comparison of accuracies for both methods. The usual numerical solutions had to be modified in such a way that slight motions of the camera had no effect on the computed ground co-ordinates from one exposure to the next. The results obtained analytically were also compared by plotting and contouring the deflections on the Wild A-5 Autograph in an affine model. Observation accuracy for photo-co-ordinates was ±5 µ. The deflections could be determined with an accuracy of ±0.5 mm., which would correspond to an error of ±10 µ in the negative. The methods employed proved to be more reliable and more accurate to assess ultimate strength design criteria than methods previously used. Strain gauges fail to register outside of the elastic range, especially near failure.
STEREOMETRIC CAMERA FOR STRUCTURAL ENGINEERING

\[ dz = -\frac{xz}{y} d\phi + y \left(1 + \frac{z^2}{y^2}\right) d\omega + x d\epsilon \]
\[ - zdby - dbz + \frac{y}{f} d\epsilon' - \frac{x}{f} df \]
\[ dy = -\frac{yz}{b} \left(1 + \frac{x^2}{y^2}\right) d\phi + \frac{xyz}{b} d\omega - \frac{yz}{b} dk \]
\[ - x \frac{dbz}{b} - \frac{y}{b} dbx + \frac{y}{bf} d\epsilon' - \frac{xy}{bf} df. \]

However no change of relative orientation elements occurs for the C-12 camera. Including only approximate terms for absolute orientation changes, the formulas become for the camera assembly:

\[ dx = -yd\phi + \frac{y}{f} (dx' - dx') - \frac{x}{f} (df_{\epsilon} - df_{\eta}) \]
\[ dz = + xdk + yd\Omega + \frac{y}{f} (dz') - \frac{x}{f} (df_{\epsilon} - df_{\eta}) \]
\[ dy = + \frac{yz}{bf} (dx' - dx') - \frac{xy}{bf} (df_{\epsilon} - df_{\eta}). \]

It is to be noted that for \( dz \) only, \( dz' \) becomes influenced, but not \( dx' \), since it is not measured in the stereocomparator. Expressions \( zd\omega, zd\kappa \) and \( xd\phi \) were omitted since they are negligible. Inserting approximate numerical values would give:

\[ dx = -7000 \cdot d\Phi + 77(dx' - dx') - 11(df_{\epsilon} - df_{\eta}) \]
\[ dz = + 100 \cdot dk + 7000d\Omega + 77(dz') - 3(df_{\epsilon} - df_{\eta}) \]
\[ dy = 450(dx' - dx') - 0.6(df_{\epsilon} - df_{\eta}). \]

This indicates that principal point errors of 30 to 100μ (caused by the unclear definition of the fiducial marks) can well be responsible for the discrepancies of co-ordinates, rather than rotations of the instrument of ±1 to 2 minutes during changes of the plates. This is partly confirmed by the use of two operators who obtained constant differences of up to 25μ for the co-ordinate readings \( x' - x_0' \) in the same model, mainly due to a different interpretation of the fiducial mark, resulting in a different \( x_0' \) reading.

It is obvious that due to these discrepancies the determination of deflections without use of control points will have large inherent errors. It therefore becomes necessary to reduce all readings or discrepancies to the plane determined by the control points. This reduction, equivalent to leveling the model, amounts to a space coordinate transformation:

\[ \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \lambda \cdot A \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \]

in which \( \bar{x}, \bar{y}, \bar{z} \) are the transformed coordinates, \( \lambda \) equals 1 since the model is to be leveled only. \( \bar{x} \) and \( \bar{y} \) are not as important as the deflection coordinate \( \bar{z} \), which can be expressed as:

\[ \bar{z} = a_{31}x + a_{32}y + a_{33}z. \]

Three control points will allow the determination of the coefficients \( a_{31}, a_{32}, a_{33} \) from three such equations. For four control points a least squares adjustment will lead to a unique solution. By the aid of the determined coefficients \( z \)-coordinates of a subsequent load stage can be referred back to the initial dead load stage.

If all points to be transformed lie approximately in one plane, a simple correction graph procedure can also be applied.

**RESULTS OF COMPOSITE BEAM TEST**

Since all points used for measurement of composite beam deflections were situated along a straight line in \( x \)-direction, and since the stability of the camera in \( k \)-direction was assured only one control point, point 22, was needed for the evaluation. Figure 9 shows the obtained deflection curves for various loads. It can be seen that the maximum deflection for load stage 800 psi. and 1,200 psi. is slightly left of half span, indicating the failure of a shear connector which was actually noticed at 700 psi. The results indicate that photo-

<table>
<thead>
<tr>
<th>Table 2</th>
<th>COORDINATE DIFFERENCES FOR POINT 22 OF COMPOSITE BEAM TEST, INDICATING THE DEGREE OF STABILITY OF THE C-12 CAMERA</th>
</tr>
</thead>
<tbody>
<tr>
<td>(mm.)</td>
<td>Zero</td>
</tr>
<tr>
<td>( Y )</td>
<td>7,093.595</td>
</tr>
<tr>
<td>( X )</td>
<td>141.634</td>
</tr>
<tr>
<td>( \Delta Y )</td>
<td>—</td>
</tr>
<tr>
<td>( \Delta X )</td>
<td>—</td>
</tr>
<tr>
<td>( \Delta Z )</td>
<td>—</td>
</tr>
</tbody>
</table>

TABLE 2

COORDINATE DIFFERENCES FOR POINT 22 OF COMPOSITE BEAM TEST, INDICATING THE DEGREE OF STABILITY OF THE C-12 CAMERA
deflections of the composite beam for varying loads.

Table 4

<table>
<thead>
<tr>
<th>X-COORDINATE DIFFERENCES BETWEEN POINTS ON CONCRETE (mm.)</th>
<th>Dead Load 800 psi</th>
<th>Difference dAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>X:1, 5</td>
<td>1,212.985</td>
<td>1,218.284</td>
</tr>
<tr>
<td>X:13, 9</td>
<td>1,218.202</td>
<td>1,223.127</td>
</tr>
</tbody>
</table>

\[ dy = n \cdot b \cdot \tan \left( \frac{1}{2} \theta + 70' \right) \]

\( y = 5 \text{ m. at a base-distance ratio } b/y \text{ of } 1:4 \)

The undisturbed stereoscopic range is limited to \( dy = 25 \text{ cm. due to the 70-second stereoscopic perception condition:} \)

\[ dy = n \cdot b \cdot \tan \left( \frac{1}{2} \theta + 70' \right) \]

\( n \) being the magnification of the stereoviewing device. The stereovision is thus severely limited to one to two rows of the 6" grid.

This very fact makes it inadvisable to plot contours on the Wild A-5 stereautograph from this type of photography, which, on account of the range, would still be possible in the way terrestrial photographs are usually plotted.

Figure 10 is an example for the evaluated deflections of load stage 12. The maximum deflection, in the center, is \(+13.8 \text{ mm.}\) The corners lift up by \(-1 \text{ to } -2 \text{ mm.}\) due to an arching effect.

Vertical photography—analytical treatment

Due to much better stereoscopic vision, the evaluation of vertical photography results in greater ease and speed of measure-
ment as well as in greater accuracy for the measurement of $px$ and of $mpx = \pm 3\mu$. At a distance of 3.5 m this leads to

$$m_z = \pm \frac{(z^2/b^2)mpx}{z^2} = \pm 0.34 \text{ mm.}$$

which on account of the larger base distance ratio is comparable to the accuracy with which deflections can be determined from oblique photographs. (For an equal object distance, vertical photography is ordinarily less accurate.)

The analytical evaluation of vertical photography is identical to that described for oblique photography, except that in the formula-systems $z$ and $y$ become interchanged.

**VERTICAL PHOTOGRAPHY—STEREOPLOTTING**

Vertical photography lends itself much easier for plotting in the Wild A-5 stereoautograph. Due to difficulties to insert the small 90×65 mm. negative plates with proper interior orientation into the picture carriers, relative orientation in $by_R$, $\Phi_R$, $\kappa_L$ and $\kappa_R$ will become necessary, as well as absolute orientation in $\Phi$ and $\Omega$ to control points.

For evaluation of the test a principal distance of $2f = 180.00$ mm. was chosen. The plotting scale was 1:5, with a model scale of 1:10 in the xy-direction and 1:5 in the z-direction. Point measurements resulted in a height accuracy of $\pm 0.3$ mm., comparable to analytical methods.

Figure 11 shows 5 mm. contours of the deflections for load stage 5 of the rectangular concrete slab, plotted on the A-5 autograph. (Actually plotting of 2 mm.-contours would have been feasible.)

Since the slab was rectangular rather than square, larger deflections of up to $+27$ mm. were encountered. An arching effect of $-0.7$ mm. was observed on the corners.

**CONCLUSIONS**

As has been shown, photogrammetric methods are capable of measuring structural model deformations. The use of the stereometric camera allows one to conduct a relat-

(Continued on page 117)

![Fig. 11. Deflection contours for the rectangular concrete slab plotted on the Wild A-5 Autograph.](image)
CLUSION THAT SUCH A RESULT IS PROMISING IF COMPARED WITH THE ACCURACY OF ±0.2" TO ±0.3" WHICH IS OBTAINED TODAY IN FIRST ORDER TRIANGULATION FOR A RELATIVE DIRECTION.

IN CLOSING THIS CONSIDERATION OF PHOTOGRAMMETRIC SATELLITE TRIANGULATION, IT CAN BE PREDICTED THAT THIS METHOD WILL NOT ONLY PROVE USEFUL FOR A WORLD-WIDE TRIANGULATION SCHEME, BUT WILL EVENTUALLY PROVIDE THE NECESSARY ACCURACY FOR INCREASING THE GEOMETRIC FIDELITY WITHIN INDIVIDUAL GEODEATIC DATUMS.

EXTENSIVE NUMERICAL ANALYSIS ON VARIOUS POSSIBLE SCHEMES FOR THE APPLICATION OF PHOTOGRAMMETRIC SATELLITE TRIANGULATION IS CURRENTLY IN PROGRESS FOR THE PURPOSE OF STUDYING THE PROBLEM OF ERROR PROPAGATION AND FOR ESTABLISHING AN OPTIMIZED FIELD OPERATIONAL PROCEDURE.

REFERENCES

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VIETIVELY FAST TEST SEQUENCE, AND PERMITS ONE TO ACHIEVE AN ACCURACY OF ABOUT ±0.5 MM. FOR DEFORMATIONS IN X OR Z. THIS ACCURACY IS SUFFICIENT FOR THE DETERMINATION OF ULTIMATE STRENGTH DESIGN PARAMETERS, WHICH CANNOT BE OBTAINED RELIABLY FROM STRAIN GAUGES. STRUCTURAL MODEL TESTING PLOTTERS [3] ARE CONSIDERABLY MORE EXPENSIVE AND ARE VERY LIMITED IN SIZE AND VERSATILITY AS COMPARED TO STEREOGRAPHIC CAMERA TECHNIQUES.

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REFERENCES

END.

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