TWO METHODS OF DETERMINING FLYING HEIGHT

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Synopsis: This article discusses and illustrates two basic theories for solving the problem of determining the flying height of a vertical aerial photograph.

The article shows quite conclusively the necessity for broadness in the teaching of the basic theory of photogrammetry by illustrating the advantage of one basic concept over another for solving a common problem.

It should be of particular interest to educators and students of the theory of photogrammetry and also to the persons who have the problem of determining the flying height of aerial photographs mathematically.—*Publication Committee.*

THE article "Ideas Relative to Education in Photogrammetry" published in the September 1947 issue of PHOTOGRAMMETRIC ENGINEERING should be of inestimable value to the advancement of photogrammetric education.

Part III of this article outlined in a unique and concise manner—"A Discussion of Some Basic Principles of Photogrammetry" by Earl Church, Professor of Photogrammetry, Syracuse University.

The writer was privileged to be a student under Professor Church and recommends these basic principles for a fundamental education in photogrammetry. However, in the light of later experience, the writer believes that it would be of practical value to include, in a broad educational program, some of the new concepts originated by Mr. R. O. Anderson, Mathematician, Tennessee Valley Authority. As an illustration of the value of these principles, the efficiency of handling certain problems by means of the equivalent elevation theory will be demonstrated in an example to follow.

The problem of determining the flying height of a vertical photograph from one control line will be outlined for both the Church and Anderson Methods. To prove a method, it is difficult to assume a fictitious photograph so that its elements do not work more to the advantage of one method than to the other. In this regard, Photo 1 of the fictitious set of photographs as compiled by the 30th Engineers, Fort Belvoir, Virginia will be used as the test case.

PHOTOGRAPHIC AND CONTROL DATA

Given:

Point	x	y	X	Y	h
a	00.000 mm.	+76.531 mm.	5,000′	25,000′	400′
b	+78.947 mm.	+78.947 mm.	15,000′	25,000′	1,000′

Focal Length, $f = 150.000$ mm.

Photo Distance, $P = 78.984$ mm.

Ground Distance, $D = 10{,}000'$

To find: H (flying height).

The line, ab, was chosen so as to result in an h_c correction that is common to the Anderson Dropped Perpendicular Method. If the line were taken radially from the principal point there would be no h_c correction. Since point a lies on the $+y$ axis and there is relatively little y difference between point a and b, the mathematical computations are made easier for the rectangular coordinate system of the Church Method. Even though "square rooting" in this particular problem can be done visually, the four terms will be listed for they may be of any conceivable magnitude.

The correct flying height for the given photographic and control data is

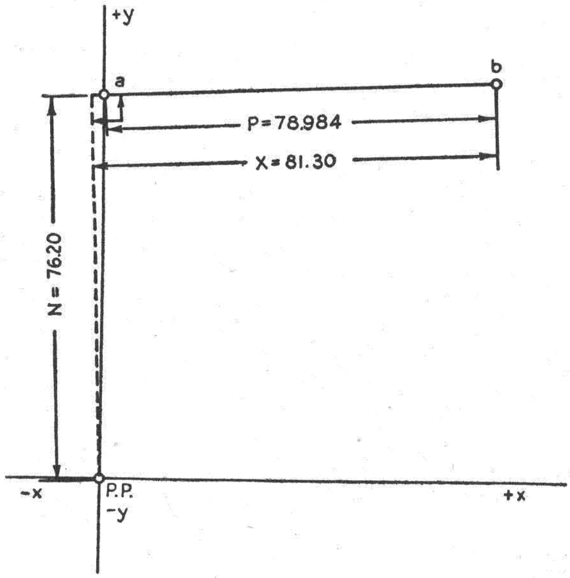

FIG. 1. Photographic Measurements.

20,000′. Although this is the answer, it is given now to aid the reader in following the successive determinations.

CHURCH METHOD[1]

$$H_1 = \left(\frac{D}{P}\right)(f) + \frac{h_a + h_b}{2} \qquad \text{Eq. (1C)}$$

$$H_1 = \frac{(10{,}000)(150.000)}{78.984} + \frac{400 + 1{,}000}{2}$$

$$H_1 = 18{,}991.2 + 700 = 19{,}691.2'$$

This is the first approximate flying height based on the photographic scale times the focal length, plus the average elevation of the control points.

First Determination

$$X_a = \frac{H - h_a}{f} \cdot x_a \qquad Y_a = \frac{H - h_a}{f} \cdot y_a \qquad \text{Eq. (2C)}$$

$$X_b = \frac{H - h_b}{f} \cdot x_b \qquad Y_b = \frac{H - h_b}{f} \cdot y_b \qquad \text{Eq. (3C)}$$

[1] PHOTOGRAMMETRIC ENGINEERING, July 1947, page 389, par. (3).

The ground coordinates are solved for using a revised H by trial, until the computed ground distance equals the correct length of the control line.

$$X_a = \frac{(19{,}691.2 - 400)(0.000)}{150.000} \qquad Y_a = \frac{(19{,}691.2 - 400)(76.531)}{150.000}$$

$$X_a = 0.0' \qquad Y_a = 9{,}842.5'$$

$$X_b = \frac{(19{,}691.2 - 1{,}000)(78.947)}{150.000} \qquad Y_b = \frac{(19{,}691.2 - 1{,}000)(78.947)}{150.000}$$

$$X_b = 9{,}837.4' \qquad Y_b = 9{,}837.4'$$

$$D_1 = \sqrt{(9{,}837.4 - 0.0)^2 + (9{,}837.4 - 9{,}842.5)^2}$$

$$D_1 = 9{,}837.4'$$

Correct H_1 proportionally to the change required in the computed distance, where h is the average elevation of the control points:

$$\frac{D}{D_1} = \frac{H_2 - h}{H_1 - h} \qquad \text{Eq. (4C)}$$

$$\frac{10{,}000.0}{9{,}837.4} \qquad \frac{H_2 - 700}{19{,}691.2 - 700}$$

$$H_2 = 20{,}005.1$$

This concludes the first determination.

Second Determination

$$X_a = \frac{(20{,}005.1 - 400)(0.000)}{150.000} \qquad Y_a = \frac{(20{,}005.1 - 400)(76.531)}{150.000}$$

$$X_a = 0.0' \qquad Y_a = 10{,}002.7'$$

$$X_b = \frac{(20{,}005.1 - 1{,}000)(78.947)}{150.000} \qquad Y_b = \frac{(20{,}005.1 - 1{,}000)(78.947)}{150.000}$$

$$X_b = 10{,}002.6' \qquad Y_b = 10{,}002.6'$$

$$D_2 = \sqrt{(10{,}002.6 - 0.0)^2 + (10{,}002.6 - 10{,}002.7)^2}$$

$$D_2 = 10{,}002.6'$$

$$\frac{10{,}000}{10{,}002.6} = \frac{H_3 - 700}{20{,}005.1 - 700}$$

$$H_3 = 20{,}000.1'$$

Anderson Method[2]

$$h_e = h_a + \frac{X}{P}(h_b - h_a) \qquad \text{Eq. (1A)}$$

$$h_e = 400 + \frac{81.30}{78.984}(1{,}000 - 400)$$

[2] "Applied Photogrammetry," Edwards Bros., Inc., Ann Arbor, Mich.

$$h_e = 1017.6'$$

h_e is termed the equivalent elevation.

The convergence correction, h_c, is found by interpolation of Table 1 as follows:

$$\frac{N}{P}(h_b - h_a) = \frac{76.20}{78.984}(1{,}000 - 400) = 579 \qquad \text{Eq. (2A)}$$

$$\frac{D}{P}(f) = \frac{(10{,}000)(150.000)}{78.984} = 18{,}991.2 \qquad \text{Eq. (3A)}$$

The corresponding value of the convergence correction is:

$$h_c = 8.9'$$

$$H = \frac{D}{P}(f) + h_e - h_c \qquad \text{Eq. (4A)}$$

$$H = 18{,}991.2 + 1{,}017.6 - 8.9$$

$$H = 19{,}999.9'$$

Conclusion

The prime difference in the approach to this problem is:

(a) *Church Method:*—The average elevation of the control points is used as the argument in determining an approximate flying height. Then H is corrected proportionally by successive determinations, until two successive values of the flying height agree closely. This method is purely analytical.

(b) *Anderson Method:*—The equivalent elevation corrected for convergence is used in lieu of an average elevation and successive determinations of H. The convergence correction is taken directly from a tabulation. In short, the flying height is solved for as an unknown in a direct solution, thereby eliminating successive determinations. This is possible because the equivalent elevation is the elevation at which the photographic scale D/P is effective. This method is semigraphical in that it requires the measurement of the drafted functions X and N.

Professor Church and Mr. Anderson have presented this country with two excellent methods of determining the elements of photographic orientation. One of the first "crossroads" contrasting these two methods has been the "average elevation vs. equivalent elevation." The use of the h_c correction in the past has been accepted with reluctance or caused widespread confusion in photogrammetric circles. To alleviate this condition, a table was prepared for direct interpolation of the values of h_c. It is hoped that this tabulation will materially aid the student in photogrammetry.

Due to the present increased interest in the teaching of photogrammetry, it is believed that parallel solutions demonstrating at least one basic problem will be helpful to prospective teachers who are not yet fully aware of the techniques of available methods.

TABLE 1. VALUES OF h_c

$\frac{N}{P}(h_1-h_2)\rightarrow$ $\frac{D}{P}(f)\downarrow$	100	200	300	400	500	600	700	800	900	1,000
5,000	1.0	4.0	9.0	16.0	25.1	36.1	49.2	64.4	81.7	101.0
6	0.8	3.3	7.5	13.3	20.9	30.1	41.0	53.6	67.9	83.9
7	0.7	2.9	6.4	11.4	17.9	25.8	35.1	45.9	58.1	71.8
8	0.6	2.5	5.6	10.0	15.6	22.5	30.7	40.1	50.8	62.7
9	0.6	2.2	5.0	8.9	13.9	20.0	27.3	35.6	45.1	55.7
10,000	0.5	2.0	4.5	8.0	12.5	18.0	24.5	32.1	40.6	50.1
11	0.5	1.8	4.1	7.3	11.4	16.3	22.3	29.1	36.9	45.5
12	0.4	1.7	3.8	6.7	10.4	15.0	20.4	26.7	33.8	41.7
13	0.4	1.5	3.5	6.2	9.6	13.8	18.9	24.6	31.2	38.5
14	0.4	1.4	3.2	5.7	8.9	12.9	17.5	22.9	29.0	35.8
15,000	0.3	1.3	3.0	5.3	8.3	12.0	16.3	21.3	27.0	33.4
16	0.3	1.2	2.8	5.0	7.8	11.2	15.3	19.9	25.3	31.3
17	0.3	1.2	2.6	4.7	7.3	10.6	14.4	18.8	23.8	29.4
18	0.3	1.1	2.5	4.4	6.9	10.0	13.6	17.7	22.4	27.8
19	0.3	1.1	2.4	4.2	6.6	9.5	12.9	16.8	21.3	26.3
20,000	0.3	1.0	2.3	4.0	6.3	9.0	12.3	16.0	20.3	25.0
21	0.2	1.0	2.1	3.8	6.0	8.6	11.7	15.2	19.3	23.8
22	0.2	0.9	2.0	3.6	5.7	8.2	11.1	14.5	18.4	22.7
23	0.2	0.9	2.0	3.5	5.4	7.8	10.6	13.9	17.6	21.7
24	0.2	0.8	1.9	3.3	5.2	7.5	10.2	13.3	16.9	20.8
25,000	0.2	0.8	1.8	3.2	5.0	7.2	9.8	12.8	16.2	20.0
26	0.2	0.8	1.7	3.1	4.8	6.9	9.4	12.3	15.6	19.2
27	0.2	0.7	1.7	3.0	4.6	6.7	9.1	11.8	15.0	18.5
28	0.2	0.7	1.6	2.9	4.5	6.4	8.7	11.4	14.5	17.9
29	0.2	0.7	1.6	2.8	4.3	6.2	8.4	11.0	14.0	17.2
30,000	0.2	0.7	1.5	2.7	4.2	6.0	8.2	10.7	13.5	16.7
31	0.2	0.6	1.5	2.6	4.0	5.8	7.9	10.3	13.1	16.1
32	0.2	0.6	1.4	2.5	3.9	5.6	7.7	10.0	12.7	15.6
33	0.2	0.6	1.4	2.4	3.8	5.5	7.4	9.7	12.3	15.1
34	0.1	0.6	1.3	2.4	3.7	5.3	7.2	9.4	11.9	14.7
35,000	0.1	0.6	1.3	2.3	3.6	5.1	7.0	9.1	11.6	14.3
36	0.1	0.6	1.3	2.2	3.5	5.0	6.8	8.9	11.2	13.9
37	0.1	0.6	1.2	2.2	3.4	4.9	6.6	8.6	10.9	13.5
38	0.1	0.5	1.2	2.1	3.3	4.7	6.4	8.4	10.7	13.2
39	0.1	0.5	1.2	2.1	3.2	4.6	6.3	8.2	10.4	12.8
40,000	0.1	0.5	1.1	2.0	3.1	4.5	6.1	8.0	10.1	12.5