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OBLIQUE PHOTOGRAPHS AND THE PHOTOALIDADE

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A NEW photoalidade for use in mapping from oblique aerial photographs was described in an article published in the *Military Engineer* of November-December, 1937. The instrument was exhibited at the Annual Meeting of the American Society of Photogrammetry, January 24 and 25, 1938, and was also the subject of a paper read at that meeting. The mathematical discussion of the principles upon which the photoalidade works did not seem to have a place either in the published article or in the paper. However, the operation of the instrument requires a working knowledge of the related mathematics of oblique photographs, which will be outlined as briefly as possible in the paragraphs to follow. An abstract of the description of the photoalidade is given first, for the convenience of those who have not seen either the instrument or the published article.

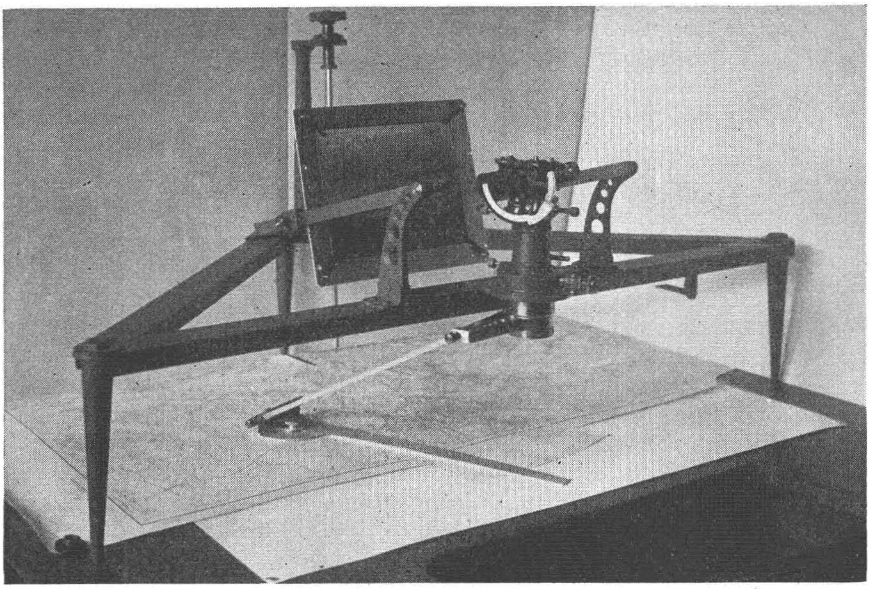
The well-known and much used plane-table, with a telescopic alidade, is a device for plotting lines of direction to sighted objects directly upon a map or work sheet, and for measuring vertical angles to those objects. The photoalidade serves the same purpose, the principal difference being that the plane-table is used on ground stations in the field, sighting to objects or features in the actual landscape, whereas the photoalidade is designed for use in the office to sight features shown in photographs of the landscape. Each instrument has a telescope with cross wires through which the selected features may be sighted accurately, with an arc and vernier for measuring vertical angles to them, and each has also a straightedge to guide the drawing of lines of direction corresponding to the pointings of its telescope.

The photoalidade may be adjusted to use oblique photographs taken with the camera axis at any angle of inclination from 10° above to 50° below the horizon. Contact prints may be used if the photographs were made with a camera having a lens with a focal length of more than 4 inches and less than 14 inches. Photographs that show the horizon near and about parallel to their upper edges are preferred, as it is possible to place them in the instrument immediately with approximate settings of the adjustments. Final settings for any photograph are obtained by a trial and correction method.

When the photograph is correctly adjusted in the instrument, observations are taken in the same way as with an ordinary plane-table. The illustration of the photoalidade shows how easy it is to imagine that the view displayed is a real scene, framed in a small open window, rather than a mere photograph. In fact, manipulation of the instrument will seem a surprisingly familiar operation to any one accustomed to the use of a plane-table.

The instrument is mounted for convenience on a triangular frame supported by three 9 inch legs. This brings the sighting telescope up to the level of the eye of the operator who sits at a table of normal height. The spread of the legs allows ample room on the table for maps or work sheets, which can be conveniently shifted into position because of the free space between table and instrument. The straightedge of the photoalidade is on a double-hinged arm, which can be folded so that the straightedge will be out of the way when not in use or lifted from time to time for inspection of the map beneath it. Thus it is possible to sketch freely on the map without disturbing its position in relation to the instrument. A centering microscope fitted with cross wires is provided, so that the station point on the map can be placed exactly in the vertical axis of the instrument. A clamp with slow-motion screw allows the telescope to be turned and set at any desired angle in relation to the straightedge, so that final orientation can be effected without rotating the map on the table. The simplicity of operation and the comparatively low cost of construction are attractive features of this instrument.

Only one photoalidade of this type has been made. The writer would like to express his very sincere appreciation for the work of construction done by



Geological Survey Photoalidade T-1

R. L. Atkinson, Chief, Division of Field Equipment, United States Geological Survey, in translating theoretical principles into this practical instrument. The remarkable near-focus telescope was designed and constructed by the Keuffel & Esser Co. of New York, to meet the requirements of this particular photoalidade. Helpful suggestions and constructive criticism by other engineers of the United States Geological Survey have aided considerably in producing the instrument and are hereby thankfully acknowledged.

Since the publication of the article in the *Military Engineer*, a few changes in the details of construction of the photoalidade have been proposed that may be of interest. The new photoalidade was first designed with a metal plate as a holder for the photographs. It is proposed to replace that plate with one made of ground glass which may be illuminated from the back, so that diapositives may be used as well as paper contact prints.

The field of view presented by a single 7×9 inch oblique photograph, taken with a lens of 12 inch focal length, subtends only about 40° of the horizon. Obviously the usefulness and efficiency of the instrument would be increased by employing photographs taken with wide angle lenses. With 70° or 80° of the horizon visible in the field of view, the resection to determine the position of the camera station could be made more accurately, and more of the foreground could be included without losing sight of the horizon. Colonel Bagley has suggested that photographs might be taken with the 3-lens (T-1) camera, holding it on its side so that the axes of the three chambers would all be approximately horizontal. The view presented by the combined photographs would then include about 140° of the horizon. It would not be necessary to transform the wing photographs to the plane of the center one. As an accessory to the instrument now in use, a holder can be made to hold the untransformed prints or diapositives in such a way as to reproduce angles between their planes equal to the angles between the corresponding focal planes in the camera. The holder would be designed also to bring the perspective centers of all three photographs

A View in Glacier Bay National Monument

together at a common point. This auxiliary holder with its combined view would be mounted as a unit upon the holding plate already provided by the photoalidade. Then the adjustment of the whole unit for tilt and swing would be the same as for a single photograph.

The following mathematical analysis is applicable generally to oblique photographs. It employs three principal reference systems. Positions of points on the photograph (see figure I) are defined by rectangular co-ordinates referred to axes, one of which is the intersection of the principal vertical plane with the plane of the photograph, the other being parallel to the horizon and intersecting the first at the center of the photograph.

Positions of points on the ground, and of the camera station (see figures II and III) are referred to a three-dimensional system of rectangular co-ordinates which may be formed by adding a vertical co-ordinate axis to any of the plane rectangular systems used in surveying. It is convenient to consider the plane containing the horizontal axes as being tangent to the earth at the point directly below the camera station, with zero elevation at that point. Curvature and refraction corrections must be applied in considering elevations at all places except at the point of tangency just mentioned, but horizontal directions and

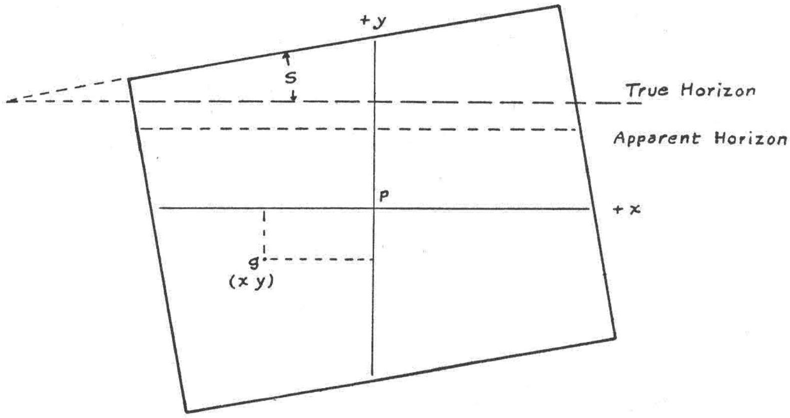


FIGURE I Reference System on the Photograph

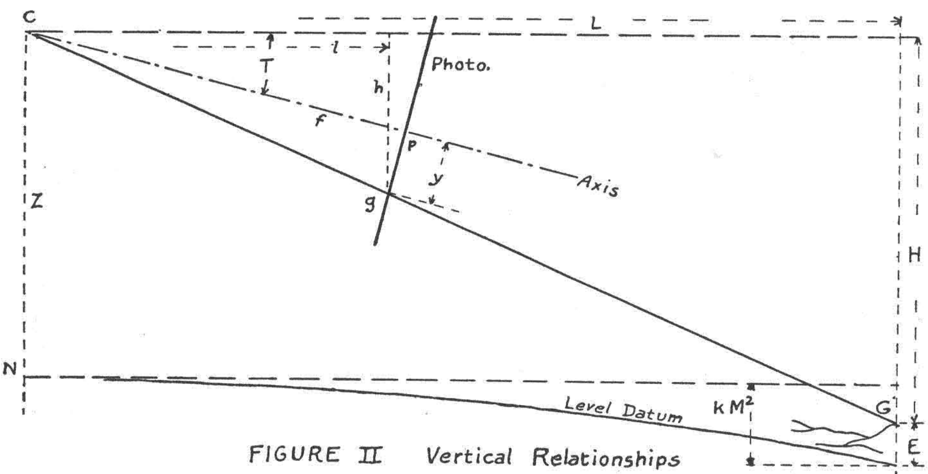


FIGURE II Vertical Relationships

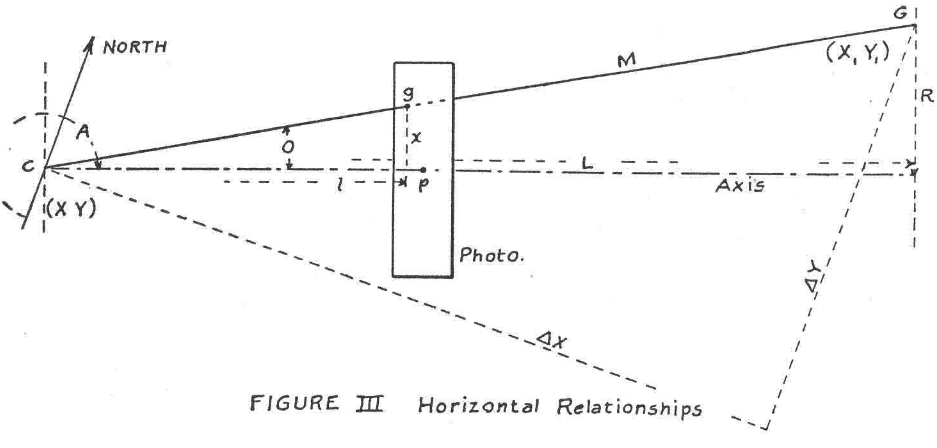
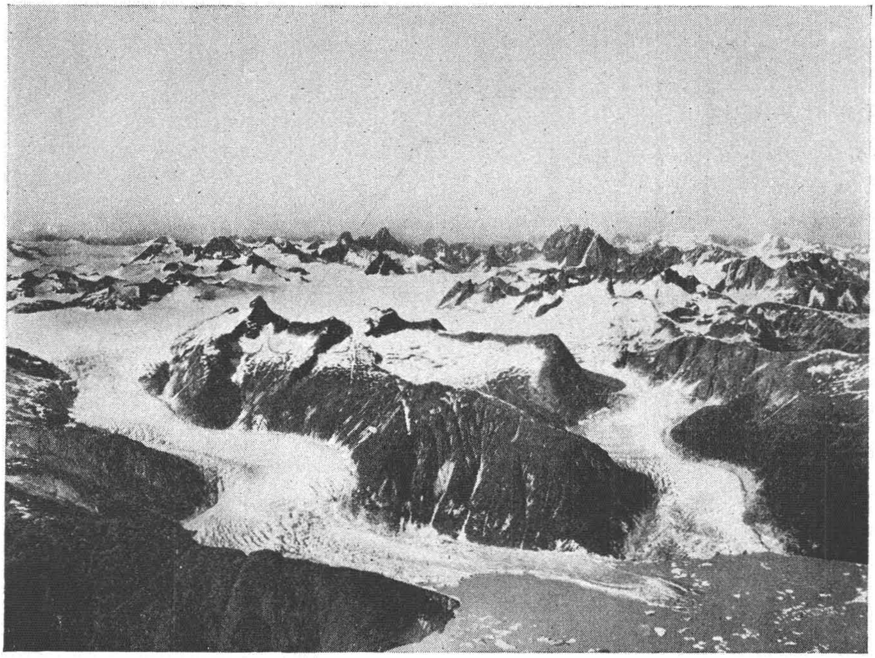


FIGURE III Horizontal Relationships

distances on the surface of the earth will be treated as if they were in the tangent plane.

Camera orientation is defined by three angles: inclination of camera axis, or tilt; direction or azimuth of the vertical plane containing the camera axis; and swing or rotation of the camera about its main axis appearing in the form of slope angle of the photograph edges with respect to the co-ordinate axes in the plane of the photograph.

The symbols used to represent the points, distances, and angles have been



Twin Glaciers from Taku River

indicated in the diagrams adjacent to the parts they represent, where that is possible, and are defined as follows:

Points

- C perspective center; camera station
- N nadir of camera station, at zero elevation
- p center of photograph, at intersection of principal axis
- G a point on ground, or G_1, G_2 - - - for specific points
- g point on photograph corresponding to G

Known quantities

- f effective focal length of camera lens
- x, y co-ordinates of g on the photograph
- X_1, Y_1 horizontal co-ordinates of G_1 or G
- E elevation of G above mean sea level
- k curvature and refraction constant

Camera orientation and position

- A azimuth of camera axis
- T tilt of camera axis from horizontal
- S swing, rotation of camera about its principal axis
- X, Y, Z co-ordinates of C

Dependent distances and angles

- O horizontal angle at C between g and p
- M horizontal distance N to G
- V vertical angle, from horizontal, of line C to G
- L longitudinal component of distance NG
- R transverse component of distance NG
- l longitudinal-horizontal component of distance Cg
- h vertical component of distance Cg
- H component of distance CG that is parallel to CN
- ΔX, ΔY differences in co-ordinates between G and N

The line CG actually is curved slightly because of refraction. In this discussion, however, it is assumed to be straight, and the correction for refraction is combined with that for curvature. Properly, there should be an angle between the directions of measurement of H, and of E and the curvature correction, but all three are considered here as being measured along the same straight line. Errors due to these approximations are negligible in practical problems.

The distances f, L, l, E, and Z are always positive; h, H, and the angles T and V are usually negative (down); R is positive toward the right as seen from C; positive directions of x and y are right and up respectively; X and Y are positive toward the east and north respectively. The inch, or other small unit, should be used in common for f, x, y, l and h. A larger unit, such as the mile, should be used in common for X_1, Y_1, E, X, Y, Z, L, R, and H. If the mile is used, $k=.574/5280$, approximately.

The following equations represent some of the fundamental relationships between the distances and angles just described:

$$\Delta X = X_1 - X \qquad (1)$$

$$\Delta Y = Y_1 - Y \qquad (2)$$

$$L = -\Delta X \sin A - \Delta Y \cos A \qquad (3)$$

$$R = -\Delta X \cos A + \Delta Y \sin A \qquad (4)$$

$$h = +f \sin T + y \cos T \qquad (5)$$

$$l = +f \cos T - y \sin T \qquad (6)$$

$$H = E - Z - [(\Delta X)^2 + (\Delta Y)^2]k = M \tan V \qquad (7)$$

$$\frac{l}{L} = \frac{h}{H} = \frac{x}{R} \qquad (8)$$

$$\tan O = \frac{x}{l} = \frac{R}{L} \qquad (9)$$

The effects upon x and y of small changes in S may be expressed thus:

$$dx = -y\,dS \qquad\qquad dy = x\,dS$$

Equations (3), (4), (5), and (6) are derived through the usual formula in analytical geometry for the transformation of co-ordinates with rotation of axes. All of these equations are general for oblique photographs, and they will be found useful in deriving special equations or formulas for many particular purposes.

Before an oblique photograph can be used for mapping purposes it is generally necessary first to determine the six elements defining the orientation of the camera and its position in space at the time the photograph was taken. The equations given provide for a simultaneous determination of all six elements, but it is not practicable to solve for them directly because the equations are not in simple linear form. The equations may be differentiated in order to solve simultaneously for corrections to tentative values assumed for the six elements. Even this method is cumbersome, however, and it need not be described in detail here. Fortunately, practical problems do not generally require a mathematical solution for all six elements at the same time.

If an oblique photograph shows the apparent horizon, a line may be drawn above and approximately parallel to it, tentatively representing the true horizon. Although that line is drawn only by rough estimation, the graphical methods used with the photoalidade then quickly determine workable values for X, Y, and A. The tentative horizon line, however, is not sufficiently close to the true horizon to be depended upon in measuring the final vertical angles. Suppose a photograph has been placed in the photoalidade by referring the instrumental settings to a tentative horizon line. Then preliminary vertical angles read to several control points are likely to yield widely different figures for Z, all of them far from the correct figure for the altitude of the camera. A method must be found to correct the tentative line and so reconcile the vertical angle observations.

Selecting the tentative horizon line is equivalent to selecting a plane, through C, that is only approximately horizontal at that point. The preliminary vertical angles are measured from this tentative plane. The value of Z computed with such a vertical angle to a control point gives the vertical coordinate, not for the camera station, but for a point in the tentative plane very nearly over the point to which the angle was read.

The slope of the tentative plane may be represented by a system of parallel lines at right angles to the direction of slope and equally spaced so as to represent equal steps of increasing distance between the tentative plane and the plane that is horizontal at N. The meaning of these lines may be visualized by regarding them as contour lines of elevation of the tentative plane.

On a diagram (see figure IV) prepared upon the same scale as the map used in making the horizontal resection for camera position, plot the positions of the control points G_1, G_2, G_3, - - -. Adjacent to these points write the values of Z_1, Z_2, Z_3, - - - respectively, as determined by reading the preliminary vertical angles from the tentative plane to the control points. Observations upon three or more properly distributed control points determine the steepness and direction of slope of the tentative plane. Accordingly, the figures for Z_1, Z_2, Z_3, - - -, indicated on the diagram in their respective places, control the spacing and direction for the parallel lines. The lines may be drawn at regular intervals representing 10 feet, 50 feet, 100 feet, or other even elevation difference as may be convenient.

If only three control points are used, there is only one way to draw the parallel lines so that the values Z_1, Z_2, Z_3 will be correctly interpolated in their respective places, provided the points are not in line. If more than three control points are used, equally spaced parallel lines usually cannot be drawn in a way to satisfy exactly the observations made upon them all because of the natural errors in the observations. Judgment is then required to select a system of lines to fit as nearly as possible all of the information available.

The system of parallel lines extended to the position of C will indicate at once the elevation, Z, of that point, correct except for errors of observation, even though the photograph is not yet adjusted to the true horizon.

Up to this point, the photograph has been considered as adjusted in the photoalidade only by reference to the tentative plane. To readjust it so that correct vertical angles may be read from the true horizontal plane at C, corrections to T and S are necessary because they were measured at first from the sloping tentative plane. Let ΔT and ΔS denote these corrections, which are to be applied in readjusting the photograph in the instrument.

The component of slope of the tentative plane in the direction of L is equal

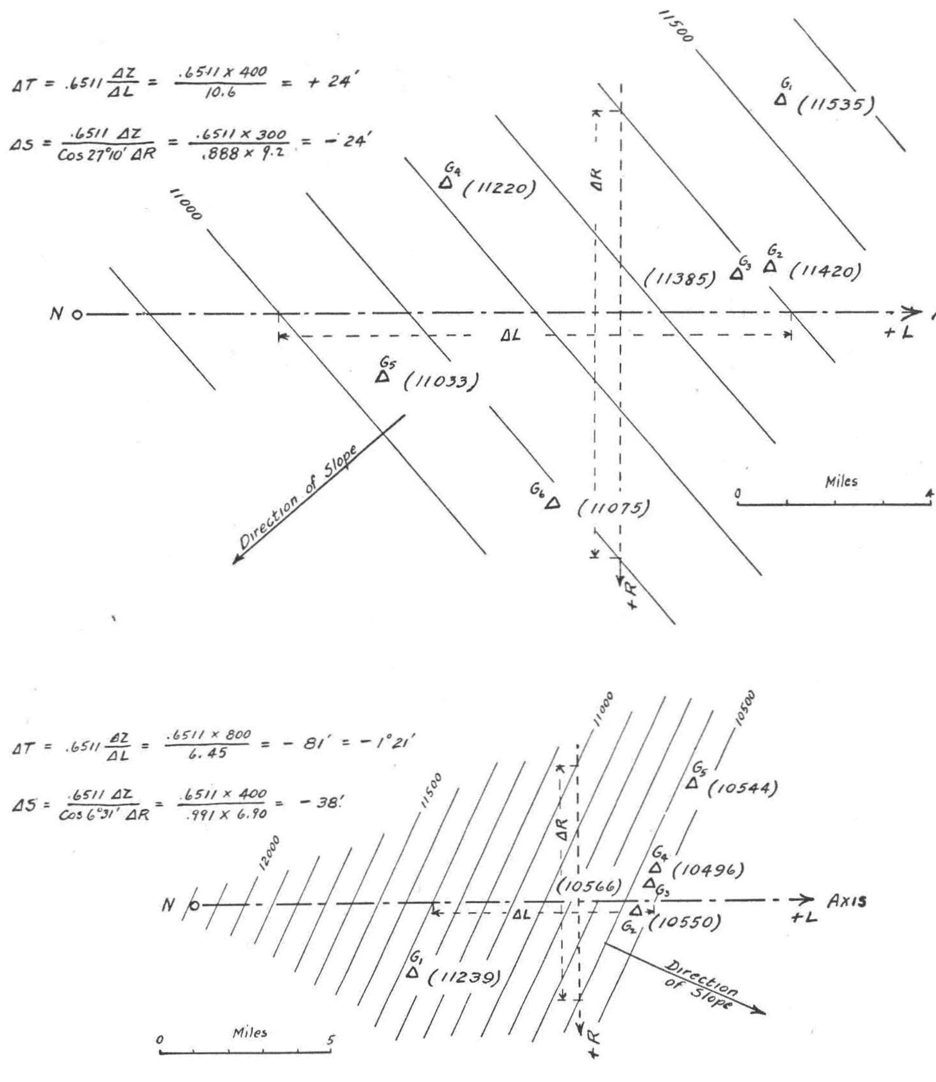


FIGURE IV Correction Diagrams

to ΔT, directly. The component in the direction of R, however, is equal to ΔS only if the camera axis is horizontal ($T=0$). This component of slope is measured in a transverse vertical plane, while ΔS is in the plane of the photograph. These two small angles have a common side in the intersection of the two planes, and the angle between the planes is T. The angle $\Delta S \cos T$ is the projection of ΔS upon the vertical plane, and is equal to the component of slope of the tentative plane in the direction of R. Let $\Delta Z/\Delta L$ and $\Delta Z/\Delta R$ represent the steepness of the components of the slope in the direction of L and R, respectively. The value of T resulting from the use of the tentative horizon line may be used for cos T. In the following formulas the numerical constant is introduced in order to express ΔZ in feet, ΔL and ΔR in miles, and ΔT and ΔS in minutes of arc.

$$\Delta T = .6511 \frac{\Delta Z}{\Delta L} \qquad \Delta S = \frac{.6511}{\text{Cos } T} \cdot \frac{\Delta Z}{\Delta R}$$

The signs of these corrections may be determined by simple rules, without having to keep account of the signs of the factors. If the tentative plane slopes downward toward the positive direction of L, ΔT is negative, so that the holder and frame carrying the photograph must be depressed. If the tentative plane slopes downward toward the positive direction of R, ΔS is negative and the photograph should be rotated clockwise about its center.

After thus readjusting the photograph to the true horizon, it may be desirable to repeat the resection for horizontal position. The horizontal angles used for the first resection may be changed by an appreciable amount, particularly if ΔT and ΔS are large.

At the present time the new photoalidade has been used only for reconnaissance mapping, which is but one of the several purposes it is designed to serve. It will possibly become a valuable auxiliary also in other established mapping methods, on larger scales, because of its ability to interpolate additional control points in areas not adequately covered by ground surveys.

The following record of reconnaissance mapping with the photoalidade has been compiled from notes furnished by Mr. T. W. Ranta in the Alaskan Branch of the Geological Survey. During the early months of 1928 he was engaged in extending the reconnaissance map of the Tonsina District, Alaska, already partly done by plane-table method, to include an area of about 355 square miles immediately north and northeast of Valdez. The field scale of the map is 1:180,000. The map of this area was made from 31 of a series of oblique photographs available through the co-operation of Mr. Bradford Washburn of the Institute of Geographical Exploration, Harvard University. The photographs are 7×9 inches in size and were taken with a lens having a focal length of 12 inches. The photoalidade was not completed until just before this work was started; therefore these notes constitute the record of its first accomplishment.

The area mapped is very inaccessible for ground methods and was represented on the existing map as an approximately circular space, entirely blank. The only available control points from which the new mapping could be developed were the more prominent mountain peaks which had been located and sketched by plane-table methods when the surrounding parts of the map were made.

Mr. Ranta worked alone and reported time amounting to 62 working days to complete the area. Of this time, at least 10 days were used in learning to identify and select points suitable for control, in testing the precision of the resections for camera position and altitude, and in forming a routine in using the instrument.

The experience gained shows that photographs should be taken from carefully planned positions. Narrow intersection angles should be avoided in the areas where points are to be located by the intersection method. The photographs should show in the foreground parts of the area to be mapped, with control points both near by and in the distance if possible. There will be a distinct advantage if photographs are taken in stereoscopic pairs; the area to be mapped may then be examined stereoscopically while critical points of the topography are being selected for location by intersection. The points should be marked and numbered on one photograph of the pair for easy identification when the photoalidade is used. After these points have been located on the map and their elevations have been determined, the drawing of contours is much surer if the features represented are kept in view through a stereoscope. It is surprising how stereoscopic observation of a pair of oblique photographs separates successive ridges of the landscape and makes them stand out individually.

The precision attained in position and elevation appears consistent with what would be expected with a plane-table and telescopic alidade under like conditions of map scale and with the same kind of control. The time necessary to obtain the final instrumental adjustments for a photograph is about half a day for the first time that the photograph is used, but if a record of the settings is made, it can be replaced in the instrument in five or ten minutes.

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