U.S. Geological Survey (USGS) - National Geospatial Program (NGP)

and the

American Society for Photogrammetry and Remote Sensing (ASPRS)

Guidelines on Geometric Inter-Swath Accuracy and Quality of Lidar Data

U.S. Department of the Interior **U.S. Geological Survey**

American Society for Photogrammetry and Remote Sensing

Guidelines on Inter-Swath Geometric Accuracy and Quality of Lidar Data - Version X

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Summary

This document provides guidelines on quantifying the relative horizontal and vertical errors observed between conjugate features in the overlapping regions of lidar data. The quantification of these errors is important because their presence estimates the geometric quality of the data. A data set can be said to have good geometric quality if measurements of identical features, regardless of their position or orientation, yield identical results. Good geometric quality indicates that the data are produced using sensor models that are working as they are mathematically designed, and that data acquisition processes are not introducing any unforeseen distortion in the data. Good geometric quality also leads to better geolocation accuracy of the data when the data acquisition process includes coupling the sensor with GNSS.

Current specifications are not adequately able to quantify these geometric errors. This is mostly because the methods to quantify systematic and non-systematic errors have not been investigated well. Accuracy measurement and reporting practices followed in the industry and as recommended by data specification documents (Heidemann 2014) also potentially underestimate the inter-swath errors, including the presence of systematic errors in lidar data. Hence they pose a risk to the user in terms of data acceptance (i.e. a higher potential for accepting potentially unsuitable data). For example, if the overlap area is too small or if the sampled locations are close to the center of overlap, or if the errors are sampled in flat regions when there are residual pitch errors in the data, the resultant Root Mean Square Differences

(RMSD) can still be small. To avoid this, the following are suggested to be used as criteria for defining the inter-swath quality of data:

a) Median Discrepancy Angle
b) Mean and RMSD of Horizontal Errors using DQM measured on sloping surfaces
c) RMSD for sampled locations from flat areas (defined as areas with less than 5 degrees of slope)

2000-5000 points are uniformly sampled in the overlapping regions of the point cloud (2000-5000 points per pair), to measure the discrepancy between swaths. Care is taken to sample only areas of single return points. Point-to-Plane data quality measures are determined for each sample point and are used to determine the above mentioned quality metrics. This document details the measurements and analysis of measurements required to determine these metrics, i.e. Discrepancy Angle, Mean and RMSD of errors in flat regions and horizontal errors obtained using measurements extracted from sloping regions (slope greater than 10 degrees).

Guidelines on Inter-Swath Geometric Accuracy and Quality of Lidar Data

Introduction: Geometric Quality, Calibration and the need to test

A dataset is said to have good geometric quality when data are produced using sensor models that are working as they are mathematically designed, and data acquisition processes are not introducing any unforeseen distortion in the data. Good geometric quality ensures greater geolocation accuracy of lidar data when the data acquisition process includes coupling the sensor with geopositioning systems. For lidar data, high geometric quality ensures that data contains consistent geospatial information in all dimensions, and across the data extents.

Geometric quality of data is ensured by proper calibration of data acquisition systems. Calibration ensures that the sensor is performing according to manufacturer specifications. In general, calibration of instruments are usually performed by the system/sensor manufacturer or the user either periodically, or based on usage. Calibration should also be performed when an instrument has had a shock or vibration (which potentially may put it out of calibration) or whenever observations appear questionable.

It is recognized that lidar systems are of many types, and each type may have different sensor models that demand different calibration philosophies. Therefore, it is not the goal of this document to discuss calibration procedures for all the instruments, but to discuss recommended processes to test the quality of calibration which is crucial to ensuring the geometric quality of data. This means testing whether the lidar point cloud data are consistent and accurate in horizontal as well as vertical dimensions. “Quality Control (QC)” is used to denote post-mission procedures for evaluating the quality of the final Lidar data product (Habib et al., 2010). The user of the data is more concerned with the final product quality, than the system level Quality Assurance (QA) procedures that may vary depending on the type of instrument in use.

The QC processes described in this document provide a method of assessing the relative horizontal, vertical and systematic errors in the data. The document introduces the following to ensure geometric quality:

a) Inter Swath Data Quality Measures (DQMs)
b) Analysis of DQMs to quantify 3D and systematic errors
c) Summarizing the three errors (relative horizontal, relative vertical and systematic)

It is expected that the following procedures outlined in this document can provide a more complete understanding of the geometric quality of lidar data. Testing lidar data based on the procedures outlined in the document will ensure that instances of poor geometric quality as shown in Figure 1 are caught in an automated manner, without having to manually view all of the delivered data, or understand the entire data acquisition process and sensor models.

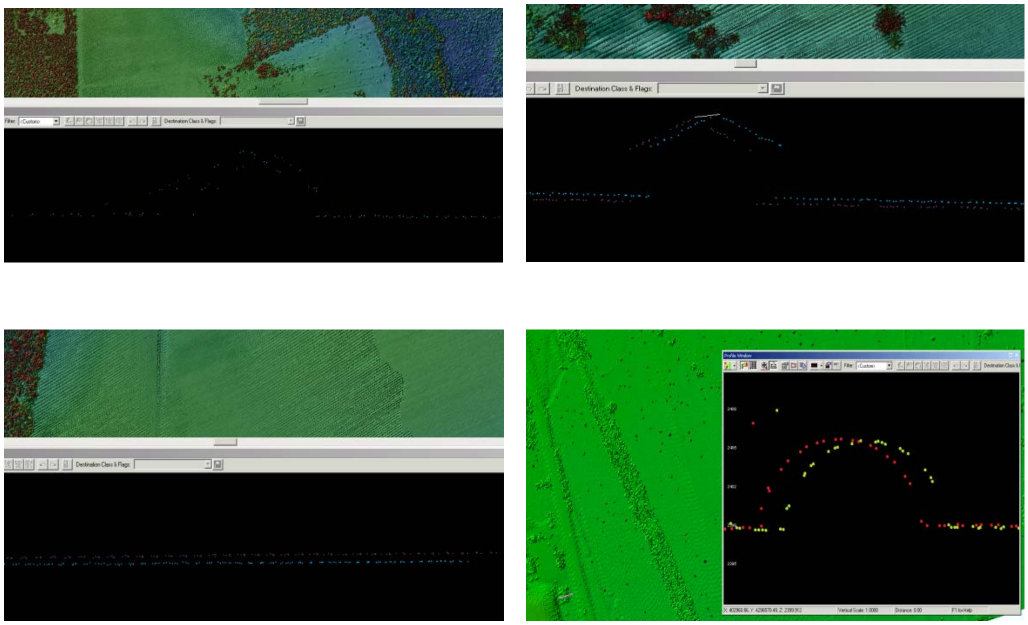

Figure 1 Errors found in swath data acquired by the US Geological Survey, indicating inadequate quality of calibration. The images show profiles of objects in overlapping regions of adjacent swaths

Data Quality Measures for Quantifying Geometric Quality

When overlapping swaths of data are available, the geometric quality of lidar data can be most easily judged by observing the area covered by overlapping swaths. Quantitative measures on the quality of calibration can be generated by analyzing these regions. The underlying philosophy is that conjugate features observed in multiple scans of lidar data are consistent and coincident.

Current Methods

There are not many documented methods available for measuring geometric quality. The USGS Lidar Base Specification and anecdotally, some data providers suggest to rasterize (or use Triangulated Irregular Model) the overlapping data, and determine raster differences. Others suggest that a few points be chosen manually or automatically in the overlapping area and the vertical differences noted. Such methods may not describe the geometric quality of data completely:

- All the measurements may not be valid. Measurements must be made on hard surfaces, and there is no mechanism to identify such surfaces in a simple manner. Measurements made in areas of rapidly changing slope must be avoided.
- Only vertical differences can be measured and horizontal errors cannot be quantified. Vertical differences alone cannot quantify geometric quality
- Systematic errors are not quantified. Systematic errors are required to estimate absolute errors in the data but according to the formulization in the ASPRS accuracy standards, they are assumed to have been eliminated
- Swaths may need to be converted to intermediate products (raster/TIN), which are not used anywhere else.

Geometric quality of data can be quantified by measuring the horizontal, vertical and systematic errors in the data. Current methods only estimate the vertical errors. Therefore they are inadequate indicators of the geometric quality.

Recommended Data Quality Metrics

A measurement of departure of the conjugate features from being coincident is termed Data Quality Measure (DQM) in this document. The DQM is a measure of registration between overlapping swaths/point clouds, after they have been calibrated and before further processing (i.e. point cloud classification, feature extraction, etc.) is done. The DQM in this document is based on a paper by Habib et al. (2010), and is based on point-to-feature (line or plane) correspondences in adjacent strips of Lidar data. The DQMs are indicators of the quality of calibration, and they are used to extract relative errors (vertical, horizontal) in the data and quantifiably estimate systematic errors in the data.

Figure 2 shows a profile of a surface that falls in the overlapping region of two adjacent swaths. The surface as defined by the swaths is shown in dotted lines while the solid profile represents the actual surface. A poorly calibrated system leads to at least two kinds of errors in lidar data. The first one is that the same surface is defined in two (slightly) different ways (relative or internal error) by different swaths, and the second one is the deviation from actual surface (absolute error). For most users of lidar data, the calibration procedures are of less concern than the data itself. However, they would like to have a process to test the quality of calibration of the instrument, because a well calibrated instrument is a necessary condition for high quality data. While data providers make every effort to reduce the kind of errors shown in Figures 1 and 2, there are no standard methodologies in current QC processes to measure the internal goodness of fit between adjacent swaths (i.e. internal or relative accuracy).

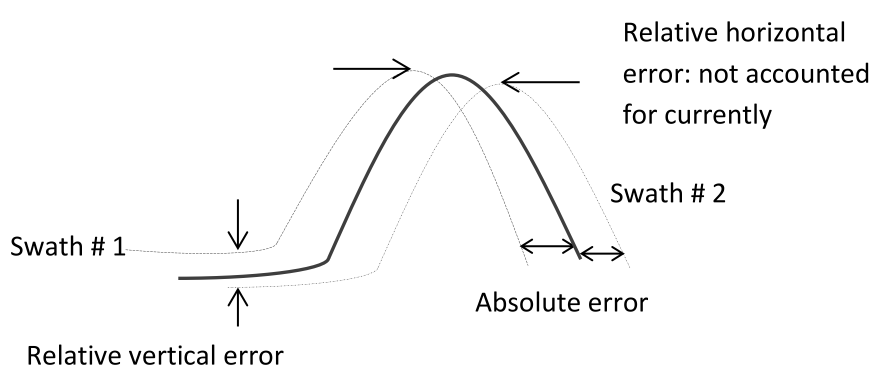

Figure 2 Surface uncertainties in hypothetical adjacent swaths. Profile of actual surface is shown as solid line while the surface defined by swath # 1 and swath # 2 are shown as dotted lines

Current specifications documents (e.g. Heidemann 2014) do not provide adequate guidance on methods to measure the inter-swath (internal accuracy) goodness of fit of lidar data. This is because there are no broadly accepted methods in use by the industry, and there are only a few scientific papers that specifically pertain to inter-swath metrics (Habib 2010;Latypov 2002;Vosselmann 2010). These methods only concern themselves with vertical error (Latypov 2002) or involve feature extraction (Habib et. al; Vosselmann) that may prove operationally difficult to achieve.

The ASPRS Cal/Val Working Group is investigating three quantities (Table 1) that measure the inter-swath goodness of fit. These measures describe the discrepancy between two overlapping point clouds and are often used to obtain optimal values of the transformation parameters.

Table 1 Data Quality Measures (DQMs) or inter-swath goodness of fit measures

Nature of surface	Examples	Data Quality Measures (DQMs)/Goodness of fit measures	Units
Natural surfaces: No feature extraction	Ground, Roof etc. i.e. not trees, chimneys etc.	Point to surface (tangential plane to surface) distance	Meters
Man-made surfaces via feature extraction	Roof planes	Perpendicular distance from the centroid of one plane to the conjugate plane	Meters
	Roof edges	Perpendicular distance of the centroid of one line segment to the conjugate line segment	Meters

The DQMs are not direct point-to-point comparisons because it is nearly impossible for a lidar system to collect conjugate points in different swaths. It is easier to identify and extract conjugate surfaces and related features (e.g. roof edges) from lidar. The DQMs over natural surfaces and over roof planes assume that these conjugate surfaces are planar, and determine the measure of separation between a point and the surface (plane). The DQM over roof edges extract break lines or roof edges from two intersecting planes and measure their discrepancy.

DQM Over Natural Surfaces: point to (tangential and vertical) plane distance

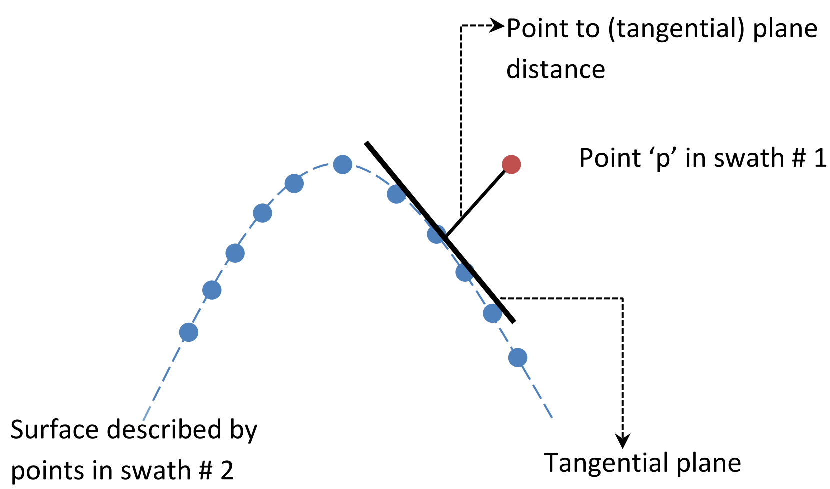

Figure 3 Representation of DQM over natural surfaces. Point ‘p’ (red dot) is from swath # 1 and the blue dots are from swath # 2

This measure is calculated by selecting a point from one swath (say point ‘p’ in swath # 1), and determining the neighboring points (at least three) for the same coordinates in swath # 2. Ideally, the point ‘p’ (from swath # 1) should lie on the surface defined by the points selected from swath # 2. Therefore, any departure from this ideal situation will provide a measure of discrepancy, and hence can be used as a DQM. This departure is measured by fitting a plane to the points selected from swath # 2, and measuring the (perpendicular) distance of point ‘p’ to this plane.

DQM over roof planes: point to conjugate plane distance

In the case where human-made planar features (e.g. roof planes) are present in the region of overlap, these features can be extracted and used for measuring the inter-swath goodness of fit. These planes can be extracted automatically, or with assistance from an operator. Assuming PL1 and PL2 to be conjugate planes in swath # 1 and swath # 2 respectively, the perpendicular distance of points used to define PL1 to the plane PL2 can be determined easily. Instead of selecting any random point, the centroid of points may be used to define PL1 can be determined. The centroid to Plane PL2 (in swath # 2) distance can be used as a DQM to measure the inter-swath goodness of fit (Habib et. al., 2010).

DQM over roof break lines: point to conjugate line distance

If human-made linear features (e.g. roof edges) are present in the overlapping regions, these can also be used for measuring discrepancy between adjacent swaths. Roof edges can be defined as the intersection of two adjacent roof planes and can be accurately extracted. Conjugate roof edges (L1 and L2) in swaths #1 and # 2 should be first extracted automatically or using operator assistance. The perpendicular distance between the centroid of L1 (in swath # 1) to the roof edge L2 (in swath # 2) is a measure of discrepancy and can be used as DQM to the measure inter-swath goodness of fit (Habib et. al., 2010).

DQM Natural Surfaces implementation

The discussion below is the result of prototype software designed and implemented by the US Geological Survey to research methods to determine inter-swath accuracy and estimate errors of calibration and data acquisition.

The US Geological Survey developed prototype research software that implements the concept of point to plane DQM. The software works on ASPRS's LAS format files containing swath data. If the swaths are termed Swath # 1 and Swath # 2 (Figure 4), the software uniformly samples single return points in swath # 1 and chooses 'n' (user input) points. The neighbors of these 'n' points (single return points) in swath # 2 are determined. There are three options available for determining neighbors: Nearest neighbors, Voronoi neighbors or Voronoi-Voronoi neighbors. However, other nearest neighbor methods such as "all neighbors within 3 m" are also acceptable.

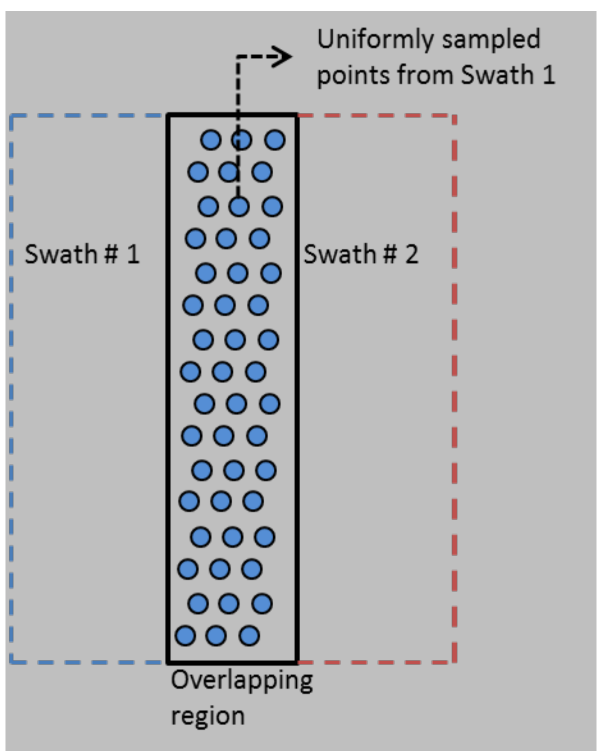

Figure 4 Implementation of prototype software for DQM analysis

A least squares plane is fit through the neighboring points using eigenvalue/eigenvector analysis (in a manner similar to Principal Component Analysis). The equation of the planes is the same as the component corresponding to the least of the principal components. The eigenvalue/eigenvector analysis provides the planar equations as well as the root mean square error (RMSE) of the plane fit. Single return points in conjunction with a low threshold for RMSE are used to eliminate sample measurements from non-hard surfaces (such as trees, etc.). The DQM software calculates the offset of the point (say 'p') in Swath # 1 to the least squares plane. The output includes the offset distance, as well as the slope and aspect of the surface (implied in the planar parameters).

The advantages of using the method of eigenvalues/PCA/least squares plane fit are fivefold:

a) The RMSE of plane fit provides an indication of the quality of the control surface. A smaller eigenvalue ratio indicates high planarity and low curvature. It provides a quantitative means of measuring control surfaces.
b) Point-to-Plane comparisons are well established as one of the best methods of registering point cloud.
c) Converting surfaces to intermediate results in (however small) loss of accuracy.
d) The arc cosine of Z component of eigenvector gives the slope of terrain
e) The normal vector of the planes are crucial to calculate the horizontal errors

Vertical, Systematic and Horizontal Errors

The DQM measurements need to be analyzed to extract estimates of horizontal and vertical error. To understand the errors associated with overlapping swaths, the DQM prototype software was tested on several data sets with the USGS, as well as against datasets with known boresight errors. The output of the prototype software not only records the errors, but also the x, y and z coordinates of the test locations, eigenvalues and eigenvectors, as well as the least squares plane parameters.

The analysis mainly consists of three parts

a) The sampled locations are categorized as functions of slope of terrain: Flat terrain (defined as those with slopes less than 5 degrees) versus slopes greater than 10 degrees.
b) For estimates of relative vertical error, DQM measurements from flat areas (slope < 5 degrees) are identified:
    a. Vertical error measurements are defined as DQM errors on flat areas
    b. For systematic errors, the distance of the above measured sample check points from center of overlap (Dco) are calculated. The center of overlap is defined as the line along the length of the overlap region passing through median of sample check points (Figure 5). The Discrepancy Angle (dS**i**) (Illustrated in Figure 5) at each sampled location, defined as the arctangent of DQM error divided by Dco, is measured
c) The errors along higher slopes are used to determine the relative horizontal errors in the data as described in the next section.

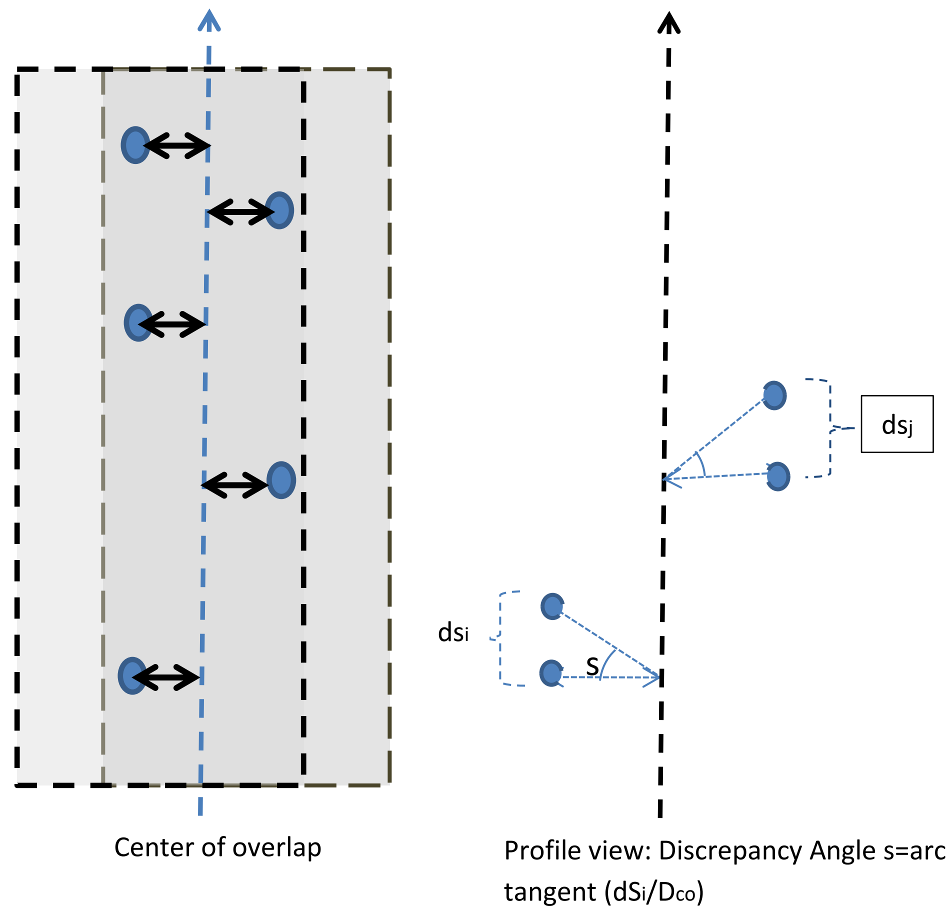

Figure 5 Analysis of DQM errors and center line of overlap

Outliers in the measurements cannot be ruled out. To avoid these measurements, an outlier removal process using robust statistics as explained here can be used:

For each category (flat and higher slopes separately), the following quantities can be calculated:

- $DQM_{Median} = median(DQM)$
- $\sigma_{MAD} = \ Median\ of\ |DQM_i - DQM_{Median}|$
- $ZDQM_i = \left|\frac{DQM_i - DQM_{Median}}{\sigma_{MAD}}\right|$
- $ZDQM_i \begin{cases} > 7, & Measurement\ is\ an\ outlier \\ \leq 7, & Measurement\ is\ acceptable \end{cases}$

Only those points that are deemed acceptable are used for further analyses. The Median Absolute Deviation method is only one of many outlier detection methods that can be used. Any other well defined method would also be acceptable.

Relative Vertical and Systematic Errors

Relative vertical errors can be easily estimated from DQM measurements made on locations where the slope is less than 5 degrees (flat locations). Figure 6 shows plot of DQM output from flat locations as a function of the distance from the centerline of swath overlap.

Positive and Negative Errors: The analysis of errors based on point-to-plane DQM can use the sign of the errors. If the plane (least squares plane) in swath #2 is "above" the point in swath #1, the error is considered positive, and vice versa. Figure 6 shows the different methods of representing vertical errors in a visual manner. Both these methods of representation of errors can be used to easily identify and interpret the existence of systematic errors also.

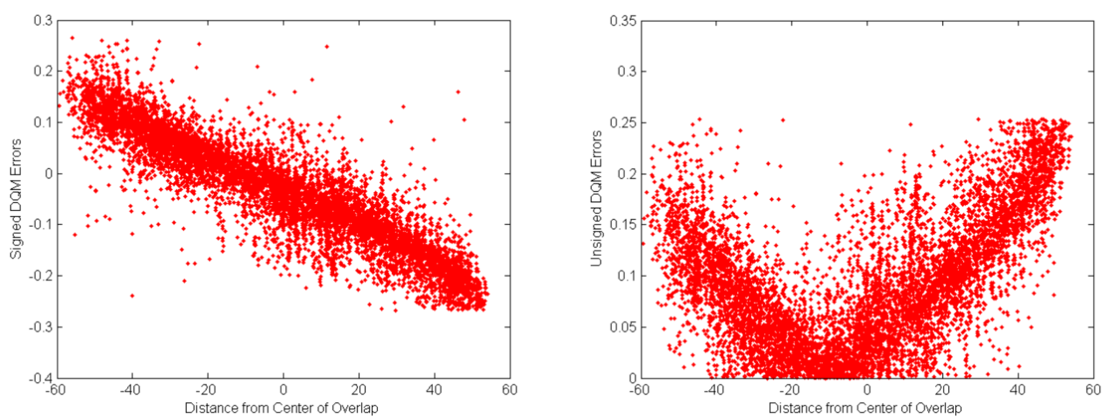

Figure 6 Visual representations of systematic errors in the swath data. The plots show DQM errors isolated from flat regions (slope < 5 degrees). 6(a) plots signed DQM Errors vs. Distance from the center of Overlap, while 6(b) plots Unsigned DQM errors vs. Distance from center of overlap.

In the presence of substantial systematic errors, relative vertical errors tend to increase as the measurements are made away from the center of overlap. In particular, errors that manifest as roll errors (the actual cause of errors may be completely different) will cause a horizontal and vertical error/discrepancy in lidar data. Therefore it is possible to observe these errors in the flat regions (slope less than 5 degrees), as well as sloping regions. In the flat regions, the magnitude of vertical bias increases from the center of the overlap. Using the sign conventions for errors defined previously, these errors can be modelled as straight line passing through the center of the overlap (where they are minimal).

In Figure 7, the red dots are measurements taken from flat regions (defined as those with less than 5 degrees), the green and the blue dots are taken from regions with greater than 20 degrees slope. The green dots are measurements made on slopes that face away from the centerline (perpendicular to flying direction), whereas the blue dots are measurements taken on slopes that face along (or opposite to) the direction of flight.

If non-flat regions that slope away from the centerline of overlap are available for DQM sampling, horizontal errors can also be observed. It should be noted that the magnitude of horizontal errors are greater than that of the vertical errors, for the same error in calibration.

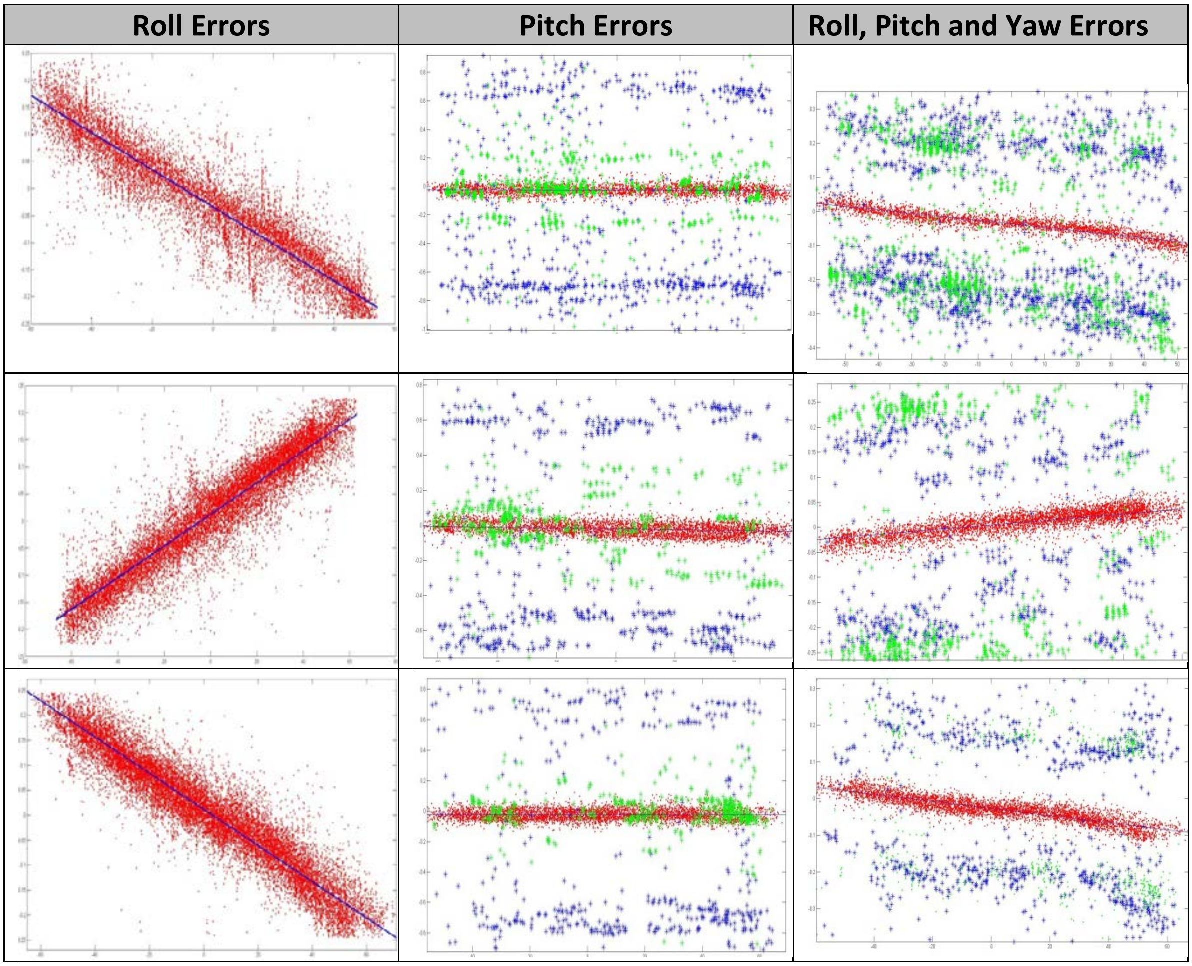

Figure 7 DQM errors as function of distance of points from center of the overlap (each column has different scales). The blue line is a regression fit on the error as function of distance from center of overlap

In Figure 7, the first column shows the plot of DQM versus distance of sample measurements from centerline of overlap. The consistent and quantifiable slope of the red dots indicates that Roll errors are present in the data. A regression line is fitted on the errors as a function of the distance of overlap. The slope of the regression line defined by the red dots in Figure 7 is termed Calibration Quality Line (CQL). The slope of the CQL corresponds theoretically to the mean of all the Discrepancy Angles measured at each DQM sample test point. In practice, the value is closer to the median of the measured Discrepancy Angles (perhaps indicating outliers).

Pitch errors cause discrepancy of data in the planimetric coordinates only, and the direction of discrepancy is along the direction of flight. The discrepancy usually manifests as a constant shift in features (if the terrain is not very steep). This requires them to be quantified using measurements made from non-flat/sloping regions. Figure 7 shows that this can be achieved (blue dots in the second column indicate pitch errors) using the DQM measurements. The blue dots indicate that there is a constant shift along the direction of flight. The presence of red dots close to the zero error line (and following a flat distribution) indicates that these errors are not measurable in flat areas. The presence of green dots closer to the zero line also indicates that pitch errors are not measurable in slopes that face away from the flight direction.

The third column in Figure 7 indicates that when these errors are combined (as is almost always the case), it is possible to discern their effects using measurements of DQM on flat and sloping surfaces.

Relative Horizontal Errors

Relative horizontal error between conjugate features in overlapping swaths is often higher in magnitude (sometimes as much as 10 times) than vertical error. Horizontal error may reflect an error in the acquisition geometry, and is an important indicator of the geometric quality of data. Currently, most methods that quantify horizontal errors in lidar data (relative or absolute) require some form of explicit feature extraction or the use of targets. However, feature extraction may be infeasible at an operational level, for every swath, when the only purpose is to validate geometric quality.

In this section, a process to summarize (i.e. generate mean, standard deviation and RMSD) the horizontal errors in the data is described. The presented process does not measure the relative displacement of every point in the overlapping region, or the sampled measurement, but provides a method of directly ***estimating*** the summary statistics (mean, RMSE and Standard Deviation) of horizontal errors. These errors can be summarized in a tabular format and used to identify suspect swaths.

Since the datasets are nominally calibrated, the errors between conjugate features are modeled as relative shifts. Any rotational error is considered small. This is not an unusual assumption as calibrated swaths are expected to match well. Also, the modeling of errors as shifts is also implicit in describing the quality of other geospatial products. We generally refer to the mean, standard deviation and root mean square errors, and estimate accuracy based on an assumption of Gaussian distribution of these errors. Implicitly, therefore, we model all errors together as a "Mean shift" and any residual errors are described by the standard deviation.

Steps to estimate Relative Horizontal Errors:

Operationally, this process is very simple to implement. The point-to-plane DQM measurements that are obtained from regions of higher slope must be identified by a filtering process (say slopes > 10˚). The normal vector components of these measurements form the 'N' matrix of equation $N_{higher\ slopes}\Delta X = \mathrm{DQM}$, the corresponding DQM measurements form the right hand side of the equation. ΔX is the estimate of average horizontal shift in the data. To solve the equation, one can use any of the libraries available for solving linear regression/least squares equations.

Mathematical Derivation:

The process of measuring the point-to-plane DQM also provides us with estimates of the normal vectors of the planar region, along with an estimate of curvature. It will be shown in the section below that if the neighborhood is large enough (this ensures that the planar fit is stable), and if the curvature is low, the normal vectors, and the point-to-plane DQM can be combined to estimate the presence of relative horizontal errors in the data.

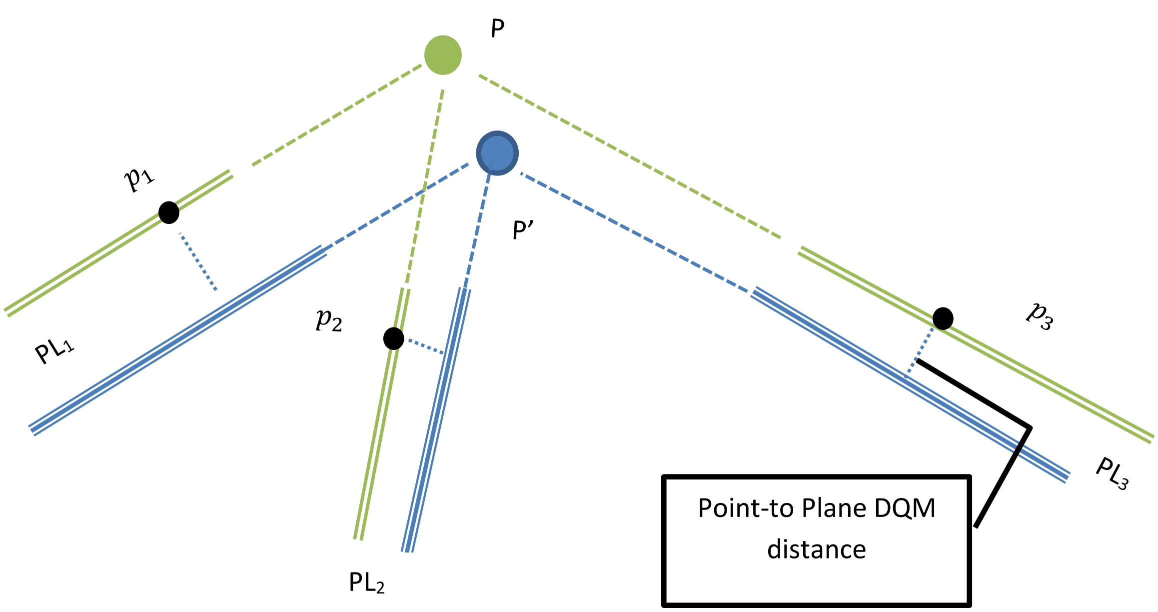

Figure 8 Three points (p_1, p_2, p_3) ,the measured DQMs, the corresponding planar patches (PL1, PL2, PL3) in Swath #1(green) and Swath # 2 (blue) and their virtual point of intersection (P' and P) are shown

Consider any three points that have been sampled for measuring DQMs in Swath # 1 (p_1, p_2, p_3 in Figure 8). Let these points be selected from areas that have higher slopes. These three points have corresponding neighboring points and three planar patches measured in Swath # 2. Let's assume that these three planar patches intersect at point P in Swath # 1(a virtual point; Point P may not exist physically). Let's assume that the conjugate planar patches in Swath # 2 intersect at P'.

Let the equations (together called **Equation 1**) of the three planes be:

$$a_1X + b_1Y + c_1Z = d_1$$

$$a_2X + b_2Y + c_2Z = d_2 \tag{1}$$

$$a_3X + b_3Y + c_3Z = d_3$$

$$Let\ N_P = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & a_1 & a_1 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

If (a_i, b_i, c_i) are the direction cosines of the normal vectors of the planar patches, then d_i is the signed perpendicular distance of the planes from the origin.

If the intersection point P is represented by $(x, y, z)_p = X_P$ then Equation 1 can be written as:

$$N_pX_P = D_{1p} \tag{2}$$

Similarly, for the point P' in Swath # 2, we have $N_p{}'X_{P\prime} = D_{1p\prime}$.

If $\Delta X = X_{P\prime} - X_P$, $\Delta N = N_p{}' - N_p$ and $\Delta D = D_{1P\prime} - D_{1P}$ we get

$$(N_p + \Delta N)(X_P + \Delta X) = D_{1p} + \Delta D \tag{3}$$

Expanding Equation 3, we get:

$$N_p X_P + N_p \Delta X + \Delta N X_P + \Delta N \Delta X = D_{1p} + \Delta D$$

However $NX_P = D_{1p}$ (Equation 2), therefore we get

$$N_p \Delta X + \Delta N X_P + \Delta N \Delta X = \Delta D \tag{4}$$

Since we are interested only in ΔX, and there are no changes to the normal vectors or displacement vectors when there is a pure shift involved, the analysis can be further simplified by shifting the origin to X_P(i.e. $X_P = (0,0,0)$).

Therefore Equation 4 becomes

$$N_p \Delta X + \Delta N \Delta X = \Delta D \tag{5}$$

Since we are measuring the discrepancy in calibrated point clouds, the expectation is that $\Delta N\ and\ \Delta X$ are small, and hence the product can be neglected, $\Delta N \Delta X$,(Typically, $\Delta N \approx .01$). Therefore Equation 5 becomes:

$$N_p \Delta X = \Delta D \tag{6}$$

Equation 6 is the equation of planes intersecting at ΔX, and at a signed perpendicular distance ΔD from the origin. Since the origin (the point ***P***) lies on all three planar patches, and again emphasizing the fact that we are testing calibrated data, ΔD values will be very close to the three measured point to plane distance errors or DQMs. This assumption models the relative errors as relative shift. In this scenario, the point to perpendicular plane distance does not change if the point P and p1 lie on the same plane, and the planes (PL1 in Swath #1 and Swath #2) are near parallel. Therefore Equation 6 becomes

$$N_p \Delta X \approx \Delta D_{DQM} \tag{7}$$

Considering the same analysis for all triplets of points of patches with higher slopes, we replace ΔX with ΔX_{mean}, N_p with *N* (which is a $n\times 3$ matrix containing normal vectors associated all 'n' DQM measurements that have higher slopes), and ΔD_{DQM} as the $n\times 1$ vector of all DQM measurements. This leads to the simple least squares equation for ΔX_{mean}

$$N\Delta X_{mean} = \Delta D_{DQM} = DQM\ Measurements \tag{8}$$

ΔX_{mean} is easily calculated and the standard deviation for ΔX_{mean} is also easily obtained by using standard error propagation techniques. ΔX_{mean} provides a quantitative estimate of the mean relative horizontal and vertical displacement of features in the inter swath regions of the data. This process can be viewed as a check of the quantification of vertical errors in the data from the previous section

Calibration Quality Report

The Calibration Quality Report should provide a visual as well as a tabular representation of the errors present in the data set. The following are suggested to be used as criteria for quantifying the quality of data:

- The median of Discrepancy Angle values obtained at sample points
- Mean and RMSD of DQM measured in flat regions
- Mean and RMSD of Horizontal Errors using DQM measured on sloping surfaces

The median angle of discrepancy is an indicator of residual roll errors. The angle of discrepancy can be determined even if there is only minimal overlap available, and hence can be very useful for the data producers as well as data users as a measure of quality of calibration or data acquisition. RMSD measurements on flat regions provide an estimate of vertical errors in the data the measurements made from sloping surfaces can be used to estimate horizontal errors.

For a visual representation, it is also suggested that plots shown in Figure 7 be part of the data quality reporting process. It should be noted that the errors described here assume that flight lines are parallel and adjacent flight lines are in the opposing direction. If that is not the case, the pattern of errors will be different. It should also be noted that not all systematic errors cannot be traced using the methods outlined here, and it is not the intention of this work to do so.

Concluding Remarks

Current accuracy measurement and reporting practices followed in the industry and as recommended by data specification documents (Heidemann 2014) potentially underestimate the inter-swath errors, including the presence of systematic errors in lidar data. Hence they pose a risk to the user in terms of data acceptance (i.e. a higher potential for of accepting potentially unsuitable data). For example, if the overlap area is too small or if the sampled locations are close to the center of overlap, or if the errors are sampled in flat regions when there are residual pitch errors in the data, the resultant Root Mean Square Differences (RMSD) can still be small. To avoid this, the following are suggested to be used as criteria for defining the inter-swath quality of data:

a) Median Discrepancy Angle
b) Mean and RMSD of Horizontal Errors using DQM measured on sloping surfaces

c) RMSD for sampled locations from flat areas (defined as areas with less than 5 degrees of slope)

2000-5000 points are uniformly sampled in the overlapping regions of the point cloud, and depending on the surface roughness, to measure the discrepancy between swaths. Care must be taken to sample areas of single return points only. Point-to-Plane data quality measures are determined for each sample point. These measurements are used to determine the above mentioned quality metrics. This document detailed the measurements and analysis of measurements required to determine these metrics, i.e. Discrepancy Angle, Mean and RMSD of errors in flat regions and horizontal errors obtained using measurements extracted from sloping regions (slope greater than 10 degrees).

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Appendix A: Calculating errors: Worked example

This section provides a detailed example of the process of measuring the vertical, horizontal and systematic accuracy of two overlapping lidar swaths, without going into the theoretical details of performing relative accuracy analysis to quantify the geometric accuracy of lidar data.

The discrepancies between the overlapping swaths can be summarized by quantifying three errors between conjugate features in multiple swaths:

- Relative vertical errors,
- Relative horizontal errors and
- Systematic errors.

The industry uses many ways to measure these errors. Traditionally, only the relative vertical errors have been quantified, however, there are no standardized processes that the whole industry uses. Other methods to quantify geometric accuracy can consist of extracting man-made features such as planes, lines etc. in one swath and comparing the conjugate features in other swaths. In case intensity data are used, the comparisons can also be performed using 2D such as road markings, specially painted targets, etc.

Vertical and Horizontal error estimation

To measure and summarize the geometric errors in the data it is suggested that the following procedure be used.

When multiple swaths are being evaluated at the same time (as is often the case) the header information in the LAS files may be used to determine the file pairs that overlap.

For each pair of overlapping swaths, one swath is chosen as the reference (swath # 1) and the other (swath # 2) is designated as the search swath. Depending on the user requirements, type of area (forested vs. urban/open), 2000-5000 points in the overlap region of the two swaths are chosen uniformly from swath # 1. These points must be single return points only. For each point that has been selected, its neighborhood in swath # 2 is selected. For the point density common in 3DEP, 25 points can be selected (although this example uses 50 points). Once the points are selected, a least squares plane is fit through the points. The distance of this plane from the point swath # 1 is the measure of discrepancy between two swaths.

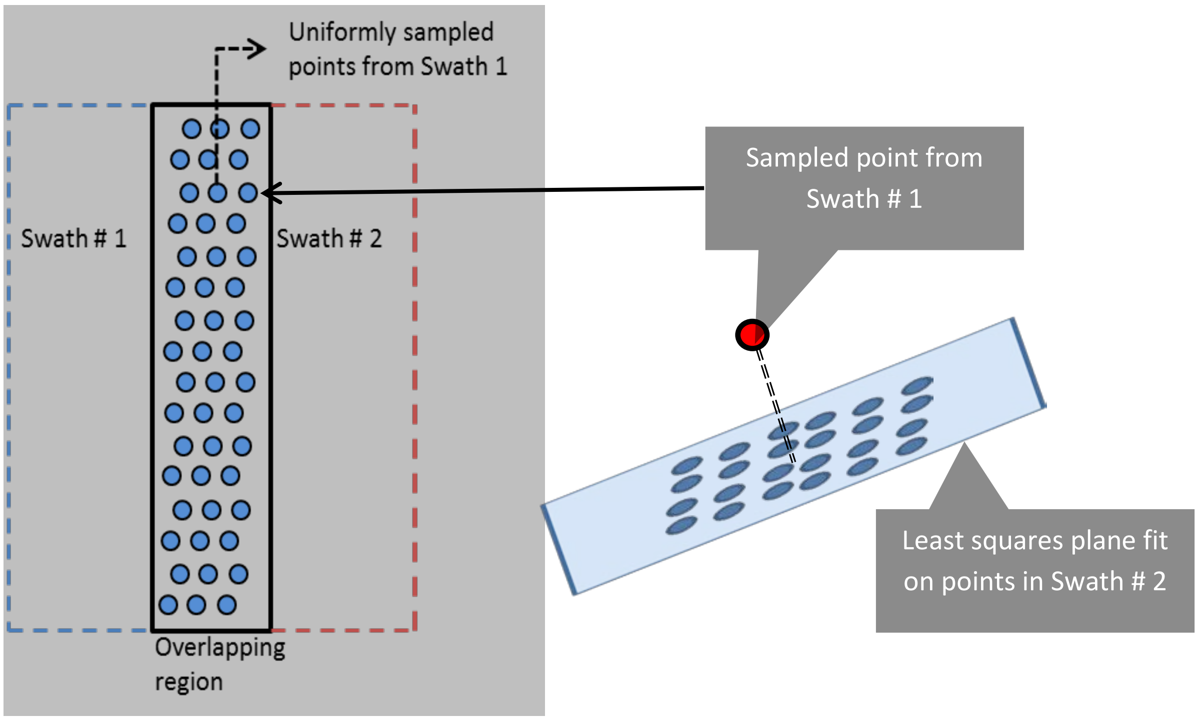

Figure A.1 Implementation of prototype software for DQM analysis. The plot on left shows how the overlap regions (single return points only) are sampled uniformly and the plot on right shows the sampled points from Swath #1 (in red), its neighbors in Swath #2

The process is explained by means of an example below. Assuming that we have a point in swath # 1 at the coordinates (931210.58, 843357.87 and 15.86), Table A.1 lists 50 nearest neighbors in swath # 2.

Table A1 Lists 50 nearest neighbors for the point chosen (931210.58, 843357.87 and 15.86) in swath # 1.

X	Y	Z		X	Y	Z		X	Y	Z
931211.7	843357.7	15.9		931213.5	843358.4	15.97		931212.6	843361.5	16
931211.7	843358.1	15.94		931212.5	843360.1	15.94		931211.7	843362	16
931211.6	843358.7	15.94		931212.7	843355.7	15.9		931214.7	843359.1	15.81
931212.2	843357.7	15.81		931213.4	843358.9	15.9		931214.3	843360.1	15.94
931211.8	843356.6	15.81		931213.1	843360.1	15.9		931214.8	843356.5	15.81
931212.8	843357.7	15.81		931214	843357.7	15.84		931213.5	843361.3	16
931212.4	843356.5	15.74		931214.1	843358.4	15.97		931212.3	843362.1	16
931212.9	843358.3	15.84		931211.6	843361.3	15.97		931213.2	843361.6	16.07
931211.9	843360.1	16		931214	843359	15.84		931215.3	843357.7	15.84
931213	843356.5	15.77		931212.2	843361.3	15.87		931215.2	843358.6	15.9
931213.4	843357.7	15.84		931213.7	843360.1	15.9		931214.9	843360	15.9
931214.7	843361.3	15.94		931214.2	843356.5	15.77		931215.2	843359.1	15.97
931215.5	843360	15.87		931212	843361.4	16.07		931212.9	843362.1	16.07
931214.4	843361.7	15.97		931214.6	843357.7	15.81		931214	843361.3	15.94
931215.9	843359.2	15.9		931212.9	843361.3	16		931215.4	843356.5	15.81
931212.2	843352.6	15.94		931214.7	843358.5	15.9		931215.8	843358.6	15.94
931214.7	843354.2	15.77						931215.9	843357.7	15.81

The first step is to move the origin to the point in swath # 1. This helps with the precision of the calculations, and allows us to work with more manageable numbers.

The next step is to generate the covariance matrix of the neighborhood points. The covariance matrix C is represented by $\begin{pmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{zy} \\ \sigma_{xz} & \sigma_{zy} & \sigma_z^2 \end{pmatrix}$ where $\sigma_x, \sigma_y, \sigma_z$ are the standard deviations of x, y and z columns, $\sigma_{xy}, \sigma_{xz}, \sigma_{zy}$ are the three cross correlations respectively. For the points listed in Table A1, the covariance matrix $C = \begin{bmatrix} 4.044 & 1.006 & -1.921 \\ 1.006 & 16.829 & -3.087 \\ -1.921 & -3.087 & 5.462 \end{bmatrix}$.

An eigenvalue, eigenvector analysis of the C matrix provides the parameters of the least squares fit. In this case, the eigenvalues (represented by $\lambda_1, \lambda_2, \lambda_3 where\ \lambda_1 > \lambda_2 > \lambda_3$) are respectively (2.48, 9.18 and 25.4).

The eigenvector corresponding to the least eigenvalue of the covariance matrix represents the plane parameters, and in this case, the planar parameters are 0.013,-0.026, 0.999, and -0.054 (represented by Nx, Ny, Nz and D).

The 'D' value (-0.054) represents the point to plane distance and is the measure of discrepancy between the swaths at that location, and is the DQM value at that location. To test whether this measurement is made on a robust surface, the eigenvalues can be used to test the planarity of the location. The ratios: $\frac{\lambda_2}{\lambda_1}$ and $\frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}$ are used to determine whether the point can be used for further analysis. The first ratio has to be greater than 0.8 and the second ratio has to be less than 0.005.If both the ratio tests are acceptable, the measurement stands.

2000-5000 DQM measurements depending on the size of the swaths can be made and recorded per pair of overlapping swaths.

Error Analysis

Once the outputs file (A portion of an example output file is shown in Table A2) is generated, the file may be analyzed to determine vertical and horizontal errors in the data. The first analysis step is to divide the output file into two sets of measurements, based on the arc cosine of the Nz column.

$$arc\ cosine(Nz)_i \begin{cases} > 10\ degrees, & Measurement\ is\ in\ Sloping\ terrain \\ \leq 5\ degrees, & Measurement\ is\ in\ flat\ terrain \end{cases}$$

Vertical Error

The vertical error can be determined by the 'D' column of all measurements from the flat terrain:

$\Delta Z_{average} \frac{\sum_{i=1}^{Nf} D_i}{Nf}$, where Nf is the number of measurements found on flat terrain and $\sigma_z = \frac{\sum_{i=1}^{Nf} \left(D_i - \Delta Z_{average}\right)^2}{Nf-1}$.

Table A2: A portion (20 measurements) of output file is shown. 10 measurements are from flat regions and 10 are from sloping surfaces.

X	Y	Z	Nx	Ny	Nz	D	λ_1	λ_2	λ_3	Number of neighbors
280283.61	3363201.64	27.01	0.0338	-0.0249	0.9991	**0.2553**	1.2774	0.7674	0.0002	16
278544.62	3363296.99	28.44	0.0083	0.0180	0.9998	**0.0552**	1.2634	0.7100	0.0002	16
275929.96	3363318.85	27.37	0.0168	-0.0034	0.9999	**0.1613**	1.0022	0.6943	0.0003	14
280581.39	3363373.07	24.28	0.0065	-0.0187	0.9998	**-0.0320**	0.6607	0.3683	0.0002	18
273856.59	3363385.56	28.66	0.0040	0.0112	0.9999	**0.0854**	0.7917	0.7423	0.0007	13
279702.17	3363395.30	27.91	-0.0729	0.0333	0.9968	**-0.2263**	0.7106	0.6454	0.0009	21
274500.56	3363410.55	29.60	-0.0384	0.0157	0.9991	**-0.0329**	1.0429	0.7272	0.0003	16
273559.31	3363424.17	28.33	0.0370	0.0365	0.9987	**0.0652**	0.9996	0.5139	0.0012	14
276223.04	3363425.71	30.11	-0.0015	-0.0118	0.9999	**-0.0258**	0.9276	0.8126	0.0002	18
275747.95	3363450.02	26.44	-0.0405	-0.0450	0.9982	**0.1056**	0.8210	0.6055	0.0003	16
The rows below have slopes greater than 10 degrees										
278928.08	3363230.97	26.86	**0.0267**	**0.2176**	**0.9757**	**-0.2744**	1.4898	0.7525	0.0002	16
278654.50	3363234.18	28.09	**-0.0541**	**-0.1723**	**0.9836**	**0.4879**	1.5222	0.7886	0.0002	18
278874.94	3363249.00	24.56	**0.0589**	**0.2005**	**0.9779**	**-0.2750**	1.4017	0.7269	0.0008	16
278339.98	3363317.26	27.76	**0.0730**	**0.1749**	**0.9819**	**-0.1913**	1.1565	0.7927	0.0018	16
278572.91	3363325.41	28.15	**-0.0710**	**-0.2039**	**0.9764**	**0.4038**	1.0060	0.8644	0.0002	16
279167.04	3363349.97	27.74	**0.1627**	**0.1049**	**0.9811**	**0.1221**	0.8732	0.6538	0.0007	16
278254.99	3363353.27	27.78	**0.0718**	**0.1823**	**0.9806**	**-0.1563**	0.9359	0.5832	0.0007	14
279152.81	3363380.18	27.60	**0.2055**	**0.0939**	**0.9741**	**0.1790**	0.9314	0.4998	0.0004	14
278134.66	3363487.79	28.17	**0.0766**	**0.1711**	**0.9823**	**-0.3347**	0.4553	0.3732	0.0002	22
277950.11	3363503.39	28.53	**0.1350**	**0.1832**	**0.9738**	**-0.3548**	0.3332	0.2164	0.0002	34

In the data shown in Table 1, the vertical errors are calculated using the values in the first 10 rows (i.e. having slopes less the 5 degrees as defined by arc cosine of the Nz column). For this example, the vertical error and the corresponding standard deviation are calculated as:

$\Delta Z_{average} = \frac{\sum_{i=1}^{10} D_i}{10}$= 0.041m and the standard deviation as 0.131 m, with a root mean square error (RMSEz) of 0.131 m.

Horizontal Error

The horizontal error is determined from measurements made from sloping terrain. It is suggested that at least 30 such measurements are available; otherwise the values may not be valid. In the example shown, for the sake of clarity, only 10 DQM measurements (as shown in Table 2) are used.

To determine horizontal error, generate a matrix as shown below:

$$N = [N_x \quad N_y]_{Measurements\ from\ sloping\ terrain\ only}$$

Calculate $D_r = D - N_z \times \Delta Z_{average}$ for all measurements from sloping terrain and solve $N \times \begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix} = D_r$ to obtain estimates of horizontal errors represented by ΔX and ΔY (as well as estimates of their standard deviation). There are several least squares open source solvers available in all languages which can be used to obtain the estimates.

In this case, the values of N and D_r are:

$$N = \begin{bmatrix} 0.0267 & 0.2176 \\ 0.0541 & -0.1723 \\ 0.0589 & 0.2005 \\ 0.0730 & 0.1749 \\ -0.0710 & -0.2039 \\ 0.1627 & 0.1049 \\ 0.0718 & 0.1823 \\ 0.2055 & 0.0939 \\ 0.0766 & 0.1711 \\ 0.1350 & 0.1832 \end{bmatrix}, D_r = \begin{bmatrix} -0.314 \\ 0.448 \\ -0.315 \\ -0.232 \\ 0.364 \\ 0.082 \\ -0.197 \\ 0.139 \\ -0.375 \\ -0.395 \end{bmatrix}$$

Using Least Squares, the solution to solve $N \times \begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix} = D_r$ is ΔX =1.43m and ΔY= -2.21m. Note that if these numbers seem excessively high, an illustration of the horizontal errors for this data is shown in Figure A2

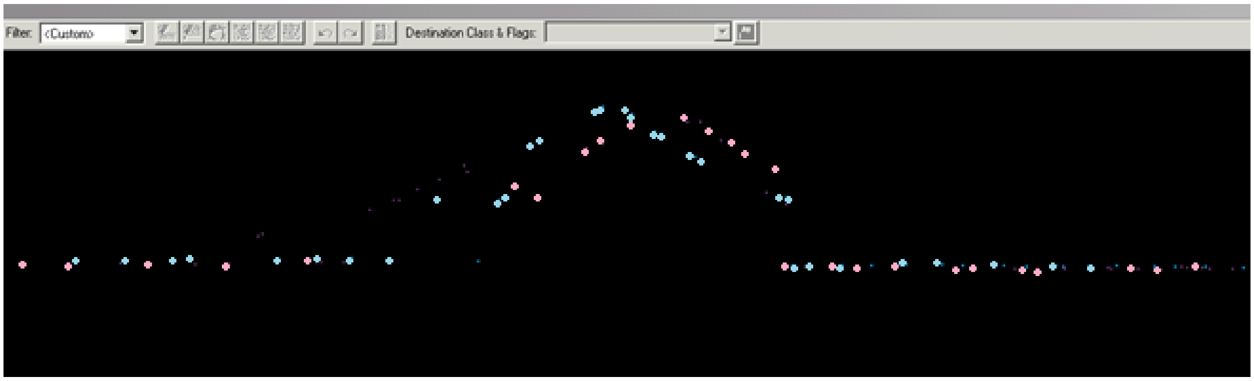

Figure A2 Horizontal Errors in the worked example data set

At the end of the process, we have summary estimates (mean, standard deviation and root mean square error, which is defined as square root of sum of squares of mean and standard deviation estimates) of error in all the data.

Systematic Error

The systematic errors in the data are quantified by the median of discrepancy angle. The discrepancy angle is calculated using the measurements made on flat regions of the overlapping data. A line is fit using the first two columns of Table 2. In this case, the parameters are: A=-0.018; B=-0.999 and ρ=-3367777.799. The distances of the points (again first two columns of Table A2) from this line are calculated as $\rho_D = \rho_{Distance\ from\ Center\ of\ Overlap} = |\mathrm{A} * \mathrm{X} + \mathrm{B} * \mathrm{Y} - \rho|$. The Mean of discrepancy angle is defined as $\mathrm{MDA} = \frac{\sum_{i=1}^{10} \mathrm{arctangent}\left(\frac{D_i}{\rho_{Di}}\right)}{10}$ where D_i are the values in the 'D' column of Table 2. In this case, the MDA works out to 0.253 degrees. Nominally, this value (for a well data set of high geometric quality) is expected to be close to zero.