CANADIAN TRIMETROGON PHOTOGRAPHY*

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IT CANNOT be doubted that proper maps are essential to the orderly exploration and development of the country's natural resources, and the trimetrogon system, from the point of view of economy and speed, has proved eminently suited for the production of these. Therefore the Royal Canadian Air Force in association with the Department of Mines and Resources, benefiting from the experience of the United States authorites, have taken up the torch of "Trimet" with the very laudable idea of applying that type of reconnaissance photography to the vast regions of Canada yet unmapped.

There is a difference, however, that must be recognized: that while the U. S. Army Air Forces were primarily interested in the production of small scale war maps, the reconnaissance mapping that would apply to Canadian needs would definitely require to conform to the standard of accuracy demanded by proper peacetime mapping practice.

The Topographical Survey of Canada Bureau of Geology and Topography, charged with the responsibility for photogrammetric mapping for federal authorities, is currently organizing a special Trimetrogon Photogrammetric Unit for the purpose of the rapid production of accurate reconnaissance planimetric and topographic maps at a publication scale not larger than 1:250,000; reconnaissance maps that will serve basic map requirements in the exploration and development of the country's great natural resources.

Methods of compilation are not yet crystallized, for obviously in a system that has not long since been introduced, utilization of new photogrammetric instruments and dictates of experience will exert an influence upon procedure so that many changes may become necessary from time to time to conform to the criterion of accuracy and speed. Several types of photogrammetric instruments, including stereoblique plotters, are in the course of construction at the National Research Laboratories for use by the Trimetrogon Photogrammetric Unit, and it is expected that these will become available shortly.

Trimetrogon photographs, when taken under weather conditions such that the horizon image is distinctly visible, contain data from which, for each set, the nadir and azimuths of the principal planes of the photographs may be computed and, since the precision of the radial triangulation framework depends upon the accuracy with which these dual requirements can be deduced, tilt analysis, accordingly, is a most important phase of the photogrammetric process.

Mr. R. B. McKay, Chief, Aerial Survey Division of the Topographical Survey of Canada, Bureau of Geology and Topography, has developed an effective method of tilt analysis and since this differs materially from that used and described[1] by the Alaskan Branch, an outline is therefore presented.

TRIMETROGON TILT ANALYSIS

Since the three cameras are maintained rigid inter-axially in their assembly mount, and exposed simultaneously, it can be conceived that their optical axes will be contained within fixed planes, of which the common trace is the optical axis of the center or vertical camera. These two planes may be called axial planes and simplification results when they are parallel or co-planar.

In such axial plane a horizontal line drawn through the air station or perspective center towards the left camera is taken both as a reference line for

* Reprinted from The Canadian Surveyor, January, 1945.

[1] Manual of Pnotogrammetry by American Society of Photogrammetry.

plotting and as a primary axis for one component of tilt—the X component of tilt of the central camera. The other component of tilt—the Y component, is expressed as a rotation about a secondary axis normal to the primary and passing through the air station, but inclined to the horizontal by the amount of the X component.

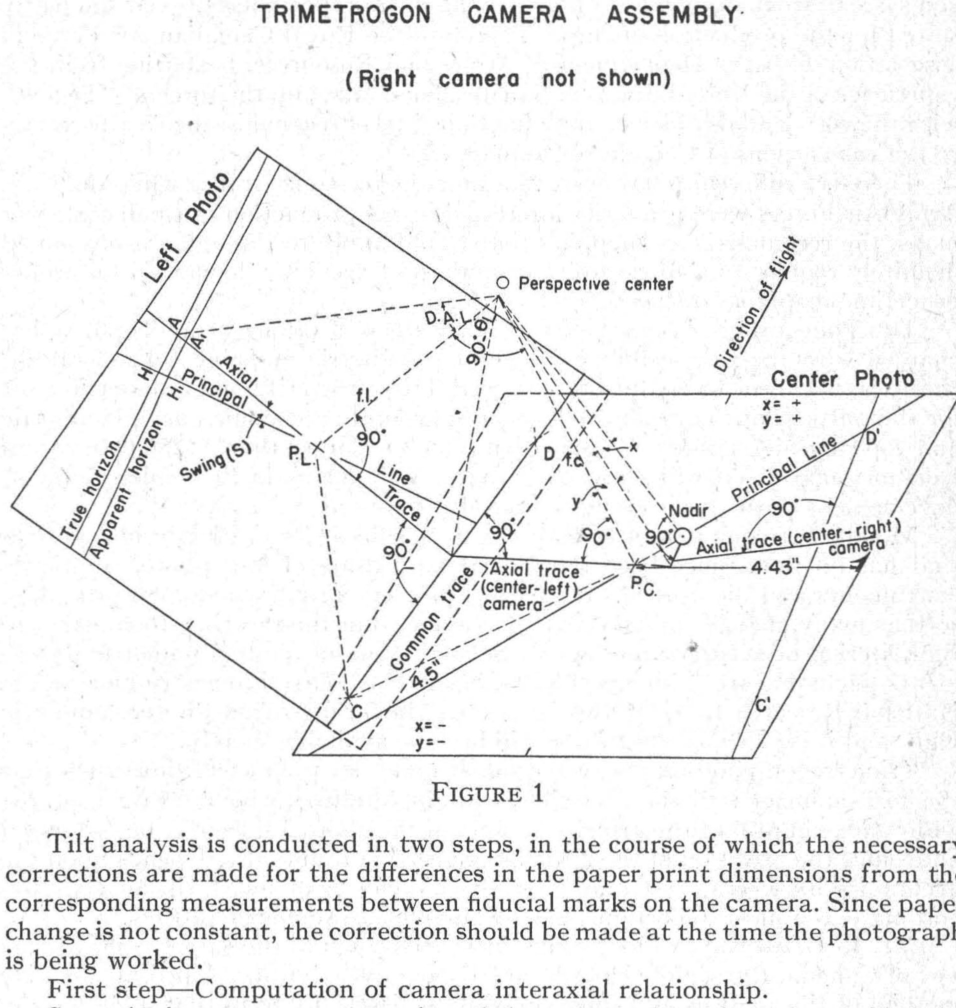

FIGURE 1

Tilt analysis is conducted in two steps, in the course of which the necessary corrections are made for the differences in the paper print dimensions from the corresponding measurements between fiducial marks on the camera. Since paper change is not constant, the correction should be made at the time the photograph is being worked.

First step—Computation of camera interaxial relationship.

Several trimetrogon sets are selected throughout the strip and, provided the values are found to be consistent, thus indicative of mount rigidity, the mean values of these are computed for appliaction to all photographs in the strip. The interaxial relationships that are computed for each trimetrogon set are:

1. Axial plane traces on vertical photograph.
2. Axial plane trace on each oblique photograph.
3. Interlocking angle, or angle between the optical axes of the vertical and oblique cameras.

All these relationships depend upon the position of the common trace, or line parallel thereto, of the picture plane of the vertical and each oblique photograph, which requires therefore to be drawn with great care.

Following are alternate methods of drawing these:

(a) The common trace.

On the vertical photograph a circle of 4.5″ radius is centered at the principal point and arcs drawn on the sides contiguous to the oblique photograph, as at C, D, C', D', Fig. 1.

On each oblique photograph a circle of radius corresponding to the focal length of the particular oblique relative to that of the vertical, and taken from tables, is centered at the principal point and an arc drawn as at C, D, Fig. 4. Arc intersections are determined by picking points of detail common to each, and the common traces are those lines joining the arc intersections.

A test of the accuracy with which these traces have been drawn is afforded by constancy of scale and by intersection of conjugate image points.

(b) The parallel substitute.

Since the determinations are all angular, time and labor may be conserved by locating lines parallel to the common traces thus:

An image point is identified upon the common overlap between vertical and each oblique; then, with the principal points as centres, arcs are drawn through each of the image points towards the opposite corners of the photographs, where the arc intersections are determined as before. The image points are joined by straight lines forming parallels to the common trace, and accuracy of position can be tested by observing if they pass through the same conjugate image points.

1. *Axial plane traces upon the vertical photograph:* Normals are drawn from the principal point to the common traces, or lines parallel thereto, and produced to intersect the marginal edges near the fiducial marks. These normals are the axial plane traces and the deflection angle at the principal point represents the horizontal angle between the axial planes. The angular values and directions of the axial traces with respect to the fiducial axes are measured and noted. Fig. 2.

2. *Axial plane trace upon each oblique photograph:* A normal is drawn from the principal point to the common trace, or line parallel thereto, on each oblique, and continued to intersect the apparent horizon and upper marginal edge. This normal is the axial plane trace and its position with respect to the fiducial axis is measured and noted. Fig. 2.

3. *Interlocking angle:* A conjugate image point on vertical and left oblique photographs lying on, or within .05″ from, the axial trace and close to the common traces is identified. On each photograph the intercept from the image point to the principal point is measured, then the angle subtended at the perspective center by each intercept may be computed from the relation $\tan^{-1}$ (intercept/focal length). The sum of the two angles is the interlocking angle between the optical axes of the center and left camera and the complement represents 0_l, the locked depression angle below the horizontal of the optical axis of the left camera at zero tilt. θ_r is obtained similarly. Figs. 1 and 2.

All axial plane traces and locked depression angles will remain constant in position and value for each trimetrogon exposure set so long as the cameras are rigidly maintained in the assembly mount. A loose camera will entail the computation of the interaxial relationship for each set to be worked, involving much additional labor and time thereby.

The trend of present practice is to draw upon each vertical and oblique photograph to be worked, short lines at the marginal edges, indicating the positions of the axial traces, in conformity with the mean values so determined from the selected sets and indicated on a diagram. Fig. 2.

Second step—Computation of tilt and azimuths of principal planes of all photo sets to be worked.

The determination of the interaxial relationship is a prelude to the location of both the nadir and the horizontal directions of the principal planes of the three photographs; data necessary for the proper laydown of the radial triangulation.

The components of tilt are reckoned from the left or right axial plane and the accuracy of their values depends primarily upon the precision with which the relating apparent horizons can be identified. These are picked carefully on both left and right oblique photographs, if possible, and the principal lines drawn. Generally, however, the horizon which appears the most reliable is selected, and nadir and azimuths of principal planes determined from that, using the other horizon as a check.

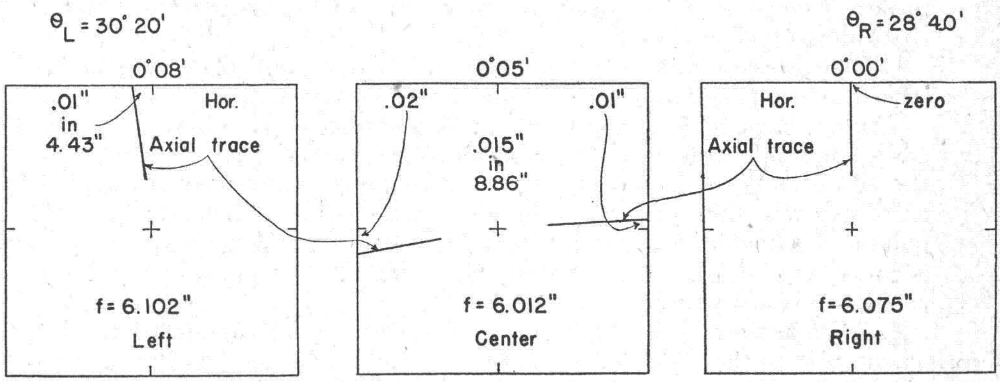

FIG. 2—Camera Assembly Interaxial Relationship

Swings is the angle measured at the principal point in the oblique photograph between the principal line and axial trace, and its value may be computed by measurement of the intercept at the fiducial marks between these two lines. Swing (and the X component of tilt) are reckoned positive when, at the true horizon on the oblique photo, the principal line is backwards of the axial plane trace relative to the direction of flight. Fig. 1.

The true depression of the left camera (D_l) is obtained from the relation:

$$D_l = \tan^{-1}(L/F_l) + Dip_l$$

where L = intercept between principal point and apparent horizon measured along principal line of left oblique photo.

F_l = focal length of left camera.

Dip_l = dip of left horizon for the height of the air station above the horizon terrain.

Similarly $D_r = \tan^{-1}(R/F_r) + Dip_r$.

The axial depressions D_{al} and D_{ar} may be obtained directly from the true depressions by the addition of corrections taken from tables, with arguments D_l and S_l or D_r and S_r.

When the swing is less than 2°, there is less than 1′ difference between the axial and true depression angles, hence depression angles may be computed within this range of error from measurements made upon the axial plane, if found more convenient. Provided the three cameras are in parallel axial planes, since both the axial depressions and the relating interaxial locking angles are measured in the same plane, the relation $\theta_l + \theta_r = D_{al} + S_{ar}$ is true and can be used not only as a check upon the correctness of the selected horizon positions but, with the swing, as a means of readily reconstructing a missing horizon.

Y-TILT

The Y component of tilt, or amount of rotation of the axial plane about a secondary axis through the perspective center normal to the axial plane may be computed from either left or right horizon, but must be referred to the proper axial plane. The value is given by the relation:

$$Y\text{-tilt}_l = \theta_l - D_{al} \text{ or } Y\text{-tilt}_r = D_{ar} - \theta_r$$

and is positive when $\theta_l - D_{al}$ is positive.

X-Tilt

The X component of tilt, or amount of rotation of the axial plane about a primary axis through the perspective center and contained in it, is computed from the measured swing angle between the principal line and axial plane trace on the oblique photograph whose true depression angle has been converted to the axial plane.

The value may be obtained from the relation:

$$X\text{-tilt}_l = \tan^{-1} (\tan \text{swing}_l \cos D_{al})$$
$$\text{or } X\text{-tilt}_r = \tan^{-1} (\tan \text{swing}_r \cos D_{ar})$$

and the sign will be the same as that of the swing.

If desired, the X component can be computed from the swing angle and true depression (instead of axial depression) by the formula:

$$X\text{-tilt}_l = \sin^{-1} (\sin \text{swing}_l \cos D_l)$$
$$\text{or } X\text{-tilt}_r = \sin^{-1} (\sin \text{swing}_r \cos D_r)$$

The values of the X_l and Y_l, or X_r and Y_r tilt components are written upon the forward margin of the center photograph, the suffixes "l" and "r" indicating whether the data was extracted from the left or right oblique photography, and, by a graphical scale the nadir is plotted from the principal point in direction normal to and parallel with the proper axial trace having regard to the signs of X_l and Y_l or X_r and Y_r.

Azimuth of Principal Plane Traces of the Oblique Photographs

When the dihedral angle between the axial planes differs from 180° as shown by the traces of these planes on the central photograph, and particularly so when large tilts are involved, it is necessary to take reference lines or primary axes, one for each axial plane and the reference lines include this same angle. If the values of components X_l and Y_l have been determined from the left photo data, the values X_r and Y_r applicable to the right photo can be determined from them by a rotation of the co-ordinate axes through the deflection angle. This can be done graphically on a large scale plot or computed.

The value of D_{ar} then is obtained from $D_{ar} = \theta_r + Y_r$ and S_r from Tan S_r $=$ Tan $X_r / \cos D_{ar}$ and so the true horizon and the principal line on the right photo, from the axial plane trace, may be established.

The bearing of each oblique principal plane relative to its respective reference line is

$$\text{Tan Bearing}_l = \text{Sin } D_l \tan S_l, \text{ or}$$
$$\text{Tan Bearing}_r = \text{Sin } D_r \tan S_r.$$

The same result may be obtained graphically by the use of diagrams giving, for various focal lengths and depressions, the distance from the true horizon to the perspective center, and associating with this distance the intercept measured along the true horizon on the oblique photograph between the axial plane traces and the principal line. Fig. 1.

The directions of the principal plane traces radiating from the nadir of the paper templet can now be laid off from the already pencilled in axial traces, in their proper directions as clearly indicated upon the corresponding oblique photographs.

Azimuth of Principal Plane of Center Photograph

In order to ensure maximum accuracy in orienting the paper templet upon the vertical photograph, the trace azimuth of the vertical photograph principal plane upon both the photo plane and the horizontal plane, with reference to the center-left or center-right axial trace, is computed. The value with respect to the templet may be determined from the expression $\tan^{-1}$ (sin X cot Y) and that with respect to the photo plane by $\tan^{-1}$ (tan X cosec Y). It should be noted that the reference axial trace referred to is the trace from which the tilt co-ordinates have been computed, and may be either left or right, according to which horizon is considered the more reliable.

Since for moderate tilts there is but little difference in these angles, it is sufficient to compute that for the paper templet and to derive the other by addition of a correction for the degree of tilt.

Example of Tilt Computation

	Method	Left oblique photo		Right oblique photo	
Elev. of air station		20,000′		20,000′	
Elev. of horizon		800′		6,000′	
Swing (S)	M.T.	−.065″	−0°50′	−.065″	−0°50′
Apparent depression	M.T.	3.145″	27°16′	3.165″	27°51′
Dip	T.	.313″	2°16′	.265″	1°56′
True depression (D)		3.458″	29°32′	3.430″	29°27′
Axial depression (D_a)	M.T.		29°32′		29°27′
Locked angle (θ)			30°20′		28°04′
Y-tilt ($\theta_l - D_{al}$)			+0°48′		+0°47′
X-tilt [$\tan^{-1}$ (tan S cos D_a)]	T.		−0°44′		−0°44′
Horizontal angle between axial and oblique principal plane trace [$\tan^{-1}$ (tan S sin D)]	G.T.	−.05″	−0°25′	−.05″	−0°25′
Angle between axial and centre principal plane trace on templet [$\tan^{-1}$ (sin X cot Y)]	C.		42°31′		
Ditto on vertical photo	T.		42°31′		

In this example the left horizon was the more reliable and the tilt computation accordingly was made from it, the right horizon being used as a check.

Subscripts Mean:
$_l$—Left oblique Photograph.
$_r$—Right oblique Photograph.
$_a$—Axial plane.

Reconnaissance Topographic Mapping

The foregoing discussion has reference altogether to planimetric mapping proposed to be done by the trimetrogon photogrammetric unit. Such maps, in themselves, serve a most useful purpose but, due to the tremendous strides made in aviation during the war with the probable large expansion in flying activities after the war, their value would be greatly enhanced by the addition of elements of relief.

Trimetrogon photography, if proper care is exercised in its execution, provides means of recapturing the camera position in space, thus permitting of the measurement of vertical angles and hence of differential elevations to detail points throughout the photographs.

For this purpose several modified versions of the Wilson photoalidade are currently being constructed by the National Research Laboratories for use by a division of the Trimetrogon Photogrammetric Unit charged with this phase of mapping. Until experience has been acquired in the use of trimetrogon photographs and the Wilson photoalidade, not much can be said respecting contour interval or speed of production. Obviously, however, much will depend upon the class of the trimetrogon photography and the density and type of the vertical control.

NEW SUSTAINING MEMBER

PHOTOGRAMMETRIC ENGINEERING is pleased to announce that the Society has accepted Kargl Aerial Surveys, Inc., Midland, Texas, as a sustaining member. Many of our older members will recall the old "Kargl Aerial Surveys" of San Antonio, Texas. This company was dissolved quite a few years ago. The new Kargl Aerial Surveys of Midland, Texas, is in no way connected with the old company.

MANUAL OF PHOTOGRAMMETRY

The Secretary of the Society has notified me that he has a number of copies of the *Manual of Photogrammetry* available for sale. The following is a review published by the Photographic Society of America:

> "Photogrammetry is the science or art of obtaining reliable measurements by aid of photography. Although not a new application of photography, it has been only in most recent years that photogrammetry has become thoroughly entrenched in surveying and map making. This new book is very timely, since in one volume the principles of the subject and the newer wartime developments are presented, which certainly will be useful in post-war as well as present engineering undertakings. The book contains 17 major chapters, each prepared by an authority in his particular field. Each subject is dealt with exhaustively, with the text well supported by numerous illustrations. The chapter on the classifying of forest and other vegetation is unique for photogrammetry texts. This is also true for the excellent chapter on teaching the subject of photogrammetry. The book is primarily for the specialist, but many of the chapters, such as those on optics, sensitized materials, stereoscopy, etc., are of value to photographers in general."

The book is available from the American Society of Photogrammetry, Box 18, Benjamin Franklin Station, Washington, D. C. The price is $5.00 to members; $8.50 to non-members.

The following is a quotation from a letter received by the Secretary of the Society:

> "I warmly welcomed the copy of the superb "Manual of Photogrammetry" issued by the Society. This is the best treatise on the subject I have known heretofore. This magnificent book is stuffed with the most complete and important information available to date and from the most reliable sources. I wish to congratulate you sincerely for the publication of such a formidable work which simply has come to fill a great need amongst those interested in photogrammetry as a new science of untold applications in both peace and war."

Yours very sincerely,
Francisco A. Foreror
Bogotá, Colombia, S. A.