Dr. Abdullah: The errors budget in a lidar system involves more than just errors caused by the position of the sensor and the laser ranging. It includes other factors such as mirror shaft encoder sensitivity and stability, atmospheric refraction models, sensor attitudes (rotation angles), thermal stresses and torsion, as well as other factors; however, most of the later error sources can be modeled accurately and minimized or removed, except for the errors affecting the sensor rotation angles, such as IMU accuracy, reliability, and encoder biases.

Lidar ranging is extremely accurate and most systems can provide ranges with precision of millimeters to a few centimeters from a distance of hundreds or even thousands of miles. Satellite Laser Ranging has been used over the last few decades, and the International Earth Rotation and Reference Systems Services (IERS) was able to use laser ranging to measure the distance to the geodetic satellite Lageos, which orbits the Earth from an altitude of 5900 km, with a precision of less than 1 cm. Such high precision laser ranging was used by the IERS to accurately determine the Earth’s orientation parameters. As for the errors caused by inaccurate boresight determination or inadequate Inertial Measurement Unit (IMU) performance, the effect is linearly proportional to the flying altitude and is significant when compared to errors caused by laser ranging. To determine the maximum error due to IMU or boresight performance, let us evaluate the error at the edge of a swath flown with a lidar system with total field of view of 40 degrees from two different flying altitudes of 3000 m and 6000 m.

The horizontal error can be estimated using equation (1):

$$\text{Horizontal – error} = (R)^2(\Delta \theta_{\text{heading}})^2 + 2H(\Delta \theta_{\text{roll}})^2$$  \hspace{1cm} (1)

The vertical error can be estimated using equation (2):

$$\text{Vertical – error} = (R)(\Delta \theta_{\text{roll}}) = (R)(\Delta \theta_{\text{pitch}})$$ \hspace{1cm} (2)

where:

- \(R\) = Distance on the ground from nadir in meters
- \(H\) = Flying altitude in meters
- \(\Delta \theta_{\text{heading}}\) = Heading accuracy in radians
- \(\Delta \theta_{\text{roll}}\) = Roll and Pitch accuracy in radians

Assuming the use of an IMU with an accuracy of 0.005 degree in the roll and pitch and 0.01 degree in the heading, the following errors, resulting from poor IMU and/or boresight performance, can be expected:

<table>
<thead>
<tr>
<th>Flying Altitude (m)</th>
<th>Max. Horizontal Error (m)</th>
<th>Max. Vertical Error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,000</td>
<td>0.40</td>
<td>0.095</td>
</tr>
<tr>
<td>6,000</td>
<td>0.80</td>
<td>0.19</td>
</tr>
</tbody>
</table>

These figures represent the maximum errors at the edge of the swath, and therefore one-sigma errors should be smaller than those shown in the table above. It is obvious from the error figures in the table that the effect of the attitude errors doubles when we double the flying altitude. This is not the case with the positioning error that is caused by inaccurate airborne GPS. Although bad GPS affects the Kalman filter performance of the IMU-measured angles, the effect of the residual errors in the GPS propagates to the ground through the sensor position with a ratio of 1:1. In other words, if we have an error of 0.10 m in the vertical component of the GPS position of the lidar system, we should expect an equal error of 0.10 m in the elevation of the calculated ground position. This is a direct result of the GPS error regardless of the flying altitude. Finally, the correlation between the degradation in accuracy with the increased flying height is mainly due to errors coming from the sensor orientation and not the laser ranging. The laser ranging errors are small and often negligible when compared to errors caused by inaccurate attitude determination.

Question 2: I have always found your column ‘Mapping Matters’ to be helpful and informative. I have been involved with Photogrammetry for 22 years. Do you keep a subscription list for your column? If so, I would like to subscribe.

Dr. Abdullah: There is no way to subscribe exclusively to the column, but you can join ASPRS and receive the journal, Photogrammetric Engineering & Remote Sensing (PE&RS) as a member benefit. Joining the Society has many benefits, including discounts on webinars and conferences, in addition to receiving PE&RS with the “Mapping Matters” column and more interesting articles. ASPRS also posts “Mapping Matters” free of charge on their website at http://www.asprs.org/mapping_matters/.