# OBLIQUE PHOTOGRAPHS IN MAPPING 

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OBLIQUE photographs may be employed in mapping with no aids other than common drafting equipment, if the principal line and angle of tilt are available If the photographs are of large field and there is sufficient control, quite accurate results can be obtained. Enlargements from original negatives on nonshrink photographic paper are desirable. The principal line of the photograph is employed as the $X$-axis, and a line at right angles to the principal line drawn through the principal point as the $Y$-axis.


Figure 1
Figure 1 is used to explain this method. Horizontal angles may be determined by employing any conveniently placed horizontal plane. In this instance either $W_{1} C_{1} M_{3}, W C M_{1}$, or $S H K$, each being a right triangle in a horizontal plane, may be employed. The angle $A$ in each case is the dihedral angle between the two vertical planes $S H V$ and $S K V$. Let $f$ denote the focal length of the pho-
tograph, and $S V$ be a plumb line swung from the air station $S$. It cuts the map plane at $W_{1}$. C, being the principal point of the photograph, $S C V$ and $S C H$ are right triangles. The coördinates of any image point $M$ are $x$ and $y$.

The simplest solution is obtained through the triangle $W C M_{1}$. In it

$$
C M_{1}=\frac{x \cdot f \tan i}{f \tan i-y} \text { (the triangles } V M_{2} M \text { and } V C M_{1} \text { being similar) }
$$

and $W C=f \sin i$, which substituted in the equation

$$
\begin{equation*}
\tan A=\frac{C M_{1}}{W C} \tag{1}
\end{equation*}
$$

gives

$$
\begin{equation*}
\tan A=\frac{x}{f \sin i-y \cos i} . \tag{2}
\end{equation*}
$$

Equation (1) may be better to use in some instances than equation (2), for it does not require the use of a protractor. It is applied by laying off a distance $W C$ along a straight line, erecting a perpendicular at $C$, laying off the computed value of $C M_{1}$, and drawing a line from $W$ to pass through $M_{1}$. The value of $C M_{1}$ may be obtained quickly and accurately enough for graphical purposes by slide rule. The value of $W C$ remains constant for each photograph, as do the values of $f \sin i$, and $\cos i$ in equation (2).

The map position of an air station (the point $W_{1}$ in Figure 1) is found by the tracing paper method. At least three (it is better to have four or more) control points properly distributed must appear in the photograph. The horizontal angle, relative to the initial line $W C$, of each image point is determined, using equation (1) or (2), and from $W$ as pivot point a line toward each control point is drawn on a sheet of tracing paper, which is used to make a three-point location. This gives the point $W_{1}$ of Figure 1, which is the map location directly beneath the air station.

The depression angle $\alpha$ (also its complement $B$ in Fig. 1) depends on the angle of tilt $i$, the focal length $f$, and the coordinates $x$ and $y$. The triangle $V S$ can be solved by first computing $S V, V M$, and the angle $D$, then proceeding with the fundamental equation of trigonometry when two sides and the included angle of a triangle are known, which in this case is

$$
\tan 1 / 2(B-F)=\frac{V M-S V}{V M+S V} \cot \frac{D}{2}
$$

This gives the value of $B-F$ and since $B+F=180^{\circ}-D$, the value of $B$ can be found. The depression angle is $\alpha=90-B$.

This procedure involves as preliminary steps the determination of the angles $E, A$, and $D$. It is laborious and therefore has little practical value.

A much less laborious method is to use the relationships existing between $x, M_{4} M_{5}, W_{1} M_{4}$, and $W_{1} M_{5}$; for $W_{1} M_{5}$ readily gives the value of the angle $B$ from the right triangle $S W_{1} M_{5}$. The equations involved are

$$
\begin{aligned}
\tan G & =\frac{y}{f}, & S M_{2} & =\frac{f}{\cos G},
\end{aligned} \quad S M_{4}=\frac{S W_{1}}{\cos (i-G)}, ~ 子 ~ M_{4} M_{5}=\frac{x \cdot S M_{4}}{S M_{2}}, \quad \text { and } \quad \tan B=\frac{W_{1} M_{5}}{S W_{1}} .
$$

Substitution of values gives

$$
\begin{equation*}
\cot \alpha=\tan B=\frac{x \cos G}{f \sin A \cos (i-G)} \tag{3}
\end{equation*}
$$

If equation (2) has been employed, the angle $A$ will have been previously determined for any image point used. In this case it is only necessary to find the value of $G$ to solve equation (3).

When the image point is above the $Y$-axis (line $C M_{1}$ in Fig. 1) the sign in the denominator of equations (2) and (3) changes to plus. It has been assumed in the foregoing that the true horizon is above the $Y$-axis, that is, the photograph is tilted downward as shown in the figure. If, however, the photograph is tilted upward, the vertical point $V$ will be above the air station; in which case the rule of signs for these equations is: When the image point is above the $Y$-axis the sign in the denominators is minus; plus, when the image is below that axis.

The elevation of an air station must of course be accurately determined before the photograph taken at that station can be used with satisfactory results to determine the elevations of located objects. Ordinarily the altimeter reading at the instant of exposure will be available from the flight record, but this altitude value seldom will be accurate enough for reliable mapping. Checks of the height of the air station above sea level, or the datum plane used, are obtained by computations of elevation differences for control points. In fact, except for use in determining the angle of tilt, it is unnecessary to have the altimeter reading if two or more control points of known elevation appear in a photograph, one to the right and one to the left of the principal line. The depression angle to each control station is determined, then the elevation difference in each instance is obtained from vertical angle tables, applying the correction for curvature of the earth and refraction of light in the atmosphere.

If the computations from two control points to determine the elevation of an air station agree within a few feet, it is safe to assume that the mean of the two is a satisfactory determination. Large discrepancies may indicate a faulty determination of the angle of tilt. If several control points are included in a photograph, three to five of the most favorably located and most distinct in image should be employed to determine the elevation of the air station. If the distances involved are not more than a few miles, the elevation determinations should agree within a few feet; if distances as great as 5 to 15 miles are employed, the differences between the individual determinations are liable to be comparatively large. It is not unusual in plane table mapping with the telescopic alidade, where map distances are large, to have differences between determinations run as high as 20 to 30 feet, but if a mean value is taken from several control points it may be quite accurate enough for small-scale mapping.

Obliques of flat ground, or ones including bodies of water, may be employed to plot details, lying in the horizontal planes concerned, by direct projection to a map plane. For example, in Fig. 1 the object, whose image is at $M$, is located at $M_{5}$ in the horizontal plane $W_{1} C_{1} M_{3}$, which can be taken as the plane of a map at the scale $S W_{1}$ divided by the height of the air station $S$ above the datum plane of the map. To obtain its location graphically, construct on a sheet of paper, which is to represent the principal plane of the photograph, the right triangle $S W_{1} C_{1}$. Lay off from $S$ the distance $f$ to obtain the point $C$ and erect at $C$ a perpendicular to the line $S C$. From $C$ lay off a distance $y$ to obtain the position of $M_{2}$. Connect $S$ with $M_{2}$ and prolong the line to meet $W_{1} C_{1}$ at $M_{4}$. On another sheet of paper (representing the map) draw a straight line and measure on it
the distance $W_{1} M_{4}$. At $W_{1}$ lay off the angle $A$ and draw the line $W_{1} M_{5}$. At $M_{4}$ erect a perpendicular to $M_{1} M_{4}$ to meet $W_{1} M_{5}$ at the point $M_{5}$, which is the location of $M$ desired. Any other object can be similarly located. If all the objects so located lie in the same horizontal plane, all will be correctly plotted with respect to one another, but any object higher or lower than this horizontal plane will not be properly located with respect to that plane.

The process of locating objects of a given horizontal plane may be carried out partly by computation and partly by plotting as follows: The angle $G$ can be computed from $\tan G=y / f$. Then

$$
W_{1} M_{4}=S W_{1} \tan (i-G), \quad \text { and } \quad W_{1} M_{5}=\frac{W_{1} M_{4}}{\cos A}
$$

The distance $W_{1} M_{5}$ may then be laid off on the map in a direction from the line $W_{1} C_{1}$ given by the horizontal angle $A$.

## Examples of Computations

Given $f=150 \mathrm{~mm} ., i=50^{\circ}, x=120 \mathrm{~mm}$., and $y=75 \mathrm{~mm}$. the point $M$ being below the $Y$-axis.
Using equation (2)

$$
\begin{aligned}
\tan A & =\frac{120}{150 \sin 50^{\circ}-75 \cos 50^{\circ}}=\frac{120}{66.758} \\
A & =60^{\circ} 54^{\prime} .72
\end{aligned}
$$

If the point $M$ were above the $Y$-axis

$$
\tan A=\frac{120}{150 \sin 50^{\circ}+75 \cos 50^{\circ}}=\frac{120}{163.176}
$$

and

$$
A=36^{\circ} 19^{\prime} .85
$$

The vertical angle in the first case would be obtained as follows, using equation (3):
whence

$$
\begin{aligned}
& \tan G=\frac{75}{150} \text { and } G=26^{\circ} 34^{\prime} \text { nearly } \\
& \cos \alpha=\frac{120 \cos 26^{\circ} 34^{\prime}}{150 \sin 60^{\circ} 54^{\prime} .72 \cos 23^{\circ} 26^{\prime}}
\end{aligned}
$$

$$
\alpha=48^{\circ} 15^{\prime} .25
$$

If the located position of the object $M$ were 4.326 miles from the located position of the station (point $W_{1}$ )

$$
\begin{aligned}
\text { the Elev. Diff. } & =-4.326 \times 5280 \tan 48^{\circ} 15^{\prime} .25+.574 \times 4.326^{2} \\
& =-25595.2+10.7=-25584.5 \text { feet. }
\end{aligned}
$$

Special vertical angle tables for obtaining differences of altitude or a calculating machine may be used to save time.

Where the point $M$ is above the $Y$-axis, the vertical angle would be obtained from
and

$$
\begin{aligned}
\cos \alpha & =\frac{120 \cos 26^{\circ} 34^{\prime}}{150 \sin 36^{\circ} 19^{\prime} .85 \cos 76^{\circ} 34^{\prime}} \\
\alpha & =10^{\circ} 53^{\prime} .3
\end{aligned}
$$

In this case the elevation difference for a distance of 4.326 miles is 4383 feet.
Horizontal angles obtained from image points near the horizon of an oblique photograph are not materially affected by an error in the angle of tilt, but those obtained from image points near the lower corners of the photograph may be in error more than the error in the angle of tilt. This can be appreciated when it is realized that the lower part of the field covered may subtend a horizontal angle greater than $180^{\circ}$ if the plumb line falls within that field. However, the distance from the pivot point to the plotted position of an object, whose image lies within that part of the field, will not usually be great, so an error in a direction line may be comparatively large in minutes without seriously affecting the results. Tests of obliques, enlarged to a focal length of about 11 inches from negatives having a focal length of 6 inches, and the angle of tilt having a probable error as great as 5 minutes, gave elevations within 10 feet of true for distances ranging up to 4 miles. Highly accurate horizontal and vertical control was available in abundance.


