LINE MATCHING IN OBLIQUE AIRBORNE IMAGES TO SUPPORT AUTOMATIC VERIFICATION OF BUILDING OUTLINES

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ABSTRACT

Airborne oblique images are increasingly acquired because of the availability of low cost sensor platforms, being equipped with multiple camera systems. These images are mainly used for visualization such as the Bird's eye in Microsoft's Bing Maps. Because of their characteristics, such as a varying scale and the need for combining several images for complete information of all sides of a 3D object, oblique images are up to now only seldomly used for automatic topographic data acquisition and update. However, side views allow better identification of imaged objects and may be useful for verification of building datasets. Verification is important for maintaining the correctness and thus usefulness of the datasets. We propose an approach for automatic building (2D cadastral data) verification using airborne oblique images. Building outlines are used to formulate hypotheses to be verified. The idea is that if a building from the database still exists in reality, wall façade edges, extracted from different oblique images and projected onto a hypothesized wall plane constructed from the 2D data will coincide. We have conducted experiments using both real data for existing buildings and simulated data for demolished. Preliminary results show a lot of line matches for existing buildings and no matches or very few erroneous matches for demolished buildings. These results imply that oblique images have a potential to efficiently being used for the purpose of building verification.

INTRODUCTION

Oblique images give to humans a three dimensional visual impression of the imaged objects. While in vertical images we see mainly a top view of objects such as roofs of buildings, in oblique images we see wall parts in addition to roofs. Although in a strict sense all aerial images are oblique because they are taken at a tilt caused by the aircraft motion, in practice oblique images are those intentionally taken at a tilt, typically larger than 5 degrees. Here we refer to images taken at angles large enough to allow recognition of building wall façade and small enough not to include the horizon.

Although oblique images contain side views of objects, obtaining information for the whole building requires multiple images taken from different positions. Sensor systems for airborne image acquisition have evolved from traditional capture of vertical images where along track overlap is used for stereo analysis to different types of systems for acquisition of oblique image in addition to vertical ones. Some systems capture one vertical and one oblique image, at one point, such as forward view or side view. Examples are FLI-MAP (Fugro-Aerial-Mapping-B.V.) for vertical and forward, the PFIFF camera system (Grenzdörffer et al., 2008) for side views and the 3K camera system developed by the German Aerospace Center (DLR) for vertical and side views at a larger angle such as 31^{o} (Kurz et al., 2007a; Kurz et al., 2007b). Pictometry Inc has developed a system which captures five images at the same time (Wang et al., 2008). These images are captured, at nadir (vertical images), left, right, back and forward orientations. Considering forward and side overlap, a scene is therefore captured in multiple overlapping images. Pictometry data is available for cities of Europe with over 50,000 inhabitants, is updated after every two years (Lemmen et al., 2007; Wang et al., 2008) and is available as Bird's View in Microsoft Bing Maps.

Oblique images can be overlaid with existing 2D data if orientation information and additional height information is available. In Figure 1 oblique views of the same scene are overlaid with a 2D cadastral map. The overlay is visually accurate indicating potential for verification of existing data using the images.

Figure 1. Different oblique views of buildings overlaid with 2D GIS data. Images:©Blom

Methods have been proposed for using vertical images for verification of building data. The data used is either airborne or high resolution satellite images. Some proposed approaches are image classification for simple buildings and for updating medium scale maps (Olsen et al., 2002; Knudsen, 2007). Other methods use DSM from stereo images (Olsen, 2004; Champion, 2007; Matikainen et al., 2007; Rottensteiner, 2007; Champion et al., 2008; Champion et al., 2009) or LIDAR (Vosselman et al., 2004). Most buildings are correctly verified but there are still errors caused by low quality in areas with shadows or trees. Vertical images give only information about the color and texture of roofs and DSM give information on height of captured scene, whereas Oblique images contain additional information on building walls that may be useful for recognising buildings.

Oblique images have attracted research in recent years. For cadastral purposes existing data was overlaid on Pictometry oblique image (Lemmen et al., 2007) and were recognized as a potential source of information for real estate management and taxation. Measurements in single oblique images using height information of the imaged place are also done. For this purpose, the system of Pictometry called Electronic Field Study (EFS) uses an existing digital terrain model (DTM) for making 3D measurements in oblique images (Höhle, 2008). The accuracy of the measurements, however, depends on the quality of the DTM and the orientation information provided by Pictometry. Since the façade information is quite limited in the nadir images oblique images are also used for texturing 3D models (Frueh et al., 2004; Grenzdörffer et al., 2008; Wang et al., 2008).

Image matching using oblique images has also been studied. Although with different scale it has been shown that they can be matched with good results (Le Besnerais et al., 2008; Gerke, 2009). A study has also been conducted (Mishra et al., 2008) for verification of roads in an existing dataset, where a typical classification was done using an oblique image and then the result is overlaid with the vector data (roads) to detect inconsistencies.

A method has been proposed for using oblique images for verification of buildings in existing data (Nakagawa and Shibasaki, 2008). The method requires three accurate datasets: firstly, old 3D building models to be verified, secondly old high resolution oblique images containing the buildings and thirdly new oblique image representing the current situation. The strategy used is projecting the old 3D models into both old and new oblique images to obtain textures of the models in both images and then analyze the textures using image cross correlation. Most buildings are verified. The method, however, verifies 3D models and relies on the availability of old and new images.

In this paper we propose a method for building (2D cadastral data) verification using airborne oblique images. While façade information available in oblique images may be interesting for identifying the number of floors or usage of a building such as industrial or apartments, we currently concentrate on identifying whether the building represented in existing vector data still exists on the site, has been demolished or is changed.

The proposed method requires oblique images for extraction and use of straight line edge on wall façade of a building being verified. Lines can be reliably extracted from images (Hough, 1962; Canny, 1986; Förstner, 1994; Christoudias, 2002; Gioi et al., 2008). The 2D image lines are projected to a plane containing the building wall in order to reconstruct 3D object lines and then these 3D lines are compared for images taken from different perspectives. Two sets, each containing some existing and demolished buildings, are used for testing the developed method. The first is a training data set. The trend obtained for the existing and demolished walls in the training dataset is used to assess results for the second dataset, the validation set.

In the following section we describe the building data being verified and the images used. We then present the method for projecting building wall planes to different images, edge extraction, back projection and matching to verify walls and the combination of wall verification to verify buildings. Results are finally analyzed.

DATA SOURCES

Buildings Verified

For this study 2D cadastral data of buildings of the city centre of Enschede, The Netherlands is used. The vector data is detailed, suitable for large scale maps. Figure 2 shows a part of the vector data. The corner points have a location accuracy of 0.25 m. Some buildings tested are obtained through simulation of demolished buildings. The buildings shown in Figure 2 are those used for the experiments. These buildings still exist and are visible in the images. Green are existing buildings used for training, blue are existing buildings used for validation, red are demolished (simulated) buildings used for training and yellow are demolished buildings used for validation.

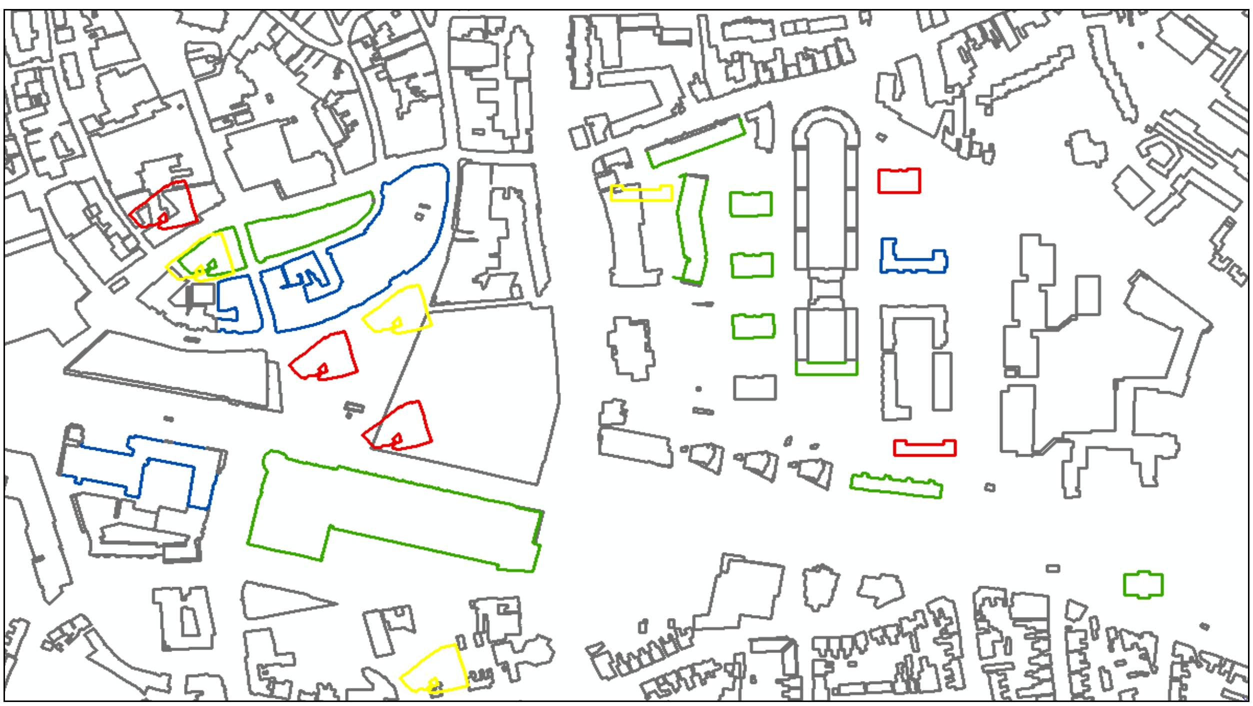

Figure 2. Part of 2D data of buildings of the city centre of Enschede: green are existing buildings used for training, blue are existing buildings used for validation, red are demolished (simulated) buildings used for training and yellow are demolished buildings used for validation.

Oblique Images

In this work oblique images from Pictometry (BLOM Aerofilms) have been used. Some parameters of these images are shown in Table 1. One of the images used is shown in
Figure **3**.

Table 1. Some parameters of the dataset used

Parameter	Value
Flying height (m)	920
Focal length of camera (mm)	85
Sensor size (mm)	36x24
Pixel size (µm)	9
Tilt (degrees)	50
Ground sample distance -GSD (cm)	10-16

Figure 3. One of the oblique images used. Image: ©Blom

WALL FAÇADE EDGE EXTRACTION AND UTILIZATION

The proposed method uses information on building walls imaged in oblique images. It is based on the assumption that oblique images of areas with buildings contain straight line edges of imaged wall façade such as window, door and building corner edges. It is also based on the assumption that these segments can be reliably extracted from the images. Further, it is assumed that lines extracted from any two or more images of the same wall taken from different perspectives and projected to object space will match if the building still exists and will not match if the building is demolished. Equally, there will be matches for some walls of a building and no matches for others if the building is changed such as by extending or demolishing part of it.

In the following sections data preparation is first introduced then processing aspects are described.

Data Preparation

The proposed method for verification of buildings using oblique images requires accurate image exterior and interior orientation information. The orientation parameters were therefore obtained by using the method that incorporates scene constraints in triangulation of oblique images (Gerke and Nyaruhuma, 2009). The RMSE at check points in object space was around 20cm for all three components after the self-calibration bundle adjustment.

In the cadastral data used some straight line segments for continuous walls are broken into several adjoining segments. The data is acquired that way for the purpose of identifying boundaries of different apartments of the buildings. The differentiation is required for taxation and other administration purposes. We combine these segments in order to use full lines for the whole walls. This is done by removing, from the polygon representing the building, a middle point of any three consecutive nodes that are collinear.

Line Extraction from Images

For each building being verified, for each image expected to contain the building or its part, and for each building wall expected to be visible in the image, straight line edges are extracted.

Building walls are defined by the 2D vector data and approximate height as described in (Suveg and Vosselman, 2000). The region containing the wall is selected by projecting the wall plane on the image using orientation information and collinearity constraint. This region is dilated for some pixels to ensure that wall corner edges are within the region. This buffer is necessary due to the uncertainties in the 2D data used for constructing the wall plane. A bounding box for the buffered region is then computed and checked whether it is within the image boundaries. Some walls may not be within the image boundaries if part of the building was not captured in the image, when a building is on the boundary of the image.

Line extraction in this work is done using the Förstner operator (Förstner, 1994). The algorithm extracts straight lines by combining adjacent pixels with similar gradient directions and fitting a line through these pixels. Thresholds are set for the minimum gradient strength, minimum region size for accepting pixels as adjacent and the minimum length of line. The operator results to sub pixel accuracy of the position of the extracted lines.

Matching Line Segments from Overlapping Images

Line segments extracted from different images expected to contain the same wall are transformed to 3D lines required for comparison. To obtain a 3D line, the plane defined by a 2D image line and the projection centre is intersected with the hypothesized wall plane.

Once the line segments extracted from different images have been transformed to 3D line segments, they are matched (in object space) to determine any possible pair corresponding to a wall edge. Any two line segments of the same wall edge extracted from different images are, ideally, collinear and overlapping each other at least partly. Therefore, any two line segments from different images are considered to correspond to the same wall edge if they have a similar orientation and the middle point of one segment is within some distance of the segment from another image. The maximum acceptable distance is determined by considering the uncertainty in orientation, vector data and extracted lines. An image point $A_j = (X, Y, Z)$ projected to the wall plane has the uncertainties $\Delta A_j = (\Delta X, \Delta Y, \Delta Z)$ given by

$$\Delta^2 A_j = \sum_{i=1}^{n} \left(\frac{\partial A_j}{\partial p_i} \right)^2 \Delta^2 p_i \tag{1}$$

Where p_i are the parameters of the projection

Δp_i are the uncertainties in the parameters

The uncertainties are computed for the end points A_{1j}, A_{2j} of every line segment being matched and then the uncertainty of the middle point of a segment is given as

$$\Delta^2 A_j = \frac{\Delta^2 A_{1j} + \Delta^2 A_{2j}}{2} \tag{2}$$

The overall uncertainty for a segment is then the mean of $\Delta^2 A_j$ and the required distance for accepting two segments as matching is obtained as a sum of these values from both segments.

For the experiments done in this study we have used the uncertainty in the building data (ΔX_0 to ΔY_1) as 0.25m and for accuracy of image segment end points (x, y) we have assumed 0.5 of a pixel size. We did not include the impact of orientation errors so far but it is not expected to be significant. Using the described method, for each wall, all lines from different images are compared. A segment which matches at least one other segment is labeled as matched.

Determination of the Wall Existence

In the developed method each wall of a building is first verified separately and then the overall building verification is obtained as a contribution of each wall.

For each wall visible in more than one image the following results are obtained: Wall size, the total length of all extracted segments, the total length of matched segments and the ratio of the total length of matched segments to the length of all extracted segments (match ratio).

We have used fuzzy logic (Zadeh, 1965) for obtaining the degree of belief that a wall exists or is demolished given a match ratio. Using training data, the fuzzy membership to the class "wall exists" at a given match ratio is computed as the proportion of the number of existing walls to the total number of existing and demolished walls at that match ratio. The size of walls is also used as a weight.

For example, if for all walls used in experiments, a total of 30 walls result to match ratio of 0.9 with 25 of them from existing walls and 5 from demolished ones, then the fuzzy membership to class "building exists" at match ratio 0.9 is 0.83 ($25/30$) and membership to class "building demolished" is 0.17($5/30$).

Fuzzy membership curves derived from training experiments are generalized to fuzzy membership functions.

Combining Wall Verification for the Whole Building

The overall building assessment is derived from memberships of its walls. For each wall the membership is obtained by using the match ratio and the membership function and then the final verification is obtained as the mean of the wall memberships weighted by the wall sizes.

BUILDING VERIFICATION RESULTS

Sample Line Matching Results

Walls from 16 buildings have been used for constructing the fuzzy membership function. Lines are extracted in oblique images and matched using the above described method. Below are line match results for walls of some existing, demolished and changed buildings.

Figure **4** shows some results for existing buildings. Existing walls give match ratio results between 0 and 1. If all lines match then a match ratio obtained is 1.0 and if no lines match then the match ratio is 0. The errors result from uncertainties in line extraction. The line extraction is also done using a bounding box in the oblique image which includes some parts of the image outside the building and the wall height is an extreme value estimate. Other errors result from occlusion.

Figure 4. Line matching for an existing building - a) an image with the lower and upper contours and wall corner edges, b) lines extracted from several images where the walls are visible projected to one of the images and shown as blue for matched, and red for unmatched lines and c) matched lines isolated and shown in a 3D viewer. Images: ©Blom

Figure **5** and
Figure **6** are for some simulated demolished buildings. The simulation was done by shifting a building to other places. By so doing it is expected that lines extracted will not match because they will be of different places defined by a false wall planes in images of different perspectives. Many line segments were extracted from images used for these buildings because the buildings were placed on places with other buildings or the open market place with some linear marks. Despite many extracted segments there were not many matches because of the difference in the perspectives of the images. Most walls actually result to no matches at all as can be seen from
Figure **5** and
Figure **6**. Demolished walls result to match ratios smaller than 0.1 implying that none or very few of the segments extracted were matched to any segment in another image.

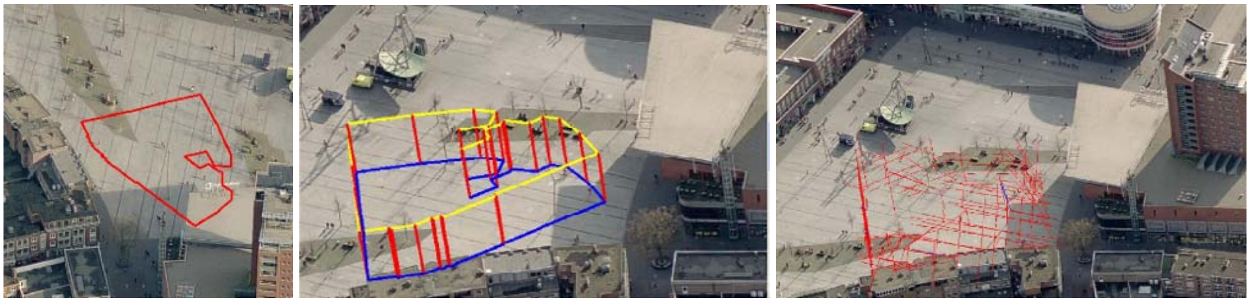

Figure 5. Simulated demolished building: left is the vector data in one image, middle is the expected position in another image right are lines extracted in images containing the building shown on one image: notice that most lines are not matched (red). Images:©Blom

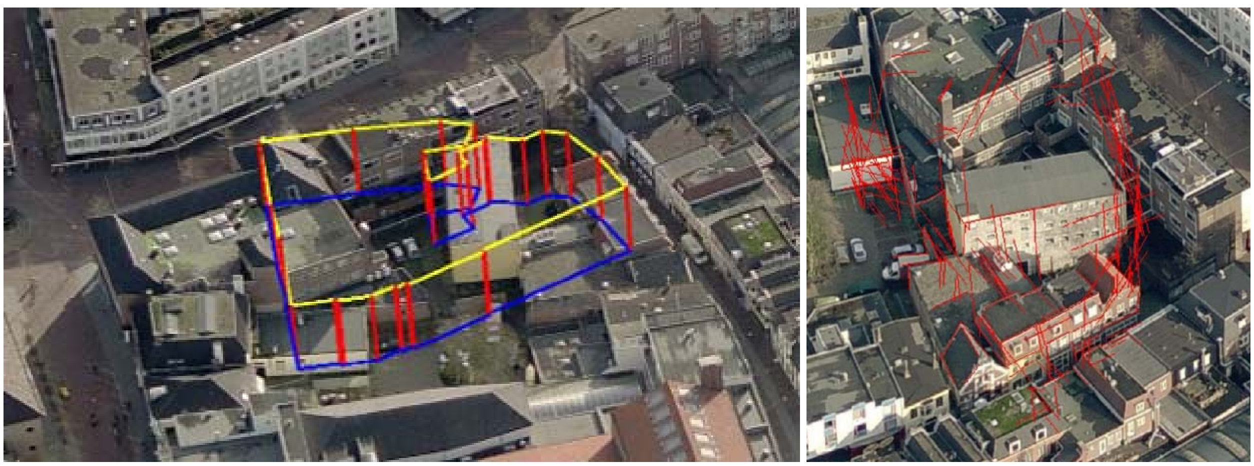

Figure 6. Another simulated demolished building. Images: ©Blom

Fuzzy Membership Function Constructed

The matching ratios for existing and demolished walls for training are used to make the fuzzy membership curves. **Figure 7** shows the memberships to class “wall exists” and “wall demolished” at different match ratios. The x-axis is the match ratio and y-axis is the membership. The Figure 7 to the right is the same data with the size of each bubble indicating the total size of walls at that ratio.

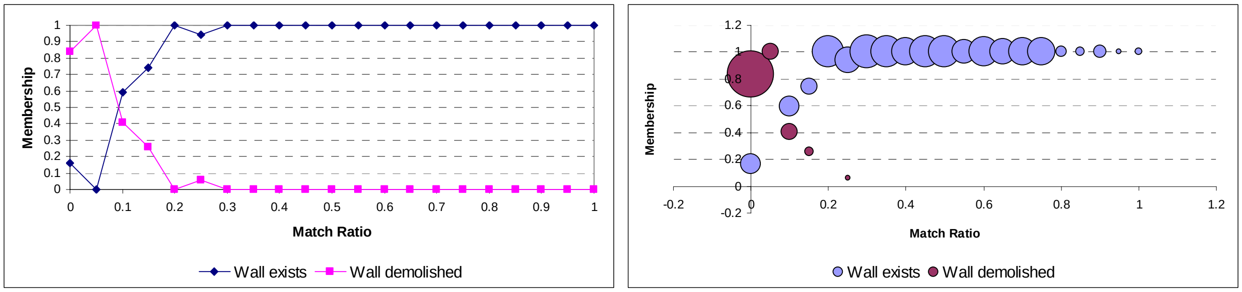

Figure 7. Fuzzy Membership - the x-axis is the match ratio and y-axis is the membership – to the right is the same data with bubble size representing size of walls.

The curves show that for the experimented walls in which the resulting match ratios were from 0.2 to 1.0, for example, the corresponding membership is 1.0. This implies that for some wall if the match ratio obtained is in that range then the wall fully belongs to class "wall exists". Note that, because we have only two classes, for any ration the membership $f(''wall\_exists'') = 1 - f(''wall\_demolished'')$.

The fuzzy membership curves have been generalized to a fuzzy membership function. A total of 16 buildings with a total of 210 walls are used for training. The membership function has been used for validation with other existing and demolished buildings (a total of 8 buildings with a total of 100 walls).

Wall Verification Results

Results of memberships for all walls used for training and validation are shown in
Figure **8**. As expected membership to "wall exists" for walls of actually existing buildings are typically close to1 while for demolished ones are close to 0. A few still existing walls in the validation dataset still result to memberships close to 0, as may be seen from the figure. These are mainly due to occlusion by other buildings, an issue which has not yet been taken care of.

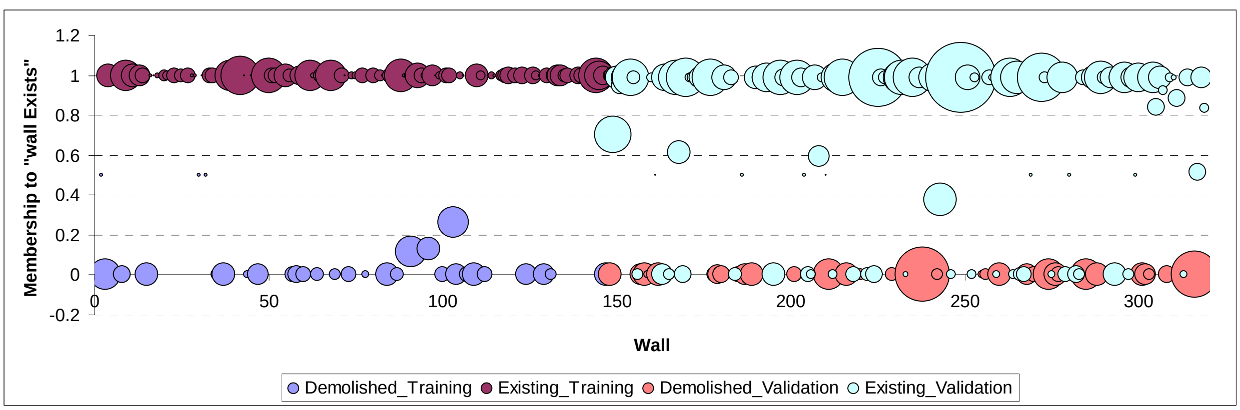

Figure 8. Wall verification results.

Building Verification Results

The overall building verification is derived from memberships of different walls as described in Section 0. Results are shown in Figure 9. The results show that for existing buildings memberships to "building exists" are generally close to 1 as expected while for demolished buildings the result is close to 0, also as expected. Matching wall façade edges detected from oblique images is useful for verifying buildings.

Building verification results obtained with this method may be used within the traffic light paradigm (Förstner, 1996); with say results above a threshold as "building exists" (green); below another threshold classified as "building demolished" (red) and in between classified as incorrect or "building changed"(yellow).

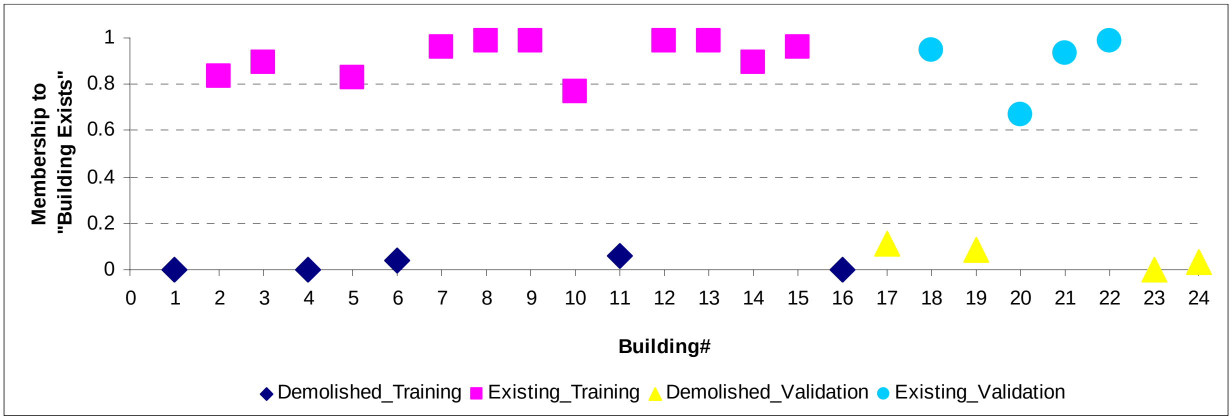

Figure 9. Building verification results.

CONCLUSION AND FURTHER WORK

In this work we have shown that oblique images can be reliably used for verification of building data. We have described our approach which uses multiple oblique images to extract and match line edges to obtain evidence of existing, demolished or changed buildings. The whole processing is done automatically and takes a few seconds for processing for each building.

The line matching results clearly show the differences of cases of demolished and existing buildings. The method developed gives results for existing, changed or demolished building and at the same time is able to tell which parts (walls) are changed in case of the a changed building. The demolished and changed buildings may be presented to a human operator for checking and extraction of changes from the images or removal from the dataset.

Future work includes using other evidence that are available from images such as image matching by using correlation coefficients or SIFT feature (Lowe, 2004) and combining or comparing the results. Occlusion will also be dealt with. The same edge extraction and matching strategy may also be extended to find new buildings.

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