PHOTOGRAMMETRIC TRIANGULATION OF 3D CUBIC SPLINES

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ABSTRACT

A common application in Photogrammetry is the 3D reconstruction of objects. Many objects are composed of curved features which are difficult to model using only point-based matching. We present an intersection method for the triangulation of 3D cubic splines from a series of measurements in several images. The method stems from the basic observation equation of a point on a 3D cubic spline; it minimizes the measurement residuals by adjusting the parameters of the spline, and the ratio of each measurement along the spline. As such, the method is flexible and can easily handle multiple projections (perspective and orthogonal) and widely varying fields-of-view. Also, the triangulation only requires that the measurements of the ends of the spline are matched between images. The intermediate measurements can be located at any point along the spline and need not match between images. We discuss the 3D natural cubic spline and the Hermite spline. The method is demonstrated with several features on a vessel imaged with a long lens in a surveillance setting.

KEYWORDS: photogrammetry, spline, triangulation

INTRODUCTION

A common task in Photogrammetry is creating a 3D model of an object from multiple images of the object. When performing this 3D reconstruction, it is generally desired to model the features of the object as closely, yet simply, as possible. The most common primitives for modeling an object are points, line segments, and triangles, especially when using point-to-point correspondences between images. However, many objects have curved and rounded features that may be difficult to model using these primitives, or may require many points to reconstruct accurately. Many such features can easily be represented by higher-level primitives such as curves.

Cubic splines are flexible, in terms of parameterization, and can accommodate a wide variety of curves found on objects. Cubic splines are also convenient in that any point along the spline can be characterized by a parameter indicating the position of the point along the spline. Natural cubic splines, a specific form of cubic spline, have been considered in context of multi-photo bundle adjustment (Lee and Yu, 2009).

We present a method for using 3D cubic splines to photogrammetrically model object features. In the work that follows, we assume that the images have been oriented; hence we are not using the splines for bundle adjustment, but are performing intersection to determine the object-space location of the spline. The method requires that several observations of each curve be made in several images. The major advantage of the method is that it does not require point-to-point correspondences between images except at the endpoints. Several points along the spline can be observed in each image without regard to individual point matches between images. It is possible however, to accommodate matching points along the spline where they can be observed.

The method is based on the projection equations of a single point, coupled with the 3D cubic spline equations. The basic principle is to adjust the spline parameters, and the position of each point along the spline, in order to minimize the residual between the observations and the projections of the points along spline. To show the flexibility of the method, we demonstrate the use of two projection models (perspective projection and scaled orthographic projection) and two spline models (natural cubic splines and Hermite splines). The model could also be extended to other projections and splines.

The method is demonstrated using surveillance images of a vessel captured with a long lens. Several different features on the vessel are modeled to show the performance of the method in general, as well as some of the strengths and weaknesses of different types of splines.

First, the two spline methods that are used will be introduced with particular attention paid to the partial derivatives. Then we introduce the two projection equations and how they can be used in the photogrammetric triangulation of a 3D spline. Finally, experiments and discussion are given.

3D CUBIC SPLINES

A 3D cubic spline is a piecewise 3rd degree polynomial which pass through n points (known as control points, knots, or joints) denoted by (X_1, Y_1, Z_1), (X_2, Y_2, Z_2), ..., (X_3, Y_3, Z_3). Each 3rd degree polynomial describes a piece of the spline between two control points. The first polynomial, or piece, describes the spline from (X_1, Y_1, Z_1) to (X_2, Y_2, Z_2). Each polynomial, is defined by a set of twelve coefficients $(a_0, b_0, c_0, a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3)$ and a parameter t which varies from 0 to 1 along the length of the polynomial. The coordinates (X, Y, Z) of a point on a spline can be found using the equation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix} + \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} t + \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} t^2 + \begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix} t^3 \tag{1}$$

where $(a_0, b_0, c_0, a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3)$ are the coefficients of the polynomial on which the point lies, and t is the parameter indicating the position of the point along the polynomial. Figure 1 shows a spline with 4 control points and three pieces

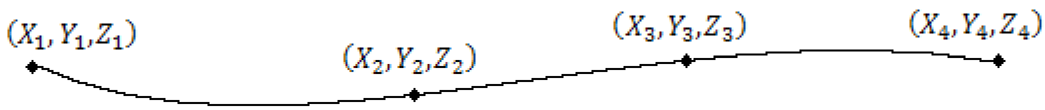

Figure 1. A spline with four control points.

Note that the point (X_3, Y_3, Z_3) is equal to the value of the polynomial for the third piece with $t = 0$ is also equal to the value of the polynomial for the second piece at $t = 1$. In other words, the polynomials are continuous. In general, the first derivatives with respect to t at the control points are also continuous. But the second derivatives are not necessarily continuous. It is possible for a spline to be a closed spline where the ends of the spline are coincident, as shown in Figure 2.

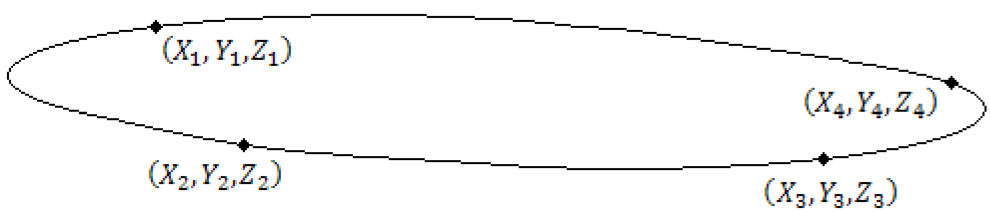

Figure 2. A closed spline with four control points.

Natural 3D Cubic Splines

A natural 3D cubic spline is a spline in which the second derivatives with respect to t are continuous at the control points and the second derivatives at the ends of the spline are 0. A natural cubic 3D spline is fully determined by its control points; or in other words, for a given set of control points, there is a unique natural 3D cubic spline which passes through them. Because of this, any point along a natural cubic spline depends upon the position of every control point.

To solve for the natural cubic 3D spline through a set of control points, there is a well-known method (Bartels, Beatty, and Barsky, 1998) where a linear system can be set up to solve for the derivatives at each control point. For the X coordinate of an open four-knot spline, the linear system used to determine the derivatives is

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = 3 \begin{bmatrix} X_2 - X_1 \\ X_3 - X_1 \\ X_4 - X_2 \\ X_4 - X_3 \end{bmatrix} \tag{2}$$

where the X terms are the X coordinates of the control points and the D terms are the X components of the first derivatives that are solved for at the control points. The Y and Z components of the derivatives can be solved similarly using the Y and Z coordinates of the control points. From the linear system above, it can easily be seen that the derivative at any given control point is dependent upon the position of every control point. As a consequence, the coefficients of each polynomial, and the position of each point along the spline, will depend on the position of every control point. In other words, a change to any control point changes the position of the entire spline curve.

Once the derivatives have been obtained, the coefficients for the each polynomial can then be obtained. For each piece, the coefficients are dependent upon the positions and derivatives of the control points at the ends of the piece. For the first piece the coefficients are

$$a_0 = X_1 \quad a_1 = D_1 \quad a_2 = 3(X_2 - X_1) - 2D_1 - D_2 \quad a_3 = 2(X_1 - X_2) + D_1 + D_2 \tag{3}$$

where (a_0, a_1, a_2, a_3) are the polynomial coefficients for the first piece, and the D and X terms are from equation (2). The coefficients for the other pieces can be found similarly using the positions and derivatives of the control points at each end. For example, the coefficients for the second piece would be solved as in equation (3) only using X_2, X_3, D_2, and D_3 in place of X_1, X_2, D_1, and D_2 respectively. The coefficients $(b_0, b_1, b_2, b_3, c_0, c_1, c_2, c_3)$ for the Y and Z coordinates for each piece can be found similarly using the Y and Z coordinates of the control points and the Y and Z components of the derivatives.

In the case of a closed spline, equation (2) becomes

$$\begin{bmatrix} 4 & 1 & 0 & 1 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 1 & 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = \begin{bmatrix} 3(X_2 - X_4) \\ 3(X_3 - X_1) \\ 3(X_4 - X_2) \\ 3(X_1 - X_3) \end{bmatrix} \tag{4}$$

and the coefficients for the final, or closing, piece are found using the last and first control points. For example, the fourth piece of the spline shown in Figure 2 would be calculated by using the fourth and first control points in equation (3) as

$$a_0 = X_4 \quad a_1 = D_4 \quad a_2 = 3(X_1 - X_4) - 2D_4 - D_1 \quad a_3 = 2(X_4 - X_1) + D_4 + D_1. \tag{5}$$

Hermite 3D Cubic Splines

A Hermite 3D cubic spline is a spline which is defined by a set of control points and control tangents (another term for derivative), at each control point. Hermite splines differ from natural cubic splines in that arbitrary tangents can be assigned at each control point, rather than solving for the tangents using equation (2) or (4).

As with the natural cubic spline, the coefficients for a piece of a Hermite spline can be solved using the position and tangents of the control points at each end of the piece. For Hermite splines, there is a useful form of the spline equation that involves the control points and tangents. By combining the first row of equation (1) with equation (3) and combining like terms, the X coordinate of a point on a spline can be written as

$$X = (2t^3 - 3t^2 + 1)X_1 + (3t^2 - 2t^3)X_2 + (t^3 - 2t^2 + t)D_1 + (t^3 - t^2)D_2 \tag{6}$$

where X_1 and X_2 are the X coordinates of the control points at the end of the piece, D_1 and D_2 are the X components of the tangents at the control points, and t is the parameter describing the position of the point along the piece. A similar equation can be constructed for the Y and Z coordinates of a point using the Y and Z coordinates of the control points and the Y and Z components of the derivatives at the control points. Note that the coordinates of a point along one piece of the spline are only dependent upon the control points and tangents at its boundaries, not the entire set of control points and tangents, as was the case with the natural cubic spline.

PARTIAL DERIVATIVES

In order to do triangulation of a spline, it is necessary to solve for (1) the partial derivatives of a point on a spline (X, Y, Z) with respect to the parameters which define the spline, and (2) the partial derivatives with respect to the parameter t which indicates the point's position along the polynomial.

Natural Cubic Splines

A natural cubic spline is defined entirely by the coordinates of its control points and the position of any point on the spline depends upon the position of all of the control points. Hence, we need the partial derivative of a point with respect to each control point.

Equations (2) and (3) do not provide a concise, closed-form equation for the coefficients as a function of the control points. We therefore use numerical approximation of the derivative. To show how this is done, we consider the partial derivative of the X coordinate of a point on a spline (X, Y, Z) with respect to the X coordinate of the first control point (X_1, Y_1, Z_1). First, we perturb the first control point's X coordinate by a small value both positively and negatively

$$\mathbf{X}_1^+ = (X_1 + dX, Y_1, Z_1) \quad \mathbf{X}_1^- = (X_1 - dX, Y_1, Z_1) \tag{7}$$

where dX is a small value and solve for the coefficients of two new splines: one passing through $\mathbf{X}_1^+$ instead of the original control point, and one passing through $\mathbf{X}_1^-$ instead of the original control point. Then, we solve for the position of a point on each of these splines using equation (1) and the t value for the original point. These two points approximate the position of the original point since the coefficients for each new spline will be only slightly different from the original spline. We denote these two points as

$$\boldsymbol{X}^+ = (X^+, Y, Z) \quad \boldsymbol{X}^- = (X^-, Y, Z) \tag{8}$$

Note the Y and Z coordinates for these points will be the same as the original point since they do not depend on the X coordinates of the control points. Using these approximate points, we obtain the partial derivative of the X coordinate of the point with respect to the X coordinate of the first control point by simple differences

$$\frac{\partial X}{\partial X_1} = \frac{X^+ - X^-}{2dX} \tag{9}$$

The partial derivative of the X coordinate of the point with respect to the X coordinate of each of the other control points can be found similarly. The partial derivatives for the Y and Z coordinates of the point with respect to the Y and Z coordinates of the control points can also be found in the same way. So, for each point on the spline, we calculate a partial derivative of its X coordinate with respect to the X coordinates of all of the control points. This is similarly done with the Y coordinate and Z coordinate. Also note that the X coordinate of a point does not depend on the Y and Z coordinates of the control points. So the partial derivatives of X with respect to Y, etc. will be 0.

The partial derivative of a point on a spline with respect to the parameter t is straightforward. Starting with equation (1) the derivatives are

$$\begin{bmatrix} \partial X/\partial t \\ \partial Y/\partial t \\ \partial Z/\partial t \end{bmatrix} = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} + 2\begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} t + 3\begin{bmatrix} a_3 \\ b_3 \\ c_3 \end{bmatrix} t^2 \tag{10}$$

Hermite Splines

A Hermite spline is defined by only the positions and tangents of its adjacent control points. Hence, for a given point, we only need the partial derivatives with respect to the positions and tangents of the control points at the ends of the piece on which the point lies.

The derivatives can be obtained directly, so the numerical approximation is not necessary. For a point on a given piece, we only need the partial derivatives with respect to the positions and tangents of the control points at its ends; the partial derivatives with respect to all other control points will be 0. The partial derivatives for the X coordinate of a point on the first piece can be obtained by directly differentiating equation (6)

$$\begin{aligned} \frac{\partial X}{\partial X_1} &= 2t^3 - 3t^2 + 1 \quad \frac{\partial X}{\partial X_2} = 3t^2 - 2t^3 \\ \frac{\partial X}{\partial D_1} &= t^3 - 2t^2 + 1 \quad \frac{\partial X}{\partial D_2} = t^3 - t^2 \end{aligned} \tag{11}$$

where X_1 and X_2 are the X-coordinates of the first and second control points and D_1 and D_2 are the X components of the derivatives at the first and second control points. The partial derivatives for points on other pieces can be found in a similar way using the coordinates and derivatives of the control points at the end of that piece. The partial derivatives for the Y and Z coordinates can be found in this way also. As was the case for natural cubic splines, the X coordinate of a point on a spline depends only upon the X coordinates of the control points and the X component of the derivatives, so the partial derivatives with respect to the Y and Z terms are 0.

PROJECTION EQUATIONS

In order to do photogrammetric triangulation of a spline, we first consider the photogrammetric projection of a spline. The projection equations for a single point by the perspective projection, also known as the collinearity equations, are

$$\begin{aligned} x &= xo - f\frac{m_{11}(X-Xc)+m_{12}(Y-Yc)+m_{13}(Z-Zc)}{m_{31}(X-Xc)+m_{32}(Y-Yc)+m_{33}(Z-Zc)} \\ y &= yo - f\frac{m_{21}(X-Xc)+m_{22}(Y-Yc)+m_{23}(Z-Zc)}{m_{31}(X-Xc)+m_{32}(Y-Yc)+m_{33}(Z-Zc)} \end{aligned} \tag{12}$$

where (x, y) are image coordinates of the point being imaged, (X, Y, Z) are object-space coordinates of the point being imaged, f is the principal distance of the camera, (xo, yo) are the image coordinates of the principal point, (Xc, Yc, Zc) are the object-space coordinates of the perspective center of the camera, and the m_{ij} terms are from the rotation matrix for the camera. The notation can be simplified by using terms to denote the numerator and denominator of these equations as

$$\begin{aligned} x &= xo - f\frac{r}{q} \\ y &= yo - f\frac{s}{q} \end{aligned} \tag{13}$$

where

$$\begin{aligned} r &= m_{11}(X - Xc) + m_{12}(Y - Yc) + m_{13}(Z - Zc) \\ s &= m_{21}(X - Xc) + m_{22}(Y - Yc) + m_{23}(Z - Zc) \\ q &= m_{31}(X - Xc) + m_{32}(Y - Yc) + m_{33}(Z - Zc) \end{aligned} \tag{14}$$

are the numerators and denominator. To show an example of another projection model, we also give the projection equations for the scaled orthographic projection

$$\begin{aligned} x &= T_{11}X + T_{12}Y + T_{13}Z + dx \\ y &= T_{21}X + T_{22}Y + T_{33}Z + dy \end{aligned} \tag{15}$$

where (x, y) are image coordinates of the point being imaged, (X, Y, Z) are object-space coordinates of the point being imaged, the T terms describe the scale and rotation of the projection, and dx and dy describe the shift of the projection.

Partial Derivatives

In order to do triangulation of a single point, the partial derivatives of the projection equations with respect to the object-space coordinates of the point being imaged are necessary. For the collinearity equations, these partial derivatives are

$$\begin{aligned} &\frac{\partial x}{\partial X} = \frac{-f}{q^2}(qm_{11} - rm_{31}) \quad \frac{\partial x}{\partial Y} = \frac{-f}{q^2}(qm_{12} - rm_{32}) \quad \frac{\partial x}{\partial Z} = \frac{-f}{q^2}(qm_{13} - rm_{33}) \\ &\frac{\partial y}{\partial X} = \frac{-f}{q^2}(qm_{21} - sm_{31}) \quad \frac{\partial y}{\partial Y} = \frac{-f}{q^2}(qm_{22} - sm_{32}) \quad \frac{\partial y}{\partial Z} = \frac{-f}{q^2}(qm_{23} - sm_{33}) \end{aligned} \tag{16}$$

where q, r, and s are as in equation (14) and all other terms are as in equation (12). For the scaled orthographic projection equations, these partial derivatives are

$$\begin{aligned} &\frac{\partial x}{\partial X} = T_{11} \quad \frac{\partial x}{\partial Y} = T_{12} \quad \frac{\partial x}{\partial Z} = T_{13} \\ &\frac{\partial y}{\partial X} = T_{21} \quad \frac{\partial y}{\partial Y} = T_{22} \quad \frac{\partial y}{\partial Z} = T_{23} \end{aligned} \tag{17}$$

where the T terms are as in equation (15).

In order to do triangulation of a point on a spline, we will need the partial derivatives of the projection equations with respect to the parameters which define the spline. To get these partial derivatives we can use the partial derivatives that we have already derived, and the chain rule. For a point observed on the first piece of a spline, the partial derivative of the x observation equation with respect to the X-coordinate of the first control point would be

$$\frac{\partial x}{\partial X_1} = \frac{\partial x}{\partial X}\frac{\partial X}{\partial X_1} \tag{18}$$

where $\partial x / \partial X$ is from equation (16) or equation (17) depending on the projection being used, and $\partial X / \partial X_1$ is from equation (9) or equation (11) depending on the type of spline being used. All other partial derivatives can be obtained in a similar manner.

PHOTOGRAMMETRIC TRIANGULATION

We now present the method for using multiple images to triangulate a 3D cubic spline. It is assumed that multiple observations of points on the spline have been made in multiple images. It is also assumed that the images have been oriented. We note that it is not necessary to collect conjugate points along the entire spline. Each image can have observations of the spline in slightly different locations along the spline. It is necessary to observe the ends of the spline in at least two images. However, it is also possible to include conjugate observations at any point along the spline if desired.

Using the projection equations and partial derivatives outlined above, a linear least-squares system can be set up to solve for the parameters of the spline. The linear system consists of two equations for each observation (one equation for the x observation and one equation for the y observation). The number of unknowns depends on the type of spline being used, the number of control points in the spline, and the number of observations. For a natural cubic spline there are three unknowns associated with the position of each control point in the spline. For a Hermite spline there are six unknowns associated with the position and tangent for each control point in the spline. For both types of splines, the t parameter which indicates the position of the observation along the spline is an additional unknown.

The least squares minimization requires initial values for all unknowns; a method for obtaining these is given below. The minimization is an iterative process where a correction to the initial, or current, values is computed each time and iteration continues until corrections become negligible. The linear system is

$$L = AX \tag{19}$$

where A is the design matrix, L is the constant term, and X is the vector of corrections.

The design matrix A has two rows for each observation and one column for each unknown. The matrix below shows the basic structure needed for the design matrix to triangulate a natural cubic spline with n control points from p observations of the spline

$$A = \begin{bmatrix} \frac{\partial_1}{\partial X_1} & \cdots & \frac{\partial_1}{\partial X_n} & \frac{\partial_1}{\partial t_1} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial_p}{\partial X_1} & \cdots & \frac{\partial_p}{\partial X_n} & 0 & \cdots & \frac{\partial_p}{\partial t_p} \end{bmatrix} \tag{20}$$

where

$$\frac{\partial_p}{\partial X_n} = \begin{bmatrix} \frac{\partial x_p}{\partial X_n} & \frac{\partial x_p}{\partial Y_n} & \frac{\partial x_p}{\partial Z_n} \\ \frac{\partial y_p}{\partial X_n} & \frac{\partial y_p}{\partial Y_n} & \frac{\partial y_p}{\partial Z_n} \end{bmatrix} \tag{21}$$

is a matrix of partial derivatives with respect to the control point coordinates and

$$\frac{\partial_p}{\partial t_p} = \begin{bmatrix} \frac{\partial x_p}{\partial t_p} \\ \frac{\partial y_p}{\partial t_p} \end{bmatrix} \tag{22}$$

is a matrix of partial derivatives with respect to the parameter t. Note that for observations of the ends of the spline, the t value is known, so there would not be a column in the matrix for the t value of these observations. Also, if there are conjugate pairs of observations along the length of the spline, these points would share the same t value. Hence, by adjusting the number of unknown t values, it is easy to accommodate both single observations and conjugate pairs of observations between images along the spline. Also note that for a Hermite spline, there would be an additional $3n$ columns for the derivatives at the control points.

The constant term L for the minimization is computed from the observations and spline parameters as

$$L = [x_1 - x'_1 \quad y_1 - y'_1 \quad \ldots \quad x_p - x'_p \quad y_p - y'_p]^T \tag{23}$$

where x_p is the observation and x'_p is the projection based on the current spline parameters, which can be computed using the projection equations. The vector of corrections X is

$$X = [dX_1 \quad dY_1 \quad dZ_1 \quad \ldots \quad dX_n \quad dY_n \quad dZ_n \quad dt_1 \quad \ldots \quad dt_p]^T \tag{24}$$

On each iteration, the linear system in equation (19) is solved and corrections X are added to the current values. Iteration stops when the corrections become negligible.

Separate Adjustment

Another approach to the least-squares minimization is to treat the control points and t parameters separately within each iteration. In this approach, the control points are adjusted first while the t parameters are held fixed, and then the t parameters are adjusted while the control points are held fixed. This approach is easily implemented using the design matrix A above, where the left portion of the matrix is used to adjust the control point and the right portion of the matrix is used to adjust the t parameters. It requires that two linear systems be solved within each iteration.

The biggest advantage to this approach is greater stability in the triangulation since fewer parameters are allowed to vary simultaneously. Essentially, when the control points are being adjusted and the t parameters are held fixed, and the points which have been observed are not allowed to move along the spline. Conversely, when the control points are fixed and the t parameters are adjusted, the points which have been observed are free to move along the spline, but the spline is fixed in space.

The disadvantages to separate adjustment is the computational cost of solving two linear systems within a given iteration. Also, the separate adjustment may lead to "thrashing" where the control points and t values bounce back-and-forth between two states. In other words, on one iteration the control points and t values are adjusted to one state, and on the subsequent adjustment, the change in the t parameters causes the control points to revert to their previous state, and an infinite cycle results.

Initial Values

One important consideration is the generation of initial values of the spline parameters. This task can be difficult in some cases, and the method presented here does rely on some assumptions that may not be valid for all splines.

Since it is necessary to have at least two observations of the ends of the spline, an intersection of these observations can be used to determine an initial value for the positions of the ends of the spline. Next, these positions can be back-projected into each image, giving an approximation of the ends of the spline in each image. Then, the total 2D distance from end-to-end through each observation can be computed on each image. Using the 2D midpoint from each image, intersection can be performed to provide an initial value of a control point in the middle of the spline. Note that this process assumes that the midpoint of a spline in 2D corresponds with the midpoint of the spline in 3D, which is never likely to be true. However, for some splines, especially simple splines which roughly lie in a plane somewhat normal to the images, this assumption is valid enough to provide adequate initial values.

Next, using the three control points (the two ends and the midpoint) a natural cubic spline can be created. A large number of synthetic points can be generated on this spline and projected into each image; then a t value for each observation can be found by assigning the t value of the closest projection.

The above method can be modified. For example, more than one point could be created between the ends of the spline by performing intersection at multiple ratios of the total 2D distance. Also, in a case where there are other point correspondences besides the ends, these points could be used as control points.

EXPERIMENTS AND DISCUSSION

Several experiments were conducted to show the effectiveness of the method on different types of model features. The image set used in the experiments was 6 images of a small vessel (approximately 2.3 meters x 6.1 meters) imaged from a distance of about 365 - 380 meters and from a slightly higher elevation (about 75 meters higher). The camera used was a Canon EOS 1Ds Mark II (4992 x 3328 pixels) with a 400mm lens (EF f/2.8 L IS USM) with a 2x extender (EF f/7.1). The lens/extender combination resulted in an effective field of view of about 0.50° - 0.65°. The six images were oriented using the perspective projection via bundle adjustment using a set of 25 manually collected points, resulting in an RMS residual value of 0.59 pixels, and a maximum residual of 1.75 pixels. These statistics are summarized in Table 1, and one of the images is shown in Figure 3.

Table 1. Characteristics of the vessel image set used for spline experiments.

Number of images	6
Vessel size	2.3 meters x 6.1 meters
Number of points used in orientation	25
Imaging distance	365 - 380 meters
Camera	Canon EOS 1Ds Mark II
Lens	EF 400 mm f/2.8L IS USM w/ EF/7.1 2x extender
Effective field of view	0.50° - 0.65°
Bundle Adjustment Residuals	RMS = 0.59 pixels, maximum = 1.75 pixels

Figure 3. An example image from the vessel data set, taken with a Canon 1Ds Mark II and a 800 mm effective focal length lens at a range of approximately 370 meters.

The first feature that was modeled using a spline was a short section of the railing on the left (port) side of the vessel. This comprised a simple curved feature that was easily identifiable in the images. Five to six observations of the railing were made in each of the images (33 observations total), with both ends of the railing being visible in most of the images. The spline was initialized as described above using the end points and one midpoint as initial control points; triangulation was then performed using a natural cubic spline. The residuals resulting from the triangulation had a mean of 1.1 pixels with a maximum of 2.8 pixels. The resulting spline was then re-triangulated treating it as a Hermite spline (i.e. the natural cubic was used as an initial approximation of a Hermite spline). When triangulation was attempted using the normal method, the adjustment became unstable due to very large changes in the t values on the first iteration. For this reason, a separate adjustment as explained above was used for the triangulation of the Hermite spline. The resulting residuals from this triangulation (mean of 0.9 pixels, maximum of 2.6 pixels) were slightly lower than for the natural cubic spline, although there was very little visual difference in the resulting splines. The results of the triangulations are given in Table 2 and the splines are shown superimposed over two of the images in Figure 4; the blue line represents the resulting natural cubic spline, the red line represents the resulting Hermite spline, the black x's represent the observations that were made, and the green dots represent the control points of the resulting natural cubic spline.

Table 2. Results of the spline triangulation for the simple, three-control-point spline.

	Mean residual (pixels)	Maximum residual (pixels)
Natural cubic spline	1.1	2.8
Hermite spline (using separate adjustment)	0.9	2.6

Figure 4. Results of simple spline triangulation from 5-6 observations in each of 6 images for both a natural cubic spline (blue) and a Hermite spline (red).

In order to test the method with a more complicated curve, a set of observations was collected along the whole railing, from the left (port) side around the front to the right (starboard) side (172 observations total). Several points were corresponded between images (the points where a vertical railing ties into the railing) as explained above and the spline was initialized using four control points in the interior (six control points total including the ends). Triangulation was then performed using a natural cubic spline. The resulting spline yielded fairly high residuals (mean residual of 3.3 pixels, maximum of 17.5 pixels). Due to the rather high residuals, triangulation was attempted again using separate adjustment; the residuals from the separate adjustment were slightly higher (mean residual of 5.0 pixels, maximum residual of 19.4 pixels) than for the first adjustment. Triangulation was also done using a Hermite spline and the natural cubic spline as an initial value. The results from this step were considerably better (mean residual of 1.1 pixels, maximum of 5.4 pixels). A comparison of the natural cubic and Hermite splines are shown for the curve in Figure 5.

Figure 5. Natural cubic spline (blue) with 6 control points (green) derived from 172 observations across six images, and a Hermite spline (red) with 6 control points (not shown).

This curve represents a good example of a feature that is modeled well by a Hermite spline using relatively few control points. Hermite splines are very flexible due to their unconstrained tangents at the control points, however the large number of free parameters requires a separate adjustment to maintain stability. The natural cubic spline, on the other hand, is not able to match the curve as well as the Hermite for the same number of control points, due to its constrained tangents, and reduced flexibility. An attempt was made to increase the number of control points for the natural cubic spline to see if a better match could be made when there are more control points, however, the best solution that could be obtained was using 10 control points (the natural cubic spline with 10 control points yielded a mean residual of 1.8 pixels, and a maximum residual of 6.5 pixels). The results from the large spline are summarized in Table 3.

Table 3. Results of several triangulations of the railing using 172 observations.

	Mean residual (pixels)	Maximum residual (pixels)
Natural Cubic, 6 control points, regular adjustment	3.3	17.5
Natural Cubic, 6 control points, separate adjustment	5.0	19.4
Natural Cubic, 10 control points, separate adjustment	1.8	6.5
Hermite, 6 control points, separate adjustment	1.1	5.4

The primary finding from the experiments is that natural cubic splines are more stable that Hermite splines, but are not as flexible in modeling curved features. The greater stability and reduced flexibility are due to fewer free parameters; i.e. a natural cubic only has three degrees of freedom for each control point while a Hermite spline has six. The triangulation method for the natural cubic splines remained stable for all of the tests, while the triangulation for the Hermite splines had to be done using separate adjustment in order to achieve a solution.

CONCLUSION

We have presented a method for the triangulation of 3D cubic splines from several observations in several images. Natural cubic splines have been considered as well as Hermite splines. These splines have been shown to be effective at representing curved features on an object imaged from multiple angles at a distance. The primary advantage of the method is that it allows for feature-to-feature correspondences rather than simply point-to-point correspondences. Observations along the curve do not, in general, need to be correlated between images, except for the ends of the curve; additional points which do correspond between images can be included.

We have shown that the additional degrees of freedom afforded by Hermite splines allow them to fit some curves more closely than natural cubic splines. However, the additional degrees of freedom can also lead to stability issues, which we have solved by adjusting the spline parameters and the t parameter (a parameter which indicates the position of a point along the spline) of each observation separately. Additional research could address the issue of stability further, and also improve the method of initialization.

ACKNOWLEDGMENTS

This research was funded by the Naval Air Warfare Center (NAVAIR) via the STTR/SBIR program with the help of Robert Hintz, NAVAIR China Lake, California. The authors would like to particularly thank T. Dean Cook (NAVAIR) and F. Brent Pack (Lidar Pacific Corporation) for their technical contributions and guidance.

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