

# A CORRESPONDENCE-BASED STRATEGY FOR AUTOMATIC REGISTRATION OF TERRESTRIAL LASER SCANNING DATA

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## ABSTRACT

The absence of explicit point correspondences among overlapping terrestrial laser scanning data limits the performance of automatic registration schemes. The popular Iterative Closest Point (ICP) and its variants, solve the correspondence problem implicitly while minimizing some distance metric. Other approaches perform low-level processing to obtain surface properties from which correspondences are established, and then they conduct the registration, which, in general is not as accurate as the ICP methods. This paper presents an approach that addresses the registration issue by dealing directly with the correspondence problem, without the use of derivative surface properties, except for local surface tangents. An Iterative Network Matching (INM) scheme is developed, in which a subset of points from one dataset comprise a 3D network. The corresponding network on the other dataset is obtained through a modified trilateration network adjustment, but without control, and a novel strategy is employed in which the local surface tangents are utilized as constraints. The main purpose of this paper is to extend the recent introduction of the INM methodology, by proving the validity of the proposed use of surface constraints for free network adjustment. From preliminary experiments with synthetic and real data, INM yielded global correspondence and registration RMSE that were as accurate as the ICP method of Chen and Medioni, 1991, and in some cases, improvements up to an order of magnitude in correspondence RMSE, were realized by INM.

**Key words:** automatic registration; correspondence problem; network adjustment; terrestrial laser scanning; iterative network matching; ICP

## INTRODUCTION

Laser scanning has matured into one of the most popular data acquisition technologies, arguably due to its rapid acquisition of object space coordinates and dense sampling of the object space. Laser scanners operate on the line-of-sight principle and often require multiple setups to complete a full scan of an object scene. The data from each setup are recorded in local coordinate frames and must be co-registered to model the entire object space.

For this registration to be done in an automatic manner there must exist a means of establishing correspondences among the local datasets. This is non-trivial, as laser data possess little semantics (Habib et al, 2004) and the pseudo-random sampling nature of the systems provide no exact point-to-point correspondence. By automatic registration we refer to methods which require no targets, but rather utilize the laser data. The two major classes of published automatic methods are feature-based and surface-based. The former extract features and objects such as lines or spheres, while the latter utilize either points or surface properties of the laser data. This paper follows along the lines of the point-based approaches, and focuses on an alternative strategy to the commonly employed Iterative Closest Point (ICP).

The difficulty of establishing point-to-point correspondences for registration of point cloud data has long been recognized. The correspondence problem has been avoided, mainly through the use of implicit correspondences from point-to-surface minimization techniques similar to the ICP method of Chen and Medioni, 1991. We describe these methods as using implicit correspondences, since point-wise coordinate transformations are performed at each iteration of their minimization process. The two sets of points being used are inherently correspondences of each other.

Chen and Medioni, 1991 projects points from one dataset along their local surface normals onto the second dataset, then fits a local tangent plane at the intersection point. The two surfaces are registered by minimizing the point-to-plane distances in an iterative fashion, and the projection points in dataset two are the implicit correspondences of dataset one. Maas, 2000 and Schenk et al., 2000 project points in dataset one along nadir (parallel to the z-direction), and then minimize the point-to-plane distance. Once more the projection points are the implicit correspondences. Park and Subbarao, 2003 introduces the Contractive Projection Point (CPP) technique, which incorporates both a projection along the viewing perspective as well as along local surface normals.

These methods and other such variants obtain point correspondences as by-products of the point-to-surface minimization process. Since automatic registration is essentially a correspondence problem, we propose an alternative approach that focuses on establishing point-to-point correspondences between pairs of terrestrial laser scanning (TLS) data. Our approach modifies the classical trilateration network adjustment, with the use of local surface geometry as constraints, to establish direct correspondences. The main contribution of this paper is to validate the use of the surface constraints in our registration strategy. The remainder of the paper develops and validates our approach and is of the following structure: Section 2 describes the proposed Iterative Network Matching (INM) strategy and Section 3 presents the proof that our surface constraints are indeed sufficient. The experimental results are given in Section 4 and final discussions and conclusions are given in Section 5.

## ITERATIVE NETWORK MATCHING: METHODOLOGY

Given two overlapping sets of TLS data (P and Q) in their local coordinate frames, the task of registering these scans can be expressed mathematically by:

$$\arg \min_{R,t} \|(RP + t) - Q\| \quad (1)$$

where  $R$  and  $t$  are the rigid-body rotation and translation parameters respectively, that align P and Q, and  $\| \cdot \|$  is the Euclidean norm operator.

By definition, the rigid-body transformation preserves scale, and thus vector magnitudes are unchanged. Therefore, given any two points from P, if the corresponding locations of these two points on Q are known, then the distance between the points on P and that of the corresponding points will be equal. This fact forms the basis of our approach, which involves the use of both condition and constraint equations.

### 2.1 Modified Trilateration Condition

If we obtain a set of points on P, our goal is to determine the locations of these points on Q, assuming here that the set of points on P lie in the overlap region with Q. For the set of points on P we can obtain the pair wise distances between these points and thus they form a network. With approximations to the transformation parameters we can also obtain initial estimates of the location of the corresponding network points on Q.

Let  $p_i$  and  $p_j$  represent two points of the network on P and  $q_i$  and  $q_j$  represent the “true” location of the corresponding points on Q. Then by scale preservation we have:

$$d(p_i, p_j) \equiv d(q_i, q_j) \quad (2)$$

where  $d(a, b)$  is the Euclidean distance between  $a$  and  $b$

Expanding the right hand side of Equation (2) with Taylor’s series expansion gives the following:

$$d(p_i, p_j) = d(q_i, q_j)|_0 + d'(q_i, q_j)|_0 \Delta + \frac{d''(q_i, q_j)|_0}{2} \Delta^2 + \dots \quad (3)$$

From Equation (3) we obtain the following modified trilateration condition equation:

$$d(p_i, p_j) - d(q_i, q_j)|_0 = d'(q_i, q_j)|_0 \Delta + \dots \quad (4)$$

Collecting all the condition equations and rewriting in matrix notation we can write:

$$f = B\Delta + v \quad (5)$$

where  $f$  contains the differences between pair wise distances,  $B$  is the Jacobian matrix containing elements of the linearized trilateration condition equation,  $\Delta$  represents corrections to the initial coordinate approximations and  $v$  contains the ignored higher-order terms of the Taylor's series expansion.

The above formulation gives an adjustment of indirect observations model (Mikhail et al., 2001), in which we assign equal weights to the distances. This non-linear model is solved iteratively and requires good initial approximations to the unknown coordinates to avoid local optima solutions. Figure 1 illustrates the use of the rigidity constraint in obtaining the modified trilateration condition equation.

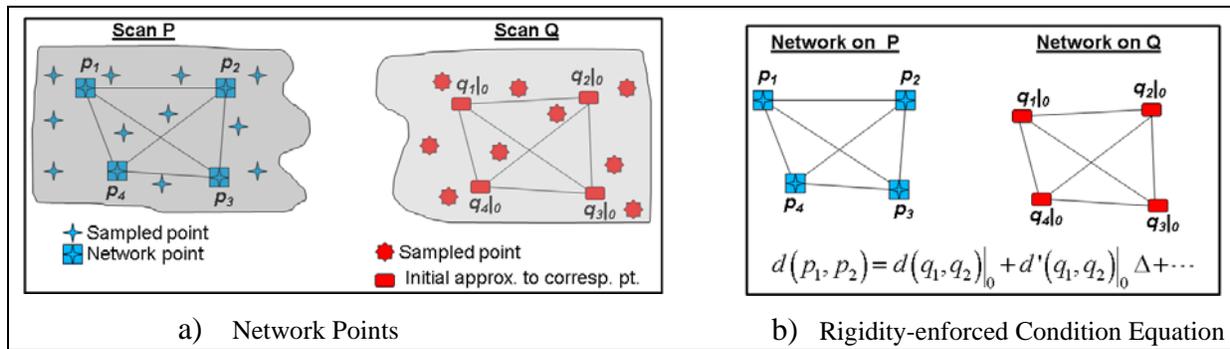


Figure 1. Overlapping Scans.

### Surface Constraint

In addition to the condition equations we enforce the final adjusted network points to remain on the surface, by setting the point-to-local tangent plane distance to be zero, through the following constraint equation:

$$(N \square \hat{q} + D)_i = 0 \quad (6)$$

where  $\hat{q}_i$  represents the  $i$ th adjusted point on Q,  $N_i$  represents the direction cosines of the local tangent at  $\hat{q}_i$ , and  $D_i$  represents the normal distance of the origin from the local tangent at  $\hat{q}_i$ . Since the adjusted point can be expressed by:

$$\hat{q}_i = q_i|_0 + \Delta q_i \quad (7)$$

we can expand Equation (6) to obtain the following:

$$\begin{bmatrix} n_x & n_y & n_z \end{bmatrix}_i \begin{bmatrix} \Delta q_x \\ \Delta q_y \\ \Delta q_z \end{bmatrix} = -(D + N \square q|_0)_i \quad (8)$$

Combining for all network points we have:

$$\begin{bmatrix} \bar{n}_1 & 0 & \cdots & \cdots & 0 \\ 0 & \bar{n}_2 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & \bar{n}_{np-1} & 0 \\ 0 & \cdots & \cdots & 0 & \bar{n}_{np} \end{bmatrix} \begin{bmatrix} \Delta \bar{q}_1 \\ \Delta \bar{q}_2 \\ \vdots \\ \Delta \bar{q}_{np-1} \\ \Delta \bar{q}_{np} \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{np-1} \\ g_{np} \end{bmatrix} \quad (9)$$

where  $\bar{n}_i$  contains the direction cosines of the local tangent at the  $i$ th point,  $\Delta \bar{q}_i$  is the correction vector for each point, and  $g_i$  is the right hand side of each point, which should be equal to zero, but can be non-zero if the current approximation to  $q_i$  does not lie exactly on the assigned local tangent plane. In general though,  $g_i$  is almost equal to zero. Now we have in matrix notation our system of constraint equations expressed as:

$$C\Delta = g \quad (10)$$

### Over-Constrained Solution

The over-constrained least squares solution, involving Equations (5) and (10) is formulated by the Lagrange minimization method, which gives the following linear system (Mikhail and Ackermann, 1976 and Strang, 2006):

$$\begin{bmatrix} N & C^T \\ C & O \end{bmatrix} \begin{bmatrix} \Delta \\ \lambda \end{bmatrix} = \begin{bmatrix} t \\ g \end{bmatrix} \quad (11)$$

and this system can be expressed more succinctly as

$$\tilde{N}\tilde{\Delta} = \tilde{t} \quad (12)$$

Now, our normal matrix  $N = B^T B$  is singular, where  $B$  is as described in Equation (5). This singularity is owing to the well known fact that a network adjustment without control (i.e. free network adjustment) has a rank-defect of six, for the trilateration case (Mikhail et al., 2001). However, the augmented matrix  $\tilde{N}$  of Equation (12) is non-singular, provided that  $C$  is a valid constraint matrix for free network adjustment. The conditions for validity will be dealt with in the Section 3.

### Rigid-body Parameter Estimation

Following the network adjustment, the closed-form rigid-body parameter estimation method of Arun et al., 1987 is used to obtain the transformation parameters. This method decouples the least-squares parameter estimation into a rotation estimation, which is done with a *Singular Value Decomposition*, followed by the translation estimation.

## VALIDITY OF SURFACE CONSTRAINTS

The surface constraints are necessary to enforce that the adjusted coordinates of the network points on Q refer to locations that are actually on the original surface. In other words, the network adjustment should not alter the surface.

Two criteria exist which ensure that the augmented matrix  $\tilde{N}$ , is full rank (Bjorck, 1996 and Lawson and Hanson, 1995). First, Equation (10) must be consistent. Second,  $\square(N) \cap \square(C) = \phi$ . Refer to Table 1 for explanation of all symbols used in this section.

### Criterion #1

Each row of our  $C$  matrix represents direction cosines of the local tangent plane at each respective point (see Equation (9)). Not only is this a sparse, structured matrix, but it is also has orthogonal rows. This means that  $C^T$  always has orthogonal columns, provided that all the direction cosines are non-zero vectors. Thus  $C^T$  is of full rank and satisfies the first criterion.

### Criterion #2

It is appropriate here to mention the sizes of the important matrices:  $B \in \square^{n,u}$ ,  $C \in \square^{np,u}$ , where  $np$  represents the number of points in the network,  $u$  is the number of unknowns, thus  $u = 3np$  and  $n$  is the number of observations, i.e. pair wise distances, and is computed from the binomial coefficient,  $n = \binom{np}{2}$ .

Now, let  $b$  represent a row in  $B$  of Equation (5). For every  $b$  there exists a  $\tilde{C}$  (a subset of rows in  $C$ , where  $\tilde{C} \in \square^{np-2,u}$ ), such that  $b \perp \tilde{C}$ . In other words, every row in  $B$  is orthogonal to a subset of  $np - 2$  rows in  $C$ .

This is easily seen since each row in  $B$  contains the contributions of a pair of points to the Jacobian matrix and every row in  $C$ , except the two rows that relate to the above-mentioned two points, are mutually orthogonal to that row in  $B$ . Further, for every row in  $B$  there are 2 linearly dependent rows in  $C$ .

Since,  $\dim \mathfrak{R}(B^T) = u - 6$  and  $\dim \square(B^T) = n - u + 6$ , then criterion#2 is satisfied *iff* at least six rows of  $C$  are obtained that are *l.d.* on rows in  $B'$ , where  $B'$  are the rows of  $B$  that are in its left-null space.

**Satisfaction of Criterion #2.** Each  $b$  involves a pair of points, and  $\square(B^T)$  involves  $n - u + 6$  distinct pairs of points. Criterion#2 is satisfied when  $\tau$  pairs are selected from the  $n - u + 6$  pairs to give at least six distinct points. To determine  $\tau$  consider the following:

- if  $r(B') \geq 3$   
then  $\exists$  at least 3 *l.i.* rows of  $B$  in  $\square(B^T)$   
 $\Rightarrow$  at least 6 rows in  $C$  are obtained  
 $\therefore \min(\tau) = 3$
- for  $\max(\tau)$  consider Table 2, where by induction,  $\max(\tau) = 11$

**Table 1.** List of Symbols

Symbol	Description
$C \in \mathbb{R}^{p,u}$	$C$ is a matrix of size p-by-u
$b \in \mathbb{R}^u$	$b$ is a vector of size $u$
$\mathfrak{R}(N)$	column space of $N$
$\dim \mathfrak{R}(N)$	dimension of column space of $N$
$\mathfrak{R}(B^T)$	row space of $B$
$\mathfrak{N}(N)$	null space of $N$
$\mathfrak{N}(B^T)$	left-null space of $B$
$r(B)$	rank of matrix $B$
$A \cap B$	matrix $A$ intersect matrix $B$
$A \perp B$	matrix $A$ is orthogonal to matrix $B$
$\phi$	null vector
$\exists$	there exists
<i>iff</i>	if and only if
<i>a l.d. b</i>	$a$ and $b$ are linearly dependent vectors
<i>a l.i. b</i>	$a$ and $b$ are linearly independent vectors
$\min(a)$	minimum value for $a$
$\max(a)$	maximum value for $a$

**Table 2.** Largest Minimal Set

# of distinct points	Largest minimal set of distinct pairs
2	1
3	2
4	4
5	7
6	11

Thus, criterion#2 is satisfied once  $\dim \square(B^T) \geq 11$ . Solving this inequality reveals that when  $np \geq 9$  criterion#2 is satisfied. We have just shown that the proposed constraint matrix  $C$  is both necessary for our registration strategy and also sufficient for free network adjustment.

## RESULTS AND DISCUSSIONS

Two types of experiments were conducted with the INM strategy to evaluate its performance in terms of correspondence estimation and registration accuracy. For the first type of experiments, a dataset (we call this P) was transformed by a known set of transformation parameters to create a new dataset (Q), and INM was used to estimate the parameters between the two datasets. P was then transformed through the estimated parameters to give  $\hat{Q}$  and the correspondence estimation was evaluated by computing the Euclidean norm between points in  $\hat{Q}$  and Q.

The second type of experiments was similar to the first, except that Q was resampled onto a regular grid by linear interpolation of Delaunay triangulation, and with grid cell size set to the average point spacing. This experiment was for evaluating registration accuracy, when there is no exact point-to-point correspondence, as occurs in real-world applications. For both types of experiments multiple scenarios were created to evaluate INM's performance with varying point densities. In each of the two types of experiments, P and Q were down sampled by various rates ranging from 100% to 25%, where a sampling rate of 50% means that P and Q both had half of the total number of points in the original dataset, and this down sampling was done randomly.

The metric which was used to evaluate the registration accuracy reflects the difference between two surfaces and does not require any knowledge of the true parameters, which are never actually available. This error metric is called the registration error and is defined as the root mean square (rms) of projection distances between points in one dataset and tangent planes in the other. The tangent planes in the second dataset are obtained from planar fitting of the closest three neighboring points to the point in the first dataset. The final registration root mean square error (rmse) values included distances, both from P to Q and Q to P, as this gives a more reliable representation of the mis-registration of the datasets.

In our experiments, three different datasets, and two different sets of parameters were used, (see Figure 2 and Table 3). A network size of 300 points was used for INM in all the experiments, and a maximum of 15 iterations were performed. In the case of the network

**Table 3.** Parameter Sets

Set#	$\omega$	$\phi$	$\kappa$	$T_x$ (m)	$T_y$ (m)	$T_z$ (m)
1	1 min	1 min	1 min	0.01	0.01	0.01
2	1 deg	2 min	2 min	0.10	0.10	0.20

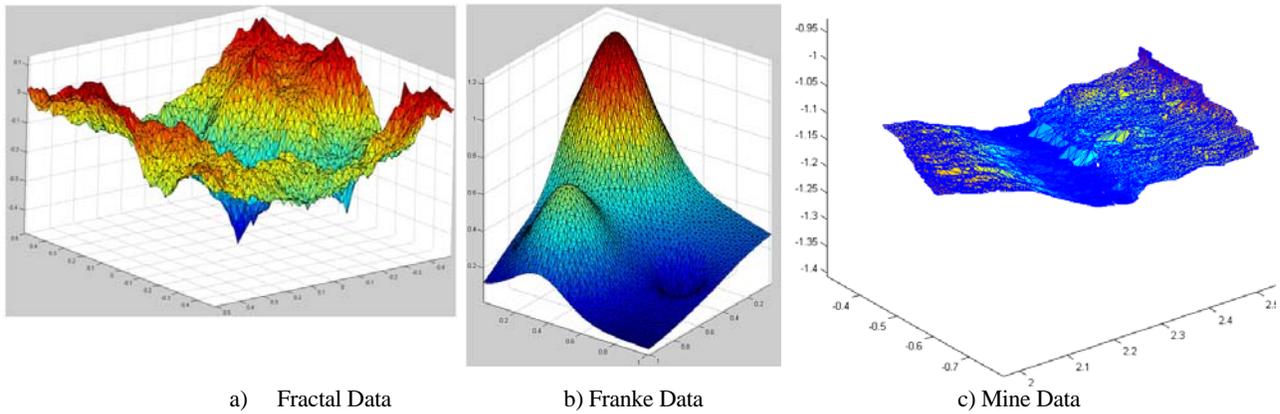
adjustment, convergence was obtained if the rms of  $\Delta$  was less than 5e-3m. The results are reported in terms of correspondence error, registration error, and INM's convergence characteristics. The first two forms of results were qualified by comparisons with the ICP method, as was developed by Chen and Medioni, 1991.

### Experimental Data

Two of the datasets were synthetic surfaces, which will be referred to as the *fractal* and *franke* data, and they represent coarse and smooth surfaces, respectively, that are typically encountered in various laser scanning projects. The third dataset was obtained from a laser scan of a mine, and is a natural, coarse surface. The fractal data have been used on previous registration research and was obtained from the Matlab® toolbox provided by Salvi et al., 2007. The franke dataset was generated from the Matlab® surface function with the same name, franke (see Table 4).

**Table 4.** Data Description  
(coordinates are in meters)

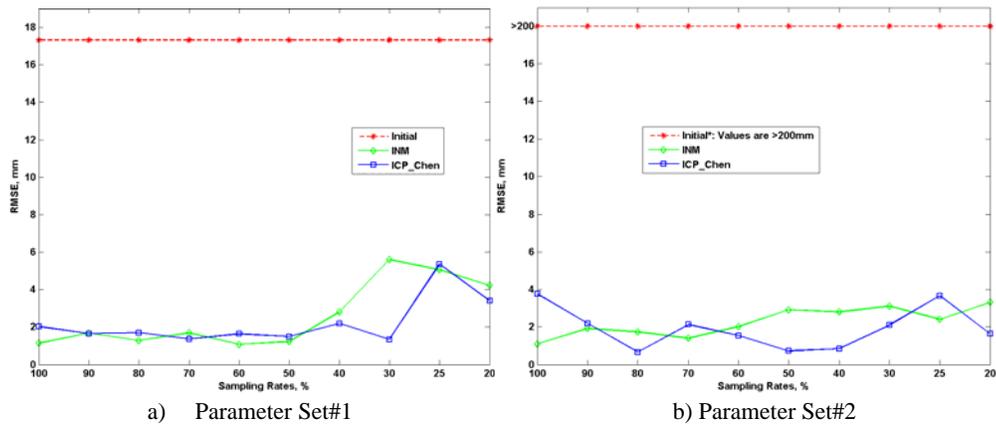
Data	#Pts	dX	dY	dZ
fractal	4096	1.000	1.000	0.607
franke	2601	1.000	1.000	1.217
mine	8380	0.613	0.460	0.112



**Figure 2.** Experimental Data.

### Correspondence Results

For the fractal data, the differences between INM and ICP\_Chen are minimal. For franke, however, INM was consistently better than ICP\_Chen, except for the first parameter set, at the 30% sampling rate. With the mine data, ICP\_Chen outperformed INM for parameter set #1, though, the difference is only noticeable for the cases with less than 50% sampling rate. However, for parameter set #2, INM showed improvements up to an order of magnitude for sampling rates greater than 50%. The two irregular results for INM at 30% and 20% sampling rates are probably due to the insufficiencies of random sampling.



**Figure 3.** Correspondence Results for Fractal Data.

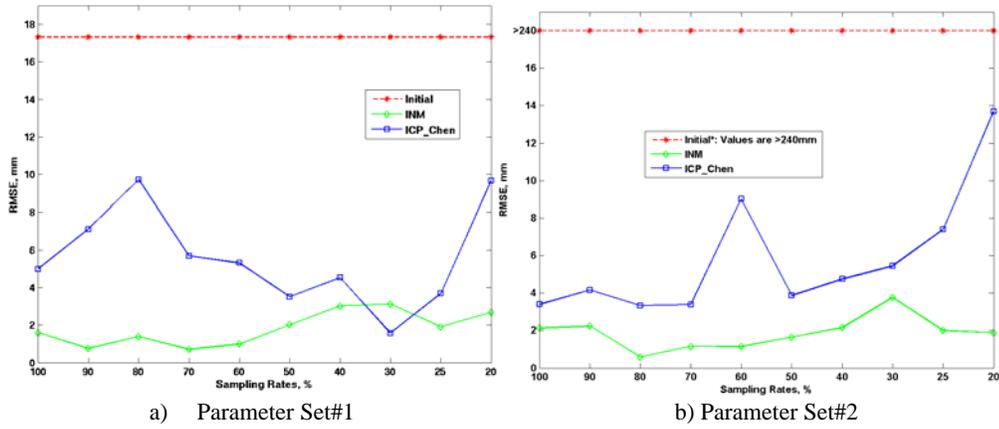


Figure 4. Correspondence Results for Franke Data.

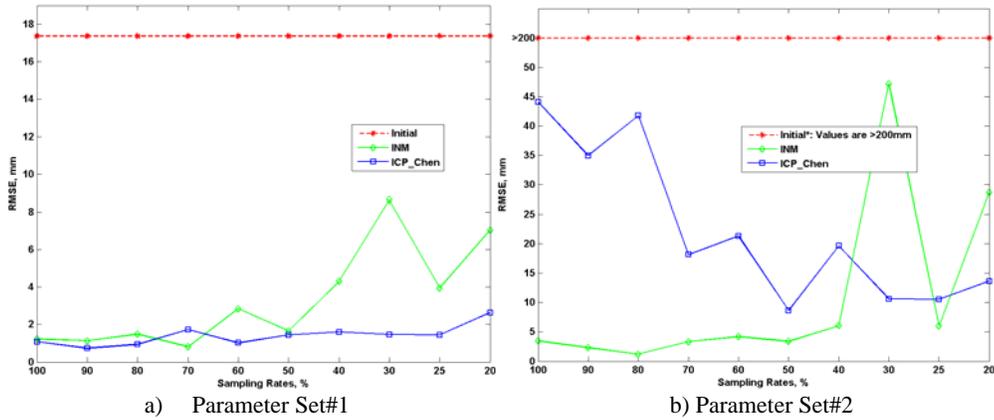


Figure 5. Correspondence Results for Mine Data.

### Registration Results

Figures 6(a, b) and 8(a) show no meaningful difference between the two registration methods. The other three figures (Figures 7(a, b) and 8(b)), however, show a consistent improvement with INM over ICP\_Chen. The improvement in registration rmse is more pronounced with the mine data, except for the irregularities at 30% and 20%, as mentioned previously.

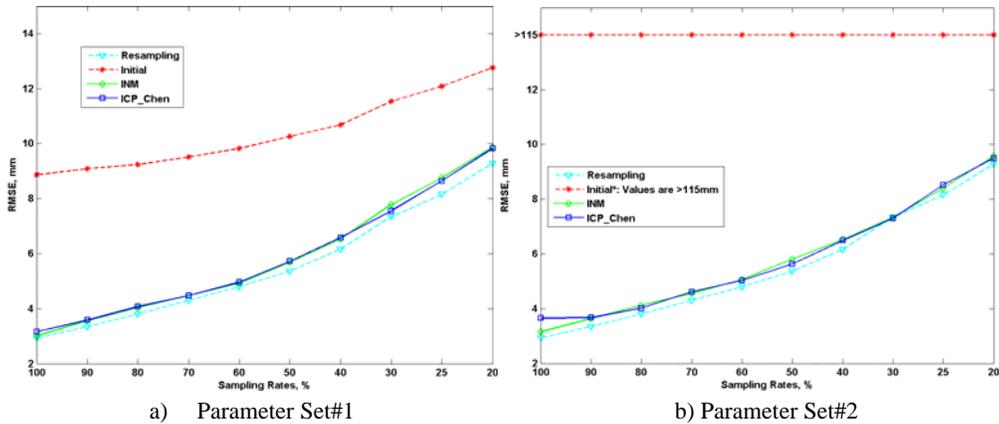


Figure 6. Registration Results for Fractal Data.

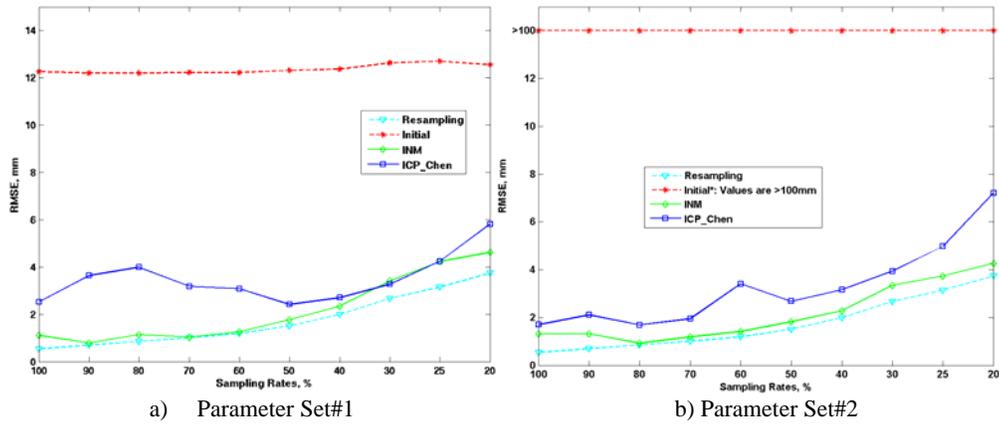


Figure 7. Registration Results for Franke Data.

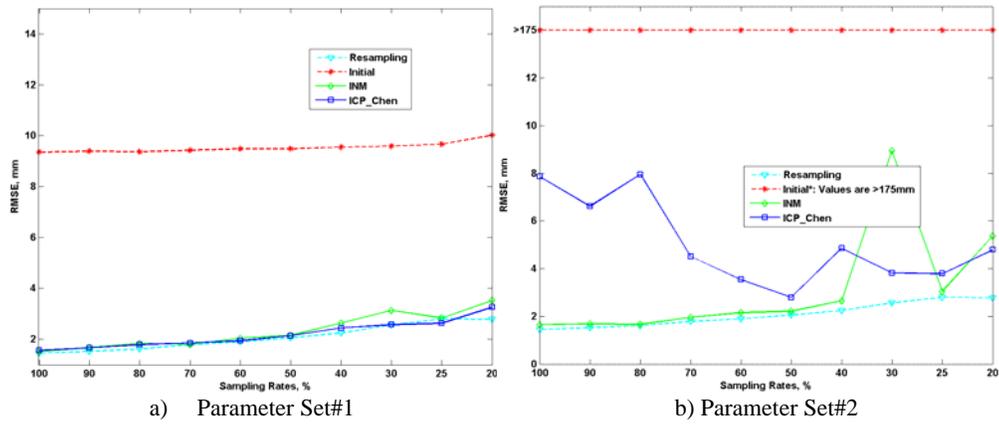


Figure 8. Registration Results for Mine Data.

### Registration Convergence

Figures 9 through 11 give the registration convergence results. Although 15 iterations were performed for each experiment, in most cases convergence was achieved approximately between 5 and 7 iterations.

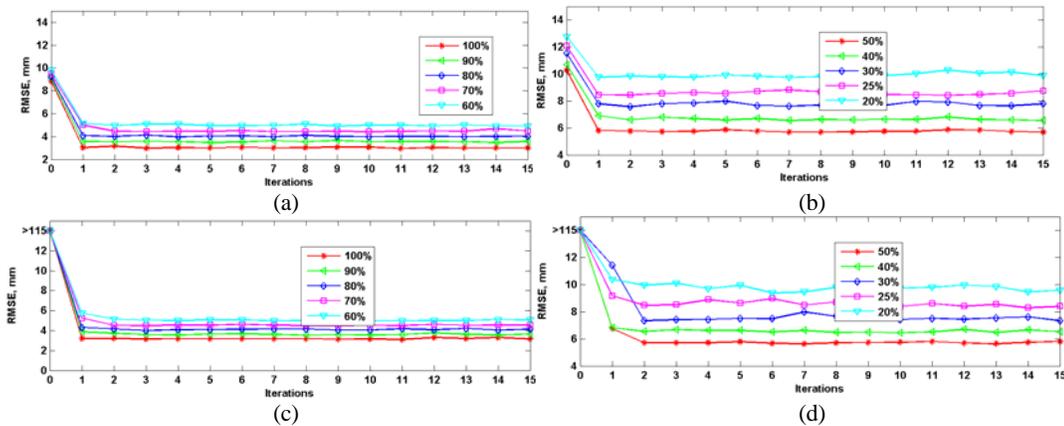
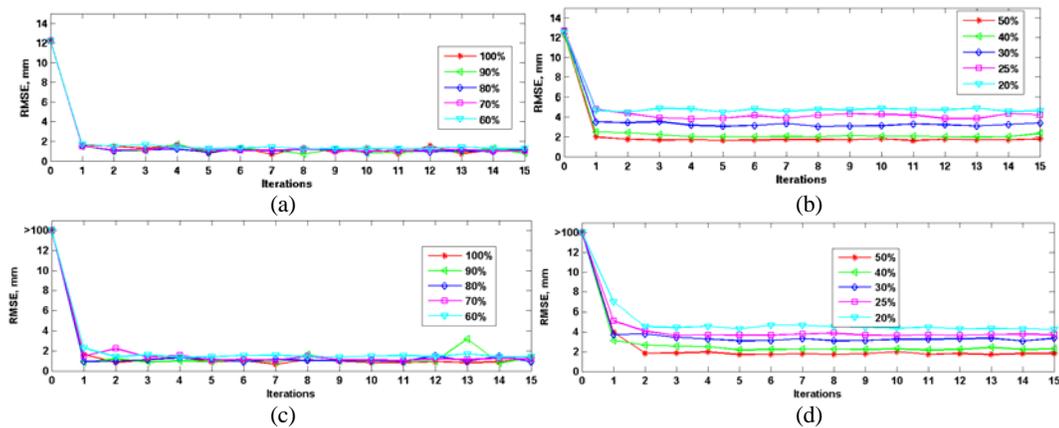
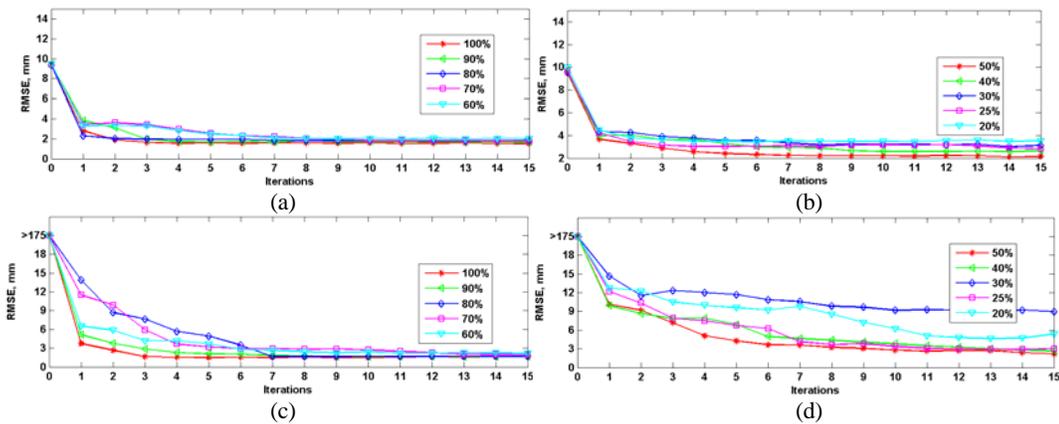


Figure 9. Registration Convergence for Fractal Data  
(a), (b): Parameter Set#1; (c), (d): Parameter Set#2.



**Figure 10.** Registration Convergence for Franke Data  
 (a), (b): Parameter Set#1; (c), (d): Parameter Set#2.



**Figure 11.** Registration Convergence for Mine Data  
 (a), (b): Parameter Set#1; (c), (d): Parameter Set#2.

## DISCUSSION AND CONCLUSIONS

An alternative strategy to the popular Iterative Closest Point (ICP) was presented in this paper, which we call the Iterative Network Matching (INM) strategy. This approach focuses on the direct estimation of point-correspondences between overlapping terrestrial laser scanning data, through an over-constrained network adjustment, which is constrained by local surface tangents. This paper showed that the proposed surface constraints are both necessary for the registration strategy and sufficient for free network adjustment.

The proposed INM strategy is able to estimate point correspondences directly, which then leads to the estimation of the registration parameters. The correspondence and registration results support the intuition that INM's registration performance is directly related to its correspondence performance. Although the correspondence results are only available with controlled experiments, our results highlight an important issue. The registration error gives an estimation of point-to-surface differences. With the ICP\_Chen method it is possible to have a relatively small final mis-registration rmse, as defined by the registration error metric, when the true correspondence error is actually quite large, (see Figures 5(b) and 6(b)).

Despite the favorable results, it must be noted that for all experiments INM used only 300 points to form the networks, and these points were randomly selected. Future work will involve developing optimal methods for determining network sizes as random selection is not always guaranteed to provide points that are optimally distributed.

Another drawback of the current INM strategy is the dimension of the system of linear equations that needs to be solved during each iteration of the over-constrained network adjustment. Further work is needed to improve the computational efficiency of this step, both in sequential and parallel programming.

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