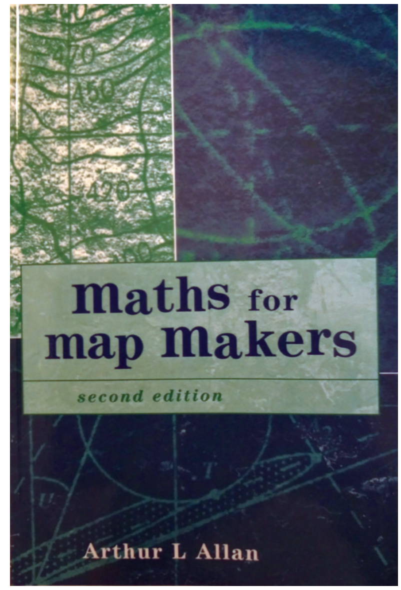

Math for Map Makers

by Arthur L. Allan. Second Edition, 2011 reprint, Whittles Publishing, Scotland, UK. Orginally published 1997.

Reviewed by Melissa J. Rura-Porterfield, Ph.D. Memphis, Tennessee.

As some folks who know me know, I tend to gravitate to indices, I might even start reading a book in the appendices and seem to as I age to find that harder and harder to abstain from. I suppose years of work on updating a glossary that has yet to see the light of day may have played a part in driving me to that but in truth often savvy authors hide their best gems in the safety of an appendix.

My first impression of "*Maths for Mapmakers*" came many years ago, being published first in 1997 while I was still a student. This impression might have been a groan or a comment something like, "***Maths***, what is that!?" but being so long ago I don't rightly recall. Needless to say since that time, reissued, twice this text has brought ***Maths*** to a large swath of mapmakers for nearly a generation.

The book consists of 394 pages, divided into 13 chapters and a reference section including 4 appendices, a summary of formulae and an index. The author encourages all readers to pause for the "How to Read this Book" section and this reviewer concurs wholeheartedly. Here the author sections his book, placing chapters 1-5, which include (1) Numbers and Calculation, (2) Plane Geometry, (3) Trigonometry, (4) Plane Coordinates, (5) Problems in Three Dimensions, into a group that should be read and the problems worked in order as if one was building or reinforcing the foundation of mathematical understanding in mapping. Whereas the later chapters 6-13, which include (6) Areas and Volumes, (7) Matrices, (8) Vectors, (9) Calculus, (10) Conic Sections, (11) Spatial Trigonometry, (12) Solutions of Equations, (13) Least Squares Estimation, can be worked more like case studies on these foundations and not necessarily in the order they are found in the book. Essentially splitting the book into lower-level and upper-level courses in mapping mathematics.

Each chapter has a list of both formulae and "key words" or vocabulary words that should have been defined within its pages. I remember as a student the frustration and relief when I finally understood the chapter 6 key word "Hero's Formula" was in the chapter 6 formula list as "$\Delta = \frac{1}{2} bc \sin A$" and that was the same as the area of a triangle I already knew from Chapter 3, $\Delta = \sqrt{[s(s-a)(s-b)(s-c)]}$, where $2s = a + b + c$ [see, Equation (3.31) and Equation (6.2)]. Of course, had I not started my homework before reading the chapter, I probably would have had less frustration. Nevertheless, I did get an opportunity to fumble around in chapter 3 to refresh my acquaintance with triangles and their areas which did me no harm in the end.

Another anecdote of floundering around in Chapter 3 comes from Section (3.6) *Coordinate Axes and Bearings*. The complete anecdote is too lengthy for this book review, but suffice to say in using Peter Dale's book[1] "*Introduction to Mathematical Techniques Used in GIS*," I came across his clarification of the difference between how a mathematician and a surveyor measure an angle [see, page 67, Figure (5.9) in Dale's book] and although, this may sound like the beginning of a bad science joke; if you want to know the difference between how a mathematician, a surveyor, a cartographer, and a geographer measure an angle that is on page 63 in Figure (4.1) of Allan's book. Dale's explanation of just the mathematician and the surveyor is better equated to Section (3.6) in Allan's book, but don't start there, go to Figure (4.1) first and use it like the Rosetta Stone for angle measurements.

Although each chapter is the source of many useful exercises used to learn mathematics in mapping, one of the chief complaints of this text is there is no answer key printed in the book for these exercises. And subsequent printings have pages with ink shortage running down the left side margin. Generally, it is the margin calculator symbol to indicate

[1]Dale, Peter "*Introduction to mathematical techniques used in GIS*," CRC Press/Taylor & Francis, [2014].

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an exercise problem that is obscured but in some places also words in a paragraph are also obscured. Moreover, the formulae and "key word" lists are in every chapter but as this reviewer looked for a list of figures for reference to her great disappointment there is none. The strength and usefulness of the many figures warranted a list that could be referenced. This reviewer understands the unnecessary extra work of naming each figure but indexing a Figure's or Table's chapter and section could have been very helpful.

List of Figures/Tables	Name	Chapter	Section
Figure (2.5 – 2.7)	Reference Grid	2	4
Figure (2.8)	Parallelogram, rectangle, square, and rhombus	2	5
Figure (2.9 - 2.10)	Pythagoras's theorem for a right angled triangles	2	6

My take-home jewel from this text is in Appendix 3, page 368, another Rosette Stone, this time for Least Squares Estimation. How many times have I heard someone say, "He uses the Ohio State least squares notation, I don't get it." Or "He must have learned that notation from Purdue, I am lost?!" Here, we are all reminded that we may speak using many different notations, to solve many similar problems using similar assumptions to find and adjust for error. We must learn to communicate. Don't give up, look-up! The answer is to be found!

Appendix A3: Notation for Least Squares

Author	Observation Matrix	Weight Matrix	Normal Equations Matrix	Dispersion Matrix
Allan[7]	$Ax + L = v$ $Ax + Cv + L = 0$	W	$Nx = b$	D
Cooper[1, 6]	$Ax = B + V$ $Ax + Bv = b$	W	$A^tWx = A^tb$ Note the lower case t	Q
American Manual of Photogrammetry[2]	$B\Delta - \ell = v$ $B\Delta - \ell = Av$	P	$B^TP\Delta = B^TP\ell$	Q
Mikhail[3]	$A(\ell + v) = d$ $Av + B\Delta = f$ $C\Delta = g$	W	N	Q
Wolf[4]	$AX = L + V$	W	$Nx = A^TL$	Q
Koch[5]	$X\beta = y + e$	P	$X^TX\beta = X^Ty$	D

[1]M.A.R. Cooper, 1987, *Control Surveys in Civil Engineering*, Collins (ISBN 0-00-383183-3,381Pages)

[2]American Society of Photogrammetry, 1966, *Manual of Photogrammetry*, Library of Congress Catalog No 65-20813 Vol 1

[3]E. M. Mikhail, 1976, *Observations and Least Squares*, Dun-Donnelley, (ISBN 0-7002-2481-5, 497 Pages)

[4]P.R. Wolf and C.D. Ghilani, 1997, *Adjustment Computations*, Wiley (ISBN 0-471-16833-5, 564 pages)

[5]K.R. Koch, 1997, *Parameter Estimation and Hypothesis Testing in Linear Models*, Springer (ISBN 3-540-65257-4 325 pages)

[6]M.A.R. Cooper and P.A. Cross, 1988, Statistical Concepts and the Application in Photogrammetry and Surveying, *Photogrammetric Record*, Vol XIII, No 73, 645-678

[7]A.L. Allan and N. Atkinson, "Back to Basics' Series Nos 14 to 24 – Least Squares Statistics and all that, Survey Review, Vols 35 and 36 Nos 272 to 282

This reviewer does recommend this text to a large swath "*of geomatics including surveying, cartography and photogrammetry, geography and civil engineering and for the use in industry or academia*" as the book's back cover suggests, but I suppose many of us have known that for years.