THE ROLE OF SURFACE COMPLEXITY IN AIRBORNE LIDAR PRODUCT ERROR CHARACTERIZATION

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ABSTRACT

There is a wide variety of data product characterization methods to describe LiDAR data quality. The most basic methods typically use a measure derived from vertical differences at known checkpoints (surface patches) to obtain the vertical accuracy, and thus, simply ignore the error contributions of the horizontal components. More advanced methods attempt to also characterize the horizontal accuracy of the LiDAR point cloud, by using measurements at LiDAR identifiable targets or other man-made objects that can be distinctly extracted from both horizontal and vertical representation in the LiDAR point cloud. However, there is a relatively limited, or no attention at all, paid to the surface complexity of LiDAR-surveyed area itself. Though, both the surface geometry macro- and microstructures and the material characteristics play a role in the error budget besides the sensor measurement errors. The objective of this study is to elaborate only on the requirements for adequate surface representation in combination with the LiDAR error characterization techniques to identify the relation between the two surfaces, the measured and reference (ideal), and thus, to support better LiDAR or in general point cloud error characterization.

KEYWORDS: Airborne LiDAR, Surface representation, Point cloud, Accuracy/error assessment

INTRODUCTION

DEM is an extremely important geospatial product that is broadly used in almost all mapping and engineering applications (Muane, 2007). For example, it is directly used in flood plane mapping or line of sight analysis for telecommunication, and indirectly in orthophoto production or 3D city modeling. While the concept of DEM is not new, the exploitation of this data structure started only with the introduction of powerful computers and softcopy photogrammetric systems that provided an affordable platform for mass surface point generation from scanned airborne imagery. The next major development occurred when active imaging sensors with direct 3D measurement capabilities were introduced in topographic mapping; in particular, when LiDAR providing a direct surface point acquisition technology became the dominant source of DEMs at local scale. In fact, image-based surface point generation lost significant market share at that time. Now with the improving performance of digital cameras and image matching techniques, the stereo-photogrammetrically created point cloud is again gaining relevance. Finally, there are a variety of terms used for describing surface model/data, including DSM, DED, DTED, DTM, DEM, etc., some of them overlap in definition, while others are unique; in the following, only the DEM is used as a general term.

The error characterization of DEM data is not an obvious task given the extremely large number of points and various characteristics of data acquisition and processing techniques. Furthermore, surface data, both terrain only and terrain with objects, have a few major data representations, including irregularly spaced point data, gridded data (raster) (both can be with and without breaklines), TIN, and contours. Standards, guidelines and product qualification methods have been developed mostly by government agencies to provide for a consistent treatment of data, usually acquired from various sources. The primary, most often used regulations are from USGS, FEMA, NGA, FGDC, ASPRS, etc. All of these standards/guidelines are mainly focused on the DEM data QA/QC, including accuracy, ground control and statistical evaluation methods, and there is no or little attention paid to the actual surface, or, in broader term, to the impact of the object space characteristics on the DEM characterization. The varying surface geometry and condition are typically considered in deciding on DEM point density or in using breaklines, etc. The objective of this study is to look into the surface/object space condition in terms of spatial sampling of the surface and surface representation to analyze its impact on the QA/QC processes of DEM data. Though the original motivation comes from using surfaces extracted from LiDAR data, the discussion will make no distinction with respect to the origin of the point cloud, as the emphasis is currently shifting from LiDAR point cloud to the broader point cloud processing, which also includes stereo or multiple ray image generated surface point clouds.

SAMPLING THEORY

Surface elevation data, S_c, with respect to a mapping plane, in general, can be considered as a two-dimensional continuous function:

$$S_c = f(x, y) \tag{1}$$

where S_c is the vertical (z) coordinate, x and y are the horizontal coordinates in a mapping plane. For practical reasons, the discrete representation of the surface should be considered, which is typically obtained by an evenly spaced two-dimensional sampling of the continuous function and by converting the continuous elevation values to discrete ones:

$$E_{ij}^d = Q_p(S_c) = Q_p(f(x_i, y_j)) \tag{2}$$

where, Q_p is the quantization function (typically a regular step-function), which maps the continuous input parameter space to 2^p discrete levels, p is the number of bit used for quantization, x_i and y_j are the coordinates at the sampling point of i and j, respectively. The fundamental question is how well the second representation (E^d_{ij}) describes the first representation (S_c). According to Shannon's information theory (Shannon, 1948), if the sampling distance satisfies some conditions, then the continuous signal S_c can be fully reconstructed from the samples E^d_{ij}. The required sampling distance is defined by the well-known Nyquist frequency (Shannon, 1949). For the two-dimensional signal case, if f^x_{max} and f^y_{max} are the highest spatial spectral frequencies for a given surface, then the sampling distances d^x_s and d^y_s are sufficient for the complete representation of this surface, and consequently, the continuous surface can be restored without any error from the discrete representation in this ideal case. The Nyquist criterion for the two-dimensional case is:

$$d_s^x \le \frac{1}{2 f_{\max}^x} \quad \text{and} \quad d_s^y \le \frac{1}{2 f_{\max}^y} \tag{3}$$

If the Nyquist criterion is satisfied, then the reconstruction of the continuous surface from the discrete samples using the required or shorter sampling distances is described by:

$$S_c(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} E_{ij}^d \frac{\sin\left(\pi d_s^x \left(x - \frac{i}{d_s^x}\right)\right)}{\pi d_s^x \left(x - \frac{i}{d_s^x}\right)} \frac{\sin\left(\pi d_s^y \left(x - \frac{j}{d_s^y}\right)\right)}{\pi d_s^y \left(y - \frac{j}{d_s^y}\right)} \tag{4}$$

In this ideal case, this reconstruction introduces no errors, as the discrete representation provides a complete description of the surface function, see Fig. 1. The sampling distances in the x and y directions could be different in some specific cases. Furthermore, the concept can be extended to non-uniform sampling; though, it has no advantages in general practice.

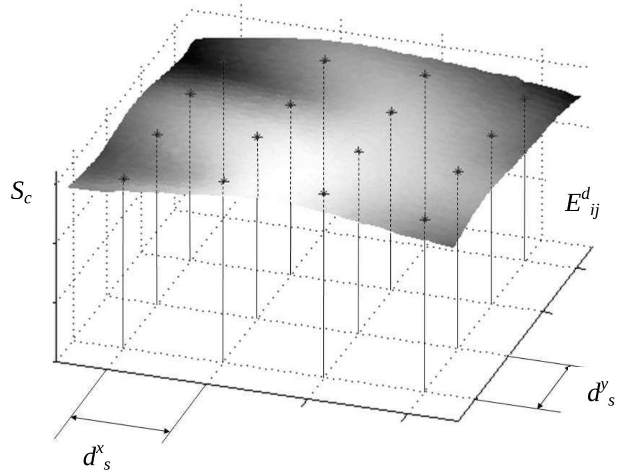

Figure 1. Surface reconstruction.

In reality, it is generally impossible to achieve this ideal situation for several reasons. First of all, the characteristics of real surfaces are not known, so the Nyquist criterion can be only estimated from samples. Second, there are inherent limitations of the measurement system, which introduce measurement errors. Although the quantization is a non-linear transformation, its impact in practice can be safely ignored, as in modern digital systems, the usual numerical representation provides high-precision representation for wide signal range, so that the error introduced by converting the continuous signal into a discrete one is negligible (Widrow and Kollar, 2008).

DISCUSSION

Though the information theory provides a clear basis for surface sampling, i.e., what the maximum sampling distance should be to fully represent a surface, it has a global character, meaning that the whole area should satisfy the Nyquist criterion. For large surfaces, this condition could be too conservative if the surface changes are different within the area. For example, an area with a river cut into smooth rolling terrain would require a higher sampling rate for the riverbank, which represents a breakline situation in mapping terms, while a moderate sampling would clearly satisfy the requirements for the remaining part of the area. Therefore, the surface area of interest should be first divided into smaller areas with nearly identical sampling requirements. This concept is practically identical to the tiling process used mapping, image compression, or, to some extent, wavelet transformation. There are several ways to perform this segmentation and most of these methods are based on estimating the surface slope.

The segmented areas can be further analyzed based on the shape of the surface segments. There could be simple surface segments that can be composed from planar patches, spheres, conical shapes, etc., which could be analytically modeled and, consequently, described by a few parameters. These segments should be handled differently from the segments of generic surfaces, and are not considered in the following.

Provided that the area is segmented into nearly homogenous areas in terms of surface undulation, the next task is the determination of the largest sampling distance that can satisfy the Nyquist criterion. Since there is practically no a priori information available on the required sampling distance, the simplest way to estimate the sampling distance is if the surface is transformed into the Fourier domain, and the power spectral density (PSD) is iteratively computed with varying sampling distances. Starting from a large value, the sampling distance is decreased in small steps and the Fourier transform is recomputed. Once there is no change in the shape of the signal in the spectral domain, the process is terminated, and the sampling rate is accepted. Unfortunately, the sampling could be limited by the applied data acquisition technology, and thus, it is possible that the surface is under-sampled (below Nyquist criterion), and thus, this approach may not work. However, the results will still indicate that (i.e., under-sampling), so it can be used as an alert signal. Based on the information from neighboring segments as well as general expectations, the implementation of this method can be optimized to reduce execution time.

Once the segments are sampled at or near the Nyquist criterion (note the sampling distance is estimated, and a multiplication factor, to decrease the sampling distance, could be also used to make sure that the criterion is met with some margin), the surface, in theory, is represented in its original shape, practically without any errors (the quantization error can be safely ignored, as discussed above). In this ideal case, the surface height is known basically with no error, provided the measurements have no error. The question is whether this perfect surface representation is really needed in applications? Clearly, even in the most demanding mapping applications, such as engineering scale mapping, there is room for an error budget. In other words, measurement errors exist and there may be no need to fully satisfy the Nyquist criterion.

In real situations, the earth surface as well as objects cannot be observed without limitations, some amount of surface details should be discerned, and thus the practical question is, how to optimize the parameters of a discrete representation, such as sampling distance (and, if needed, quantization levels), for any given accuracy requirements in terms of acceptable surface deviations in the DEM. For example, what is the minimum sampling distance to keep the differences between the two representations under a predefined threshold? The answer, in general, depends significantly on the application circumstances. For instance, for creating a topological map, road surface roughness is irrelevant, and small surface variations should not be considered. However, for road design or maintenance purposes, this information, the details of the elevation changes, is equally or even more important than the global nature of the surface, such as the road location in some mapping frame.

In most DEM applications, the measurement errors are known or relatively well-characterized, based on the technology used; for example, sub-decimeter vertical accuracy for LiDAR or sub-meter accuracy for IfSAR. The accuracy requirements of DEM products are usually defined at various accuracy levels, such as the DTED classification by NGA. In some cases, it is defined on a relative basis, to be as high as possible with respect to a given data acquisition technology. While measurement accuracy and product accuracy requirements are different, there is no difference between them with respect to surface reconstruction. In other words, the same surface distortion can be caused by either measurement error or allowance for application error. Fig. 2 shows a surface profile; for simplicity, the one-dimensional case is considered, as the generalization to 2D is straightforward. In Fig. 2, the brown line shows the ideal surface profile, and then an envelope around that profile, marked by blue boundaries, shows an acceptable error range; the error range can be defined by measurement error or product accuracy requirement in usual statistical terms, such as 1σ or CEP90. Based on the error envelope, the ideal reconstruction of the surface profile is not needed. In fact, any curve in the error envelop is an acceptable surface representation, as it meets the accuracy specification. Among the infinite number of profiles, the one that requires the largest sampling distance that meets the Nyquist criterion should be selected, and, to avoid bias, shows the smallest distance from the ideal surface profile. In practice, the first condition can be easily satisfied, while the second is typically not. The curve, marked in red in Fig. 2, shows a simple solution, which clearly meets the requirements; compared to the ideal profile, the curve is smooth.

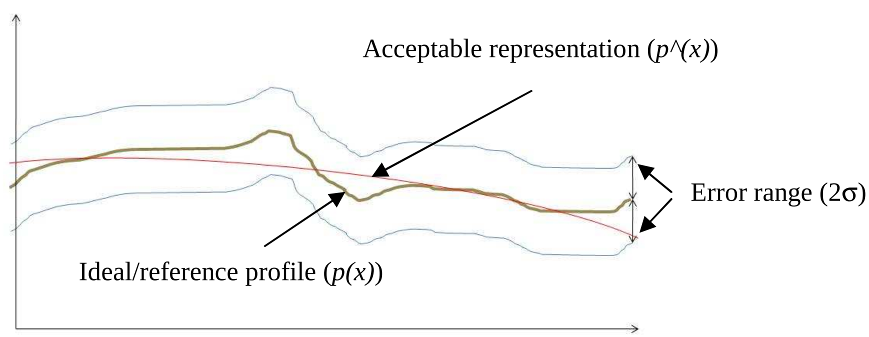

Figure 2. Allowing for surface reconstruction error.

The error between the reference and reconstructed with error profiles can be estimated by the following expression:

$$\left|\hat{p}(x) - p(x)\right| \le 2 \int_{|f|>f_s/2} \left|X(f)\right| df \tag{5}$$

where *X(f)* is the spectrum (PSD), and f_s is the spatial frequency band. The expression simply states that the reconstruction error is due to the spatial frequency components that fall outside of the frequency band defined by the sampling rate.

There are several potential approaches to find an optimal or near-optimal surface curve, provided the ideal/reference surface is known. However, it is rarely the case, though it is possible in certain situations, such as knowing the shape of the surface because it is defined by a simple geometry. Therefore, estimation is needed to determine the surface; for example, it can be done in an iterative way. To account for the estimation error, the error envelope should be reduced in this case. Though computationally intensive, simulation could be another approach to obtain the profile that meets the requirements of the accuracy specification.

The process described above can be applied in various ways to DEM representation with some accuracy specification. For example, knowing surface complexity and DEM accuracy requirements, the optimal sampling rate can be determined, and, for example, the scan rate of a LiDAR can be configured accordingly. Similarly, knowing the sampling rate and the surface complexity, the expected DEM accuracy range can be estimated. Another application is the conversion of irregularly spaced point data to a gridded format, where the optimal grid constant is defined by the sampling distance satisfying the Nyquist criterion.

CONCLUSIONS

As DEM product characterization methods to describe surfaces obtained from LiDAR and, in general, point clouds, continue to advance, the surface characteristics should be more considered to improve QA/QC performance. This study provides an initial attempt to look into one aspect of surface/object space dependency by analyzing the requirements for optimal surface representation with respect to acceptable error range. The ongoing research is focused on developing practical metrics to define the relationship between surface complexity and acceptable surface representation for a given DEM error level.

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