


By Benjamin E. Wilkinson and Henry J. Theiss

Educational Needs for Rigorous Sensor Modeling and Error Budgeting

Geomatics professionals are in a business of error analysis, or at least they should be. Acknowledging there is error (i.e., variation in observation from a "true value") is the first step towards quantifying and minimizing it. Many can mark the locations of features or measure distances in digital photographs or other geospatial products, but those measurements are made immensely more valuable when we understand the processes that led up to making them. This includes how geospatial data are collected and the metadata that comes along with them. Specifically, estimates of uncertainty, or ranges of expected magnitudes and directions of errors, are crucial to countless geospatial applications. Our understanding of the uncertainty in geospatial measurements is what sets us apart. For example, some would say a key difference between the photogrammetric and computer vision fields is photogrammetry's emphasis on geometric accuracy, uncertainty estimation, and preference for model rigor over computer vision's preoccupation with speed and simplicity. The concepts and practical applications of rigorous sensor modeling and error budgeting (i.e., how much unexplainable variation we are willing to accept from the "true value") are crucial to the professional and educational realms of the geospatial world.

An error budget can simply be a list of errors that accumulate along the collection and processing pipeline and induce error in the final product, or, more valuably, be represented in a mathematical model of the collection and processing algorithms and their accompanying errors. Central to this mathematical model is the sensor model. A sensor model is defined as the relationship linking object space coordinates and sensor space measurements. Many refer to a rigorous sensor model, meaning the model attempts to closely capture the physical phenomena occurring during acquisition, while maintaining a level of complexity that makes the model useable. This brings to mind the statistician George Box's quote:

> *"Since all models are wrong the scientist cannot obtain a "correct" one by excessive elaboration. On the contrary following William of Occam he should seek an economical description of natural phenomena. Just as the ability to devise simple but evocative models is the signature of the great scientist so overelaboration and overparameterization is often the mark of mediocrity."*

The existence and quality of sensor models and error budgeting are critical to generating accuracy reports, planning collection and processing, and data adjustment and integration. They also inform developers how to target needs in terms of hardware and software improvements. Arguably the simplest method for reporting uncertainty is the inclusion of standard deviations. There is, however, considerable value in using full error covariance matrices:

Consider for example two states (i and j), with each state having two parameters (x and y). These states could be associated with, for example, observations of x and y made at different times or locations. While it is much simpler to represent the uncertainty of each parameter independently from one another in terms of standard deviations, ignoring the "intra-state" covariances between x and y and the "inter-state" covariances between these parameters at two states i and j can have a profound effect on estimated uncertainties in many ways.

Covariance matrices enable the generation of error ellipses for locations at given confidence levels, accurate estimation of uncertainty in the calculation of distances, areas, and volumes, and also allow for rigorous adjustment and fusion of geospatial products. We will explore these concepts in more detail in a forthcoming article.

How are sensor modeling and error budgeting addressed in academia?

Educators are obliged to ensure graduates have a solid conceptual and practical foundation of error budgeting and sensor modeling. Furthermore, understanding, quantifying, and applying spatial uncertainty in photogrammetric and remote sensing products require an understanding of the hardware and algorithms used to collect and process the data in addition to their associated errors. Thus, learning these topics synergistically illuminates measurement error concepts, the

Photogrammetric Engineering & Remote Sensing
Vol. 83, No. 10, October 2017, pp. 657–659.
0099-1112/17/657–659

doi: 10.14358/PERS.83.10.657

workings of sensors to solidify student understanding of their capabilities and limitations, and illustrates product fidelity with respect to a broad range of applications.

Descriptions of error budgets are vital for many geospatial courses, and should be found at the end of the module for each given measurement system, ranging from measuring tape to laser scanners. Likewise, it is important to introduce undergraduates to basic error theory, least-squares estimation, and the special and general laws of propagation of variance/covariance. At least one full course should be devoted to these subjects since they provide the foundation for many other topics. Sensor models should be described in the undergraduate photogrammetry courses. Particularly, the fundamental photogrammetric frame camera sensor models should be described: the collinearity equations, which are so common that it has become customary for authors to refer to them as the "well-known," and the coplanarity condition equation. These, along with laser scanning sensor models, are essential since graduates are most likely to encounter them in practice, nowadays. Other, simpler models can also be described such as 2D and 3D transformations and the direct linear transformation, but collinearity is emphasized.

In addition to commercial software suites, it is the authors' experience that in-house software for coordinate transformation, space resection, relative orientation, space intersection, and bundle adjustment are extremely beneficial for the students. This software can focus more on elucidating what goes on "behind the scenes" of the photogrammetric processes. This is extremely useful, since the students and instructor know exactly what sensor models are being used, and since detailed reports can be generated highlighting the concepts described in the lecture and in the readings, including metrics not available in the often black-box commercial packages. Students can then use the known sensor model and accompanying measurement uncertainties to predict the propagated uncertainty in the products, and check against what the programs report. Similarly, in-house developed visualization programs with depictions such as those shown in Figure 1, can be used to illustrate sensor models and error propagation geometry as an additional resource to strengthen understanding. So, for example, a student could apply error propagation equations to estimate the uncertainty in a triangulated point's position, compare that uncertainty against what is reported in commercial and in-house software, and recreate the geometry using the visualization programs for a graphical representation of the error propagation, thus acquiring an expansive learning experience.

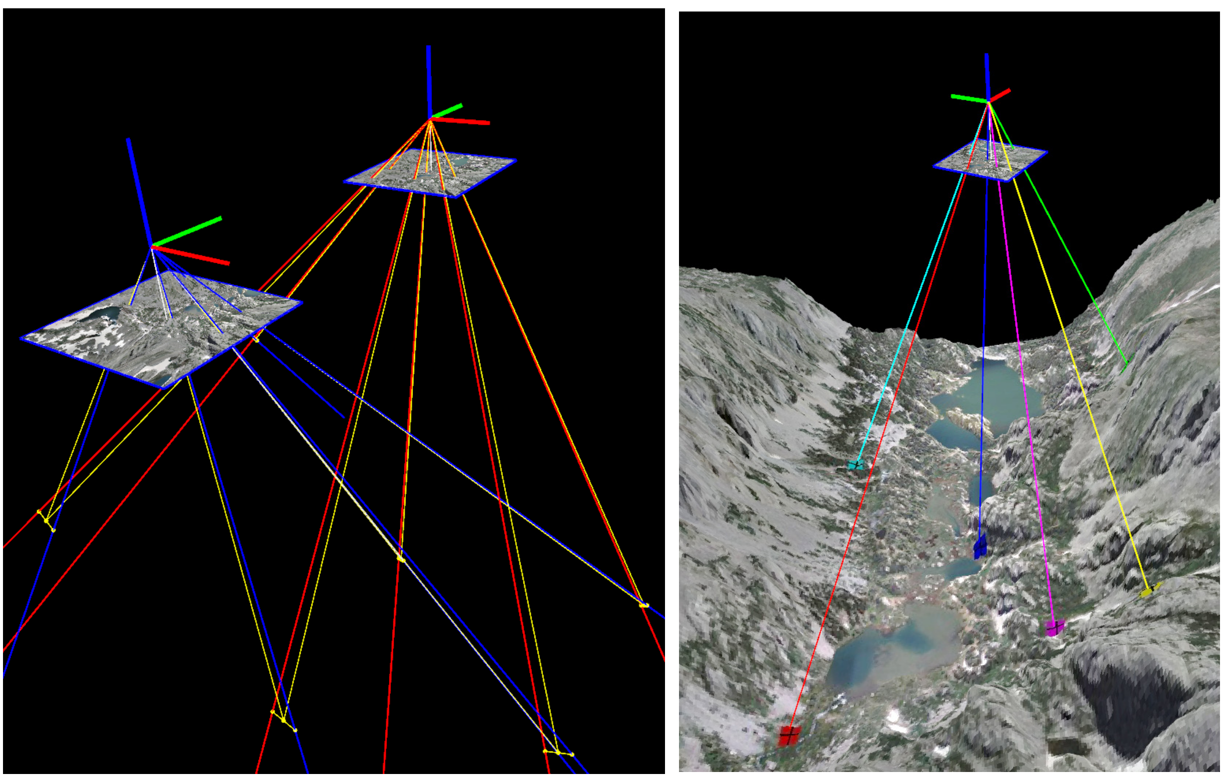

Figure 1. Interactive 3D figures, illustrating reprojection error and uncertainty of object space points using the collinearity model, used in photogrammetry courses at UF to illustrate sensor models and propagation of error. (Computer programs developed at the University of Florida using data from: Manley, W.F., Parrish, E.G., and Lestak, L.R., *High-Resolution Orthorectified Imagery and Digital Elevation Models for Study of Environmental Change at Niwot Ridge and Green Lakes Valley, Colorado: Niwot Ridge LTER, INSTAAR, University of Colorado at Boulder, digital media* [*2009*].)

Education in sensor modeling, error propagation, and data adjustment does not end with Bachelor's or Master's degree geomatics engineering courses, but is also relevant in thesis and dissertation projects as will be discussed in three examples. The first example relates to the Rational Polynomial Coefficients (RPC) sensor model that is prolific in the satellite imaging community. The RPC format allows for only two error modeling terms: one for "bias" and one for "random". PhD research has been performed to demonstrate techniques for full error covariance propagation from "physical sensor model space" to "RPC space" with generic and efficient representation and application by an end user. The second example relates to verification of consistency between errors, compared to ground truth and "predicted accuracy" when using a sensor model, and photogrammetric applications. While it is acceptable for a Ph.D. researcher to show that his/her processes are capable of improving accuracy by some measurable quantity (e.g., change in root mean square errors or change in the size of an empirically calculated error ellipse), it is even more helpful if that researcher demonstrates that the actual error dispersion (e.g., as represented by an empirically calculated error ellipse) is consistent with the error ellipse computed via rigorous error covariance propagation. A third example relates to verification of the error propagation itself, and can be used to identify issues with respect to the sensitivity (i.e., Jacobian) matrices; e.g., a mistake could have been made in the analytical computation of the partial derivatives, or the problem is so non-linear that the standard linear error propagation techniques are not a good representation. A popular technique is called "Monte Carlo Analysis," and involves writing a computer program that begins with a model with an "errorless" transformation to which known errors will be introduced for each input random variable via a random number generator. Then it evaluates the transformation thereby computing errors for each output random variable, and repeats the procedure for multiple trials (e.g., a thousand times) so that a statistical distribution for the output variables can be empirically computed and compared to that derived via error covariance propagation.

Conclusion

Error budgets, sensor models, and uncertainty estimation are all critical to developing and using geospatial products. They allow us not only to estimate the accuracy of the products, guiding us on how they should be used, but also plan missions, develop processing workflows, and rigorously fuse data. In industry, it is important for each custodian of geospatial data along their life cycle to understand and carry-forward rigorous and reliable uncertainty metadata with the products for the benefit of subsequent users; therefore, geospatial educators have a duty to include these ideas when discussing all types of measurement systems, to produce graduates with a firm understanding of them. Here, we described some fundamental ideas, but the subject is rich and there are many resources that go into depth on the subject. Furthermore, the photogrammetry community is pursuing ongoing research, and the reader is encouraged to dig deeper.

As a "Call to Action", we recommend that the interested reader pursue undergraduate or graduate studies at one of the many universities with excellent Geomatics programs. Second, we recommend reading textbooks that specialize in data adjustment and error propagation; following are few examples based on the authors' own experience:

- ***Analysis and Adjustment of Survey Measurements***, by Edward M. Mikhail and Gordon Gracie
- ***Observations and Least Squares***, by Edward M. Mikhail, with contributions by F. Ackermann
- ***Adjustment Computations***, by Charles D. Ghilani
- ***The Manual of Photogrammetry, 6th Edition***, J. Chris McGlone (ed.)
- ***Elements of Photogrammetry***, Paul Wolf, Bon Dewitt, Benjamin Wilkinson
- ***Introduction to Modern Photogrammetry***, Edward Mikhail, James Bethel, J. Chris McGlone

Finally, we invite you to read the forthcoming scenario article that will provide a deeper commentary with use cases and graphics that illustrate the impacts of rigorous error modeling.

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