PARALLAX DIAGRAMS ELIMINATE COMPUTATIONS IN FORM LINING AERIAL PHOTOGRAPHS

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Parallax Diagrams for Contouring Aerial Pictures

THE extensive exploration and construction work which has been undertaken in the last few years in the Mexican Republic has created a demand for form line maps for which aerial photography has been found to be in many cases the best solution. With the view of obtaining form line maps of the desired accuracy at a minimum cost, our company has been using very successfully Fairchild Stereocomparagraphs. When we first began to use these instruments we were confronted with the necessity of obtaining the necessary personnel for their operation as well as planning the proper procedure for their most efficient use with the idea in mind to employ personnel with good stereoscopic vision and a good knowledge of aerial photographs and mosaic work, but who were not trained photogrammetric engineers. We therefore developed some parallax diagrams which enable the user to obtain the different parallax values without any calculation whatsoever. In the last issue of PHOTOGRAMMETRIC ENGINEERING corresponding to number 1, volume IX, Mr. Revere G. Sanders of the Fairchild Aviation Corporation published a very interesting article called *Elementary Elevation Determination from Aerial Photographs.*

This article gave us the idea to submit for publication our own findings in the matter, which we trust may be of interest to the many readers of PHOTOGRAMMETRIC ENGINEERING and to members of *The American Society of Photogrammetry.*

Our diagrams have been extensively used with Fairchild Stereocomparagraphs and permit the obtaining of the different values of the parallaxes for each form line by simply scaling off the items K and Q, in which the fundamental formula has been sub-divided. They have been calculated for heights and bases between the stereoscopic pictures in meters, but can be transformed into elevations and bases in feet by simply applying their corresponding values.

The fundamental formula for the calculation of parallaxes is as follows:

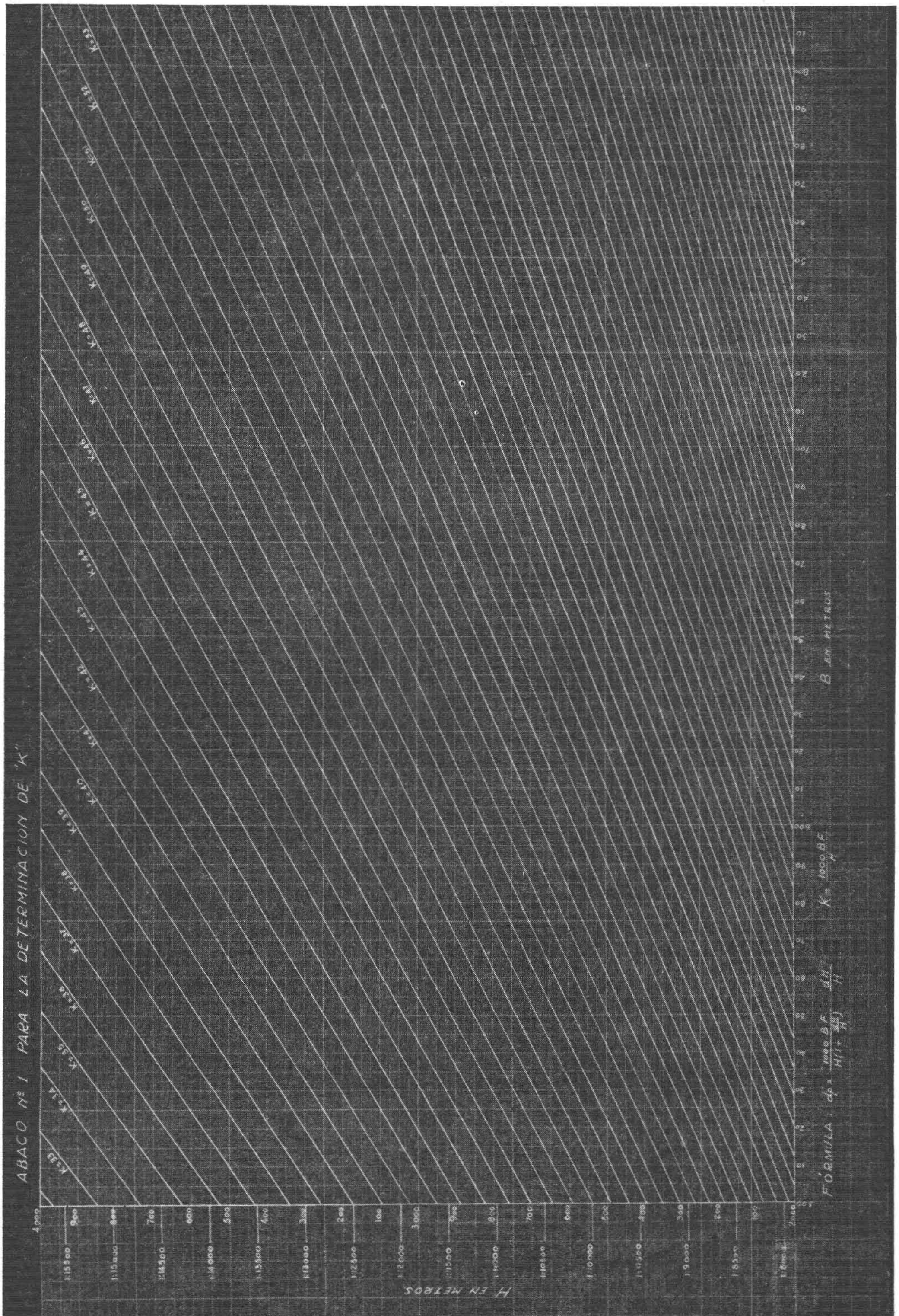
$$p = \frac{B \cdot F}{H}$$

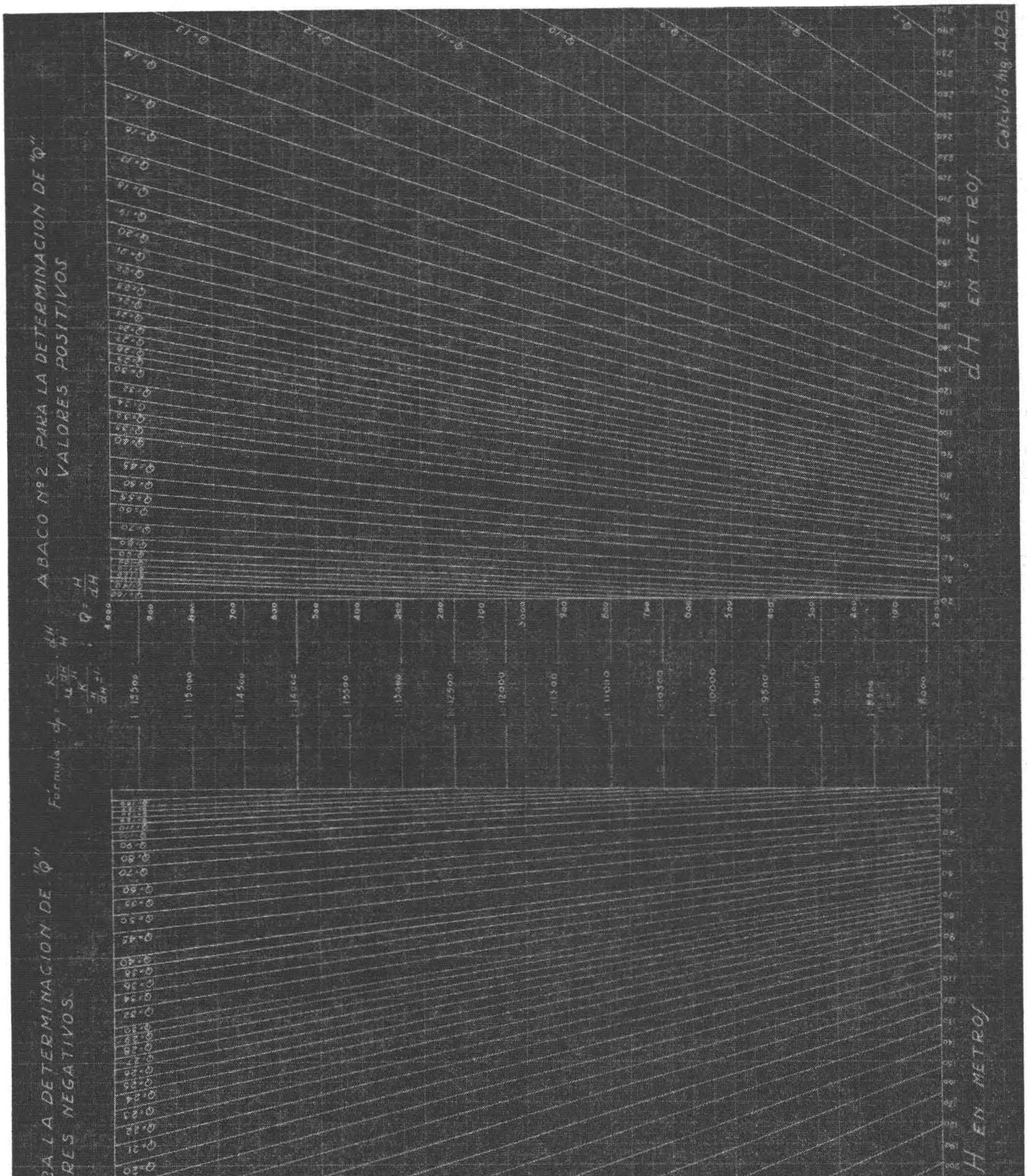
in which p is the parallax, B the base between stereoscopic pair of pictures, F focal distance and H elevation of camera above the ground at the principal point. Since we are now concerned with differential parallaxes, that is, the rate at which the parallaxes change for the corresponding difference of elevation, we will obtain this difference by the following formula:

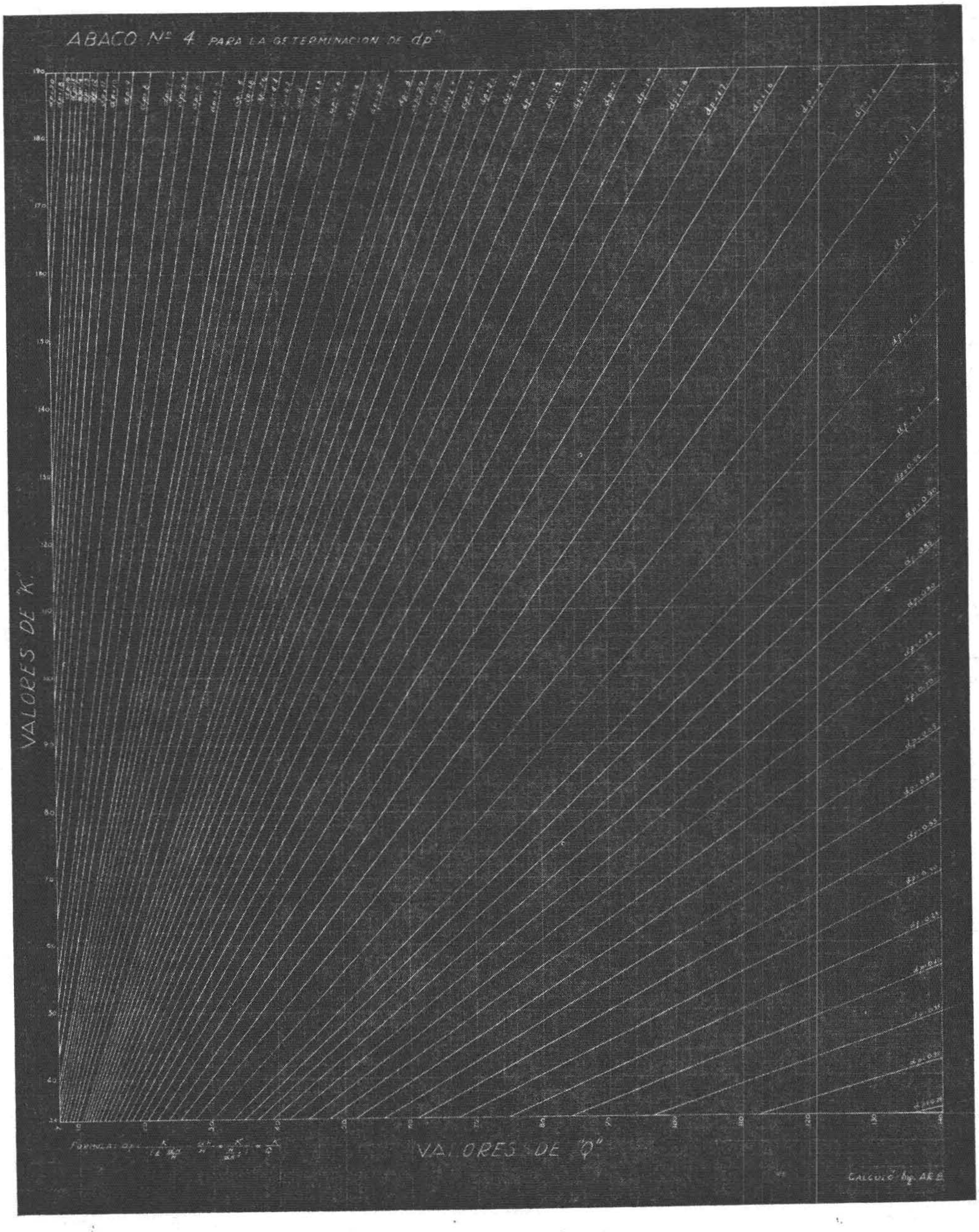
$$p - p' = \frac{BF}{H} - \frac{BF}{H'} = BF\left(\frac{1}{H} - \frac{1}{H'}\right)$$

or,

$$dp = BF\left(\frac{H' - H}{H \cdot H'}\right)$$


ABACO Nº 1 PARA LA DETERMINACION DE "K"
B EN METROS
H EN METROS


ABACO Nº 2 PARA LA DETERMINACION DE "Q"
VALORES POSITIVOS
ABACO Nº 3 PARA LA DETERMINACION DE "Q"
VALORES NEGATIVOS
dH EN METROS
dH EN METROS


ABACO Nº 4 PARA LA DETERMINACION DE dp"
VALORES DE "K"
VALORES DE "Q"
CALCULO by A.R.B.

in which $p-p'=dp$ or differential parallaxes. H and H' are the two elevations of the camera above the ground corresponding to the two planes for which the differential parallax is sought. If we call the difference between H and H' dH, that is, the differential elevation, then $H'=H\pm dH$, and substituting this value of H, in the above formula, we obtain:

$$dp = BF\frac{[(H \pm dH) - H]}{H(H \pm dH)} \quad \text{or} \quad dp = \frac{BF\cdot dH}{H^2 \pm dH\cdot H} = \frac{BF}{H\left(1 \pm \dfrac{dH}{H}\right)}\cdot\frac{dH}{H}.$$

The formula has actually a negative sign, that is, parallaxes decrease with altitude. The one thing necessary is to observe this fact when figuring the corresponding parallax values for the elevations below and above the basic reference plane.

So as to obtain the value of dp in millimeters, while the value of B, F and H, as well as dH is in meters, we multiply the formula by 1,000 obtaining the value of

$$dp = \frac{1{,}000\ BF}{H\left(1 \pm \dfrac{dH}{H}\right)}\cdot\frac{dH}{H} \qquad \text{Eq. (1)}$$

in which dp = difference of parallax for each interval of form lines in millimeters, B = base between the stereoscopic pictures in meters. F = focal distance of the aerial camera employed in meters. H = height of the airplane in meters above the ground at the principal point. The above value is obtained by measuring the distance between the two principal points of the two corresponding flight pictures on the finished mosaic, or from the rectified photographs, and H is then calculated from the scale of the pictures after the base is computed. In case the mosaic has not been made, if any accurate, sensitive altimeter is used for the flight, and the ground elevation is approximately known, the values of B and H can be obtained directly from the scale of photographs.

For the purpose of these diagrams the equation (1) was divided into two separate values named K and Q, in which

$$K = \frac{1{,}000\ BF}{H} \quad \text{and} \quad Q = \frac{H}{dH}.$$

The formula then derives to:

$$dp = \frac{K}{1 \pm \dfrac{dH}{H}}\cdot\frac{dH}{H} = \frac{K}{\dfrac{H}{dH} \pm 1} \quad \text{and for} \quad Q = \frac{H}{dH},\ dp = \frac{K}{Q \pm 1}.$$

The corresponding values of ± 1 correspond to positive and negative values of Q in the diagrams according to whether the basic plane of elevation lies above or below the corresponding form lines to be drawn. Positive values of Q are used when the form lines to be drawn are higher than the basic elevation plane; negative values of Q are used when the form lines to be drawn are lower than the basic elevation plane.

The values of K and Q are scaled off for the corresponding values of B and H.

Example: Photographs No. 96 and 97. Fairchild K-3 Aerial Camera, focal distance 10″ = 0.254 meters.
Scale 1:11,597
H = Height of the airplane 2,946 meters.
B = Stereoscopic base 922 meters.
K = Value in first diagram 79.5.
Q = Values in second diagram.
dp = Difference in parallax, value of third diagram in millimeters.

TABULATION OF VALUES

FORM LINE ELEVATIONS	Q	dp	READING IN STEREO-COMPARAGRAPH
2900	27.6	2.79	9.43
2880	35	2.20	10.02
2860	47	1.65	10.57
2840	72	1.09	11.13
2820	146	0.54	11.68
2800	0	0.00	12.22 Basic plane
2780	148	0.53	12.75
2760	74	1.06	13.28
2740	49	1.59	13.81
2720	37	2.09	14.21
2700	29.6	2.60	14.82