

THE REPUBLIC OF MADAGASCAR

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The Portuguese discovered the island of Madagascar in 1500, which was in the same era as their explorations of the entire eastern coast of Africa. The French established some colonial stations temporarily in the 17th century, and re-established their presence in the 18th century. The local native Hovas group signed a treaty with France in 1868, and France declared that Madagascar was a French protectorate in 1882. The French reorganized the administrative system of Madagascar in 1924 and independence was achieved in 1960.

Madagascar is the fourth largest island in the world; its length is almost 1,600 km (995 miles) and its maximum width is 576 km (360 miles). Tsaratanana Massif in the northern part of Madagascar is the highest point at 2880 m (9449 ft). The island is obliquely oriented to the cardinal directions on a north-east to southwest angle, and this fact has had a whopping affect on the development of mathematical cartography for the island. The centrally located capitol of the Republic of Madagascar is Antananarivo, and other major cities include Toliara in the south, Mahajanga in the northwest, Antsiranana in the north, and Toamasina in the east. The port of Hell-Ville is on the island of Nosy Bé off the northwest coast.

Original survey observations were performed by Owen of the British Royal Navy in 1824-25, and French Navy observations followed in 1830 and 1850. Geodetic triangulation for a hydrographic survey of the Bay of Diego Suarez was commenced by Colonial Engineer M. Grégoire in 1887. The baseline was

measured for a length of 1,044.295 meters, and a repeat measurement differed by a millimeter. This is typical quality for the French in the late 19th century. The Antsirana Datum of 1887 was established at the Antsirana pillar where: $\Phi_0 = -12^\circ 16' 25.5''$ South, $\Lambda_0 = 46^\circ 57' 36.2''$ East of Paris. The defining azimuth was determined from Antsirana to signal station Oronjia as: $\alpha_0 = 79^\circ 12' 19''$. Antsirana 1887 Datum is referenced to the Clarke 1880 ellipsoid where $a = 6,378,249.145$ meters, and $1/f = 293.465$. The Hellville Datum of 1888 was established at the meridian pillar where: $\Phi_0 = -13^\circ 24' 20.7''$ South, $\Lambda_0 = 45^\circ 57' 05.0''$ East of Paris. The defining azimuth was determined from the observation tower in Hellville to the fire beacon at station Tany-Kely as: $\alpha_0 = 197^\circ 25' 18''$. The Hellville 1888 Datum is also referenced to the Clarke 1880 ellipsoid. The Mojanga Datum of 1888 was established at the pillar in Mojanga where: $\Phi_0 = -15^\circ 43' 24.2''$ South, $\Lambda_0 = 43^\circ 58' 54.4''$ East of Paris. Although the Clarke 1880 ellipsoid was referenced, no record exists of the defining azimuth for this tiny Datum. Note that all three of these local Datums in northwest Madagascar also defined Hatt Azimuthal Equidistant Grids for the respective hydrographic surveys. The usual rectangular coordinate systems (SystPmes Usuel), were implemented in quadrant systems instead of in false origin systems. Simultaneously, the French Army started using the Bonne pseudo-conic equal area projection for topographic surveys and plane table mapping of Madagascar. Note that the Bonne was the projection "de rigueur" in Europe at the time, and it was the basis of the Depot de la Guerre map series of France. The quadrant system was used for the topographic

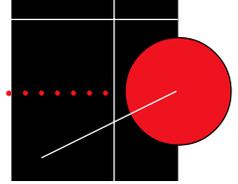
maps of Madagascar as well as for computation of the triangulation.

The Antsirana Datum of 1906 was an update established by M. Lesage of the French Navy at the Antsirana pillar where: $\Phi_0 = -12^\circ 16' 20.3''$ South (determined by Driencourt in 1904), and $\Lambda_0 = 46^\circ 57' 36.2''$ East of Paris (no change from Favé in 1887). A later report (date unknown) gives a local Hatt Grid in the vicinity having an origin where: the Central Meridian, (λ_0) = $46^\circ 55' 34.669''$ East of Paris, the Latitude of Origin, $\phi_0 = -12^\circ 16' 26.148''$ South, a False Easting of 80 km, and a False Northing of 30 km. Considering the coordinate precision reported, the International 1924 ellipsoid was presumably used where: $a = 6,378,388$ m, and $1/f = 297$.

The Nosy Vé Datum of 1907 was established for the hydrographic survey of the waters off the port of Tuléar (Toliara) in southwest Madagascar. The origin is at the meridian pillar where: $\Phi_0 = -23^\circ 39' 00.20''$ South, $\Lambda_0 = 41^\circ 16' 44.2''$ East of Paris. The Hellville 1888 Datum and the Hatt Azimuthal Equidistant Grid also used the quadrant system.

In 1926, the Hellville 1888 Datum was re-defined in terms of longitude where $\Lambda_0 = 48^\circ 16' 28.95''$ ("1.5") East of Greenwich. Presumably, this change was identified as the Hellville Datum of 1926. Although the original meridian pillar was gone, the new origin was observed to have the coordinates of $X = -28.1$ m, $y = -2.6$ m (note: X, y *s.i.c.*).

In 1928 Laborde, a colonel in the French artillery, wrote that the Bonne projection chosen for the Service Géographique de Madagascar was as bad a choice as it had been for France. "Beginning in 1924, the defects of the Bonne Projection became totally inconvenient for use on the first part in geodetic work for the calculation of geodetic coordi-



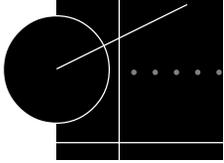
nates on the ellipsoid, and on the other part the rectangular topographic surveys accumulated a substantial error. In 1925, regions quite removed from the Bonne Axis of projection, notably the delta region of Mangoky in the southwest part of Madagascar, attained errors of 22' for angles and $\pm 1/580$ for lengths. It was decided to find another solution urgently so that the quality would not be compromised for the survey campaign of 1925. Pressed for time, a local provisional projection was adopted in which the distortions were practically negligible within the region of application. Furthermore, difficulty was encountered in recruiting specialized personnel, and in 1926, the triangulation operations were constrained for lack of officers. Since training had to be furnished to the personnel because of the isolation of the survey parties do to the topography; the

field calculations had to be simple to employ and therefore rectangular coordinates were used. Since the Bonne Projection was useless, provisional local coordinates systems were improvised. Between 1925-26 four local projections were put into service based on the Gauss Conformal Projection. (*Ed. note: The actual math model used for all four was the Gauss-Schreiber Transverse Mercator.*) The Laborde Projection, adopted in 1926, was put into full use for the reduction of the calculations of the 1926 geodetic survey campaign and for the topographic survey campaign of 1927."

The Laborde Projection is a triple conformal projection, designed to have two lines of zero scale error oblique to the meridian. The projection of the ellipsoid onto the plane is carried out in three steps; the first is the projection of the ellipsoid on the sphere. The second

step is a Gauss-Schreiber Transverse Mercator projection of the sphere onto a cylinder that is symmetrically secant about the central meridian of the projection. The third step in forming the Laborde Projection is a conformal distortion of the plane through a rotation in the secant case. Beginning with the first step (the conformal projection of the ellipsoid on the sphere), Laborde followed Gauss. Gauss showed that the ellipsoid could be mapped conformally on the sphere in such a way that the distortions in the neighborhood of some chosen point are of the third order or higher. Also note that the indirect or double projection through the sphere is not exactly the same as the direct one in that they differ concerning the higher order terms of the expanded transformation formulae. The conformal projection of the ellipsoid on the

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sphere was used by the Prussian Land Survey in 1876-1923 as the foundation for a double projection, and it is currently used in computations for cadastral retracement. This triple projection of Laborde's, using the Gaussian Sphere to achieve an Oblique Mercator, is fundamentally different from the method used some 20+ years later by Brigadier Martin Hotine of the UK. Most commercial software houses that I have noticed are abysmally ignorant on this point. (I'll grouse on this later.)

In reference to the four provisional coordinate systems mentioned earlier by Laborde, he later commented on the relation of these local systems to the Laborde Projection. His discussion referred to the 1925-1926 local triangulation situated in the region of Bemolanga-Morafenobe. The work was carried out over 4,200 km², and was comprised of 127 geodetic triangulation points calculated on a provisional Gauss-Schreiber Transverse Mercator coordinate system. Two points of an old reconnaissance triangulation were chosen as a base for a local grid. This grid was "une projection conforme de Gauss limitée," a Gauss conformal projection of limited extent. Laborde further commented that the transformation from the provisional system to the Laborde Oblique Mercator was one translation, one rotation, and a change of scale without any deformation, "sans aucune déformation." This is an implementation of one of the most valuable properties of conformal projection: the ability to maintain the undistorted shape of surveys and mappings through simple 3-parameter (translation in X, translation in Y, and rotation) transformations! In my past column on Belgium, I discussed how the U.S. Army Map Service implemented the European Datum 1950 by such 3-parameter transformations

(on the complex plane) with the UTM as a unifying tool. Laborde did this back in the 1920's, and the Russians do it in an attempt to obfuscate the relation between their "System 42 Datum" (Pulkovo origin) and the WGS 84 Datum. The Russians still do it in the 21st century by using their screwy "System 1963" series of local coordinates which are merely translated and rotated Gauss-Krüger Grids. However, Laborde's implementation for Madagascar was successful in *simplifying* the task!

Laborde's command of the Service Géographique de Madagascar also resulted in the establishment of the unifying coordinate system for the colony. After long observation, the coordinates of the observatory in the capital of Antananarivo were used as the origin of the Madagascar Datum of 1925 where:

$\Phi_0 = -18^\circ 55' 02.10''$ South,
 $\Lambda_0 = 47^\circ 33' 06.45''$ East of Greenwich. Of course, the ellipsoid of reference is the International 1924. The defining parameters were published by the retired General Jean Laborde in the seminal work, "Traité des Projections des Cartes Géographiques a L'Usage des Cartographes et des Geodèsiens," by L. Driencourt & J. Laborde, Paris, 1932. Remember Driencourt? He performed the 1904 final astronomical observations for the longitude of Antsirana Datum of 1906. Driencourt later became the Hydrographer of the French Navy (long before the four volume book was published). Those parameters are: Latitude of Origin (ϕ_0) = -21° South ($-18^\circ 54'$), Central Meridian (λ_0) = 21° East of Paris ($46^\circ 26' 13.95''$ East of Greenwich), Scale Factor at Origin (m_0) = 0.9995, False Easting = 400 km, False Northing = 800 km, Azimuth of Axis of Strength (θ) = 21° , and the False Easting at the False Origin = 1,000 km. For example, for geodetic station Morondava, $\phi = -20^\circ 17' 34.2443''$ South, $\lambda = 44^\circ$

$16' 52.6884$ East of Greenwich, Easting (Y) = 174896.590 m, Northing (X) = 644433.000 m, (H = 31.421 m).

Back in September of 1992, the late John P. Snyder asked for a copy of my 1982 stuff on the Laborde Projection of Madagascar. After John diddled a while with trying to see how close the Hotine Rectified Skew Orthomorphic (RSO) Oblique Mercator would fit, he concluded that he could match my transformations to only "about 0.08 meters for my part of Madagascar ... worse of course away from the island." This approximation is of interest for U.S. Geological Survey medium scale cartographic applications, but it is useless for geodetic work and for large scale engineering mapping. I have seen examples of much (apparent) hand wringing done by software organizations (U.S. and elsewhere), that have attempted to foist this sort of "work-around" on government agencies in Madagascar. The Grid is commonly used at great distances from the mainland because of the existence and official publication of a **False Easting at False Origin!** The RSO is *not* a valid substitute for the Laborde of Madagascar.

The U.S. National Imagery and Mapping Agency (NIMA) publishes a parameter set for transforming from the Madagascar Datum of 1925 ("Tananarive 1925") to WGS 84 Datum. However, they offer no information on accuracy of, or the origin of, the parameters. I tried a nine point solution and I obtained slightly different results as:
 $\Delta X = -191.745$ m, $\Delta Y = -226.365$ m, and $\Delta Z = -115.609$ m. The resultant rms for Latitude = 0.215 m, for Longitude = 0.323 m, and for ellipsoid height (h) = 9.958 m. The WGS 84 Datum observed coordinates for Morondava are: $\phi = -20^\circ 17' 39.079425''$ South, $\lambda = 44^\circ 16' 51.545430$ East of Greenwich, and h = 31.421 meters.

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