not be big enough to cause alarm and promote search for the cause. To correct the positions of resected points, necessitated by small residual tilts in the constructed templets, recourse is had to perspectometers. Used in conjunction with a squared grid for 1 mile scale, the paper templet is placed on the perspectometer and the tilted resected points thereon rectified on a transparency placed on the squared grid.

The acetate is finally cut into quadrangles of 30′ of latitude and 1°00′ of longitude for photographic reductions for compilation and final draughting at 5.33 miles to 1 inch. This size suits the convenience of our camera copy board, and filing of acetate for record.

EUROPEAN FIRST ORDER TRIANGULATION AND ITS ADJUSTMENT*

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THE measurement of distances between points on the surface of the earth through the method of triangulation is not of recent origin. Nearly four centuries ago, in the year 1579, the Danish Astronomer, Tycho Brahe established an observatory on an island near Copenhagen. There he determined an azimuth and an astronomic position in latitude and longitude from observations on the stars. A base line was measured on the ground to furnish his initial distance. The angles to a new station were then measured at the ends of the base line. The distances to the new point and its latitude and longitude were computed, and the azimuth carried forward, through the application of the well known elementary principles of trigonometry. The geographic positions of additional stations were likewise determined from succeeding angular measurements. A check base line was even measured on the ice during the winter months. This procedure is recognized as the familiar one still employed by geodetic engineers when surveys are initiated in new areas. The old Danish triangulation arc is all the more remarkable when we consider that it antedated by half a century the invention of the telescope by the Hollander, Hans Lippershey, in 1608. Brahe observed his angles by sighting distant church spires or similar targets on clear days with the unaided eye.

2. Following this initial arc in Denmark, and particularly during the past century and a half, the Europeans have extended their basic triangulation over the entire continent. The lower order control, fitted within the basic first-order triangulation, is likewise very extensive in Europe, much more so than in the United States. This minor control with its dense coverage is primarily for mapping and cadastral purposes. It is not uncommon, especially in Germany, to find a horizontal control density of one or two points for each square mile. The standard German requirement, which we consider excessive, was not less than 22 horizontal control points per map sheet of 1:25,000 scale.

3. This paper deals with the adjustment only of basic European triangulation of first-order accuracy. This accuracy has been defined by the International Association of Geodesy as that represented by an agreement between the value of a baseline measured with a probable error of 1:1,000,000 and the computed value of the same line carried through the triangulation of not more than one part in 25,000 of the length of the base, after the angle and side equations have

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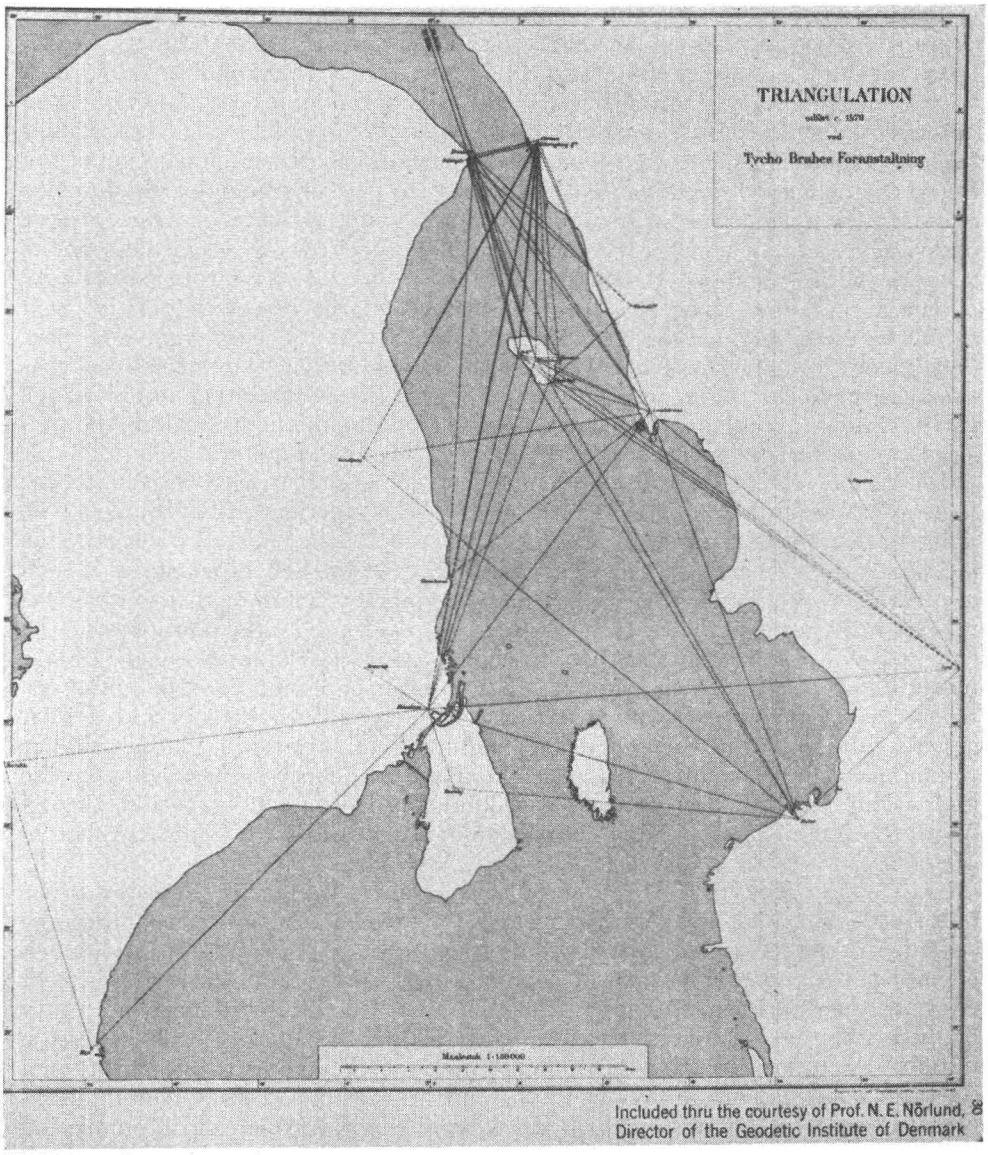
been satisfied in the least squares adjustment. The probable error of the direction of a line should not exceed 0.6 of a second of arc.

4. Instrumental equipment used on first-order field work in Europe and the United States can be said to be generally comparable in workmanship and accuracy. While the first-order triangulation of the United States is well above the minimum standard of first-order accuracy, much of the basic European triangulation is even superior. This may be explained by the fact that there never has been the urge for a rapid rate of progress in the small countries of Europe that is an economic necessity in a country the size of ours. Hence, the Europeans have not been loath to occupy their basic triangulation stations each night for a



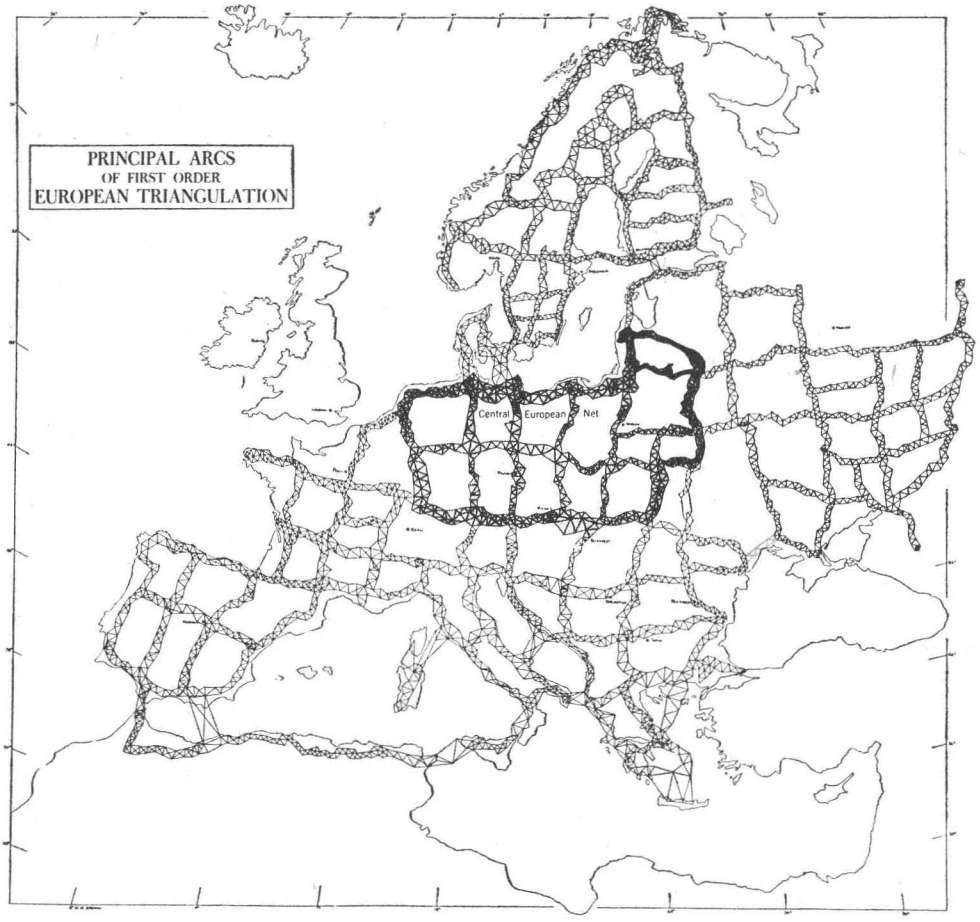
Included thru the courtesy of Prof. N. E. Nörlund, Director of the Geodetic Institute of Denmark

week, two weeks, or more, whereas our practice is to complete the observations within a few hours on one night. In addition, much of their control has been measured and remeasured several times over the years. The natural result of these more deliberate European methods, with their obvious tendency to reduce to a minimum the observational and horizontal refraction errors, has been all to the good from the standpoint of accuracy.

5. The first-order triangulation of Europe has been carried on, of course, by each of the 20 sovereign countries working independently of each other except for certain mutual agreements to connect their filled work at common points along the frontiers. Each country has selected a point, usually in its capital city, for the origin or initial point from which its own triangulation has been computed geodetically and radially to its own boundaries, some in an areal network of triangles and others by polygonal arcs. The datum point or origin of each country has been determined by extensive astronomical observations for latitude, longitude and azimuth. While additional first-order astronomic observations have been generously made throughout the triangulation, it has seldom been the practice to average out the various differences in astronomic and geodetic values for latitude and longitudes at all of these points and to revise the position of the initial datum accordingly. Neither has it been the custom to employ the Laplace equation for the purpose of removing to a great extent the twist which almost invariably occurs whenever triangulation is carried forward. Regardless of the accuracy with which the astronomic positions of the various datum points were observed, patently there remains the relatively large errors in position and azimuth due to the slope of the geoid at the datum point. The differences in these deflections, combined with the twist of the triangulation, the normal observational errors in the angles and the employment of three different spheroids in the computations, are adequate to cause discrepancies at the many national boundaries amounting in some cases to as much as 500 to 700 meters.

6. We have then in Europe an incongruous situation with respect to the first-order triangulation in that it is largely of a superior quality but of little value to the geodesist as yet for the scientific study of the figure of the earth, due to its lack of homogeneity. An analogy would exist in the United States if each two states of the Union were to have an astronomic datum of their own, separate and distinct from every other pair of states. Imagine the confusion at the state boundaries. Fortunately the European countries have included in their natural control an adequate number of first-order bases, Laplace azimuths, astronomic stations and common stations along the frontiers to permit a least squares adjustment of the entire continental triangulation in a manner similar to that accomplished in North America. Here, the basic triangulation of the entire continent has been computed geodetically from a single station, that of Meades Ranch, Kansas, and on a common figure of the earth, Clarke's Spheroid of 1866.

7. Geodesy, the study of the shape and size of the earth and of the variations of terrestrial gravity, is a science to which the great mathematicians and geodesists of Europe have contributed a major share toward our present day knowledge of this important branch of geophysics. It is due to no lack of desire or effort on their part that the basic triangulation of Europe is still on some twenty or more different datums but rather to the political jealousies of 20 sovereign nations which until recently have made it impossible to come to agreement on a scientific achievement of obvious importance to world geodesists. A notable exception to this lack of agreement is the work of the Baltic Geodetic Commission begun in 1937, through which there was completed last year the adjustment of the first-order triangulation around the Baltic Sea. The positional closure of this



Baltic Ring was remarkable from the standpoint of accuracy. This closure, as well as the various loop closures of the recently completed Central European Area, attests the statement previously made as to the superior quality of the field observations in the European first-order triangulation.

8. In April, 1945, the American forces captured the German town of Friedrichsroda in the Province of Thuringia. A geodetic intelligence team from the Office of the Chief of Engineers, U. S. Army, found in Friedrichsroda on April 11th a group of German geodesists which constituted the Trigonometrical Section of the Reichsamt für Landesaufnahme. The latter was the chief survey and mapping organization of Germany recently moved from Berlin to Friedrichsroda. Heading this group was Diploma Engineer Erwin F. Gigas, an internationally known German geodesist. Gigas, with a nucleus of his staff consisting of his best geodesists and mathematicians, was moved with his survey records and equipment from Friedrichsroda, soon to be in the Russian Area of Occupation, seventy miles south to Bamberg in the United States area. The Burgomeister at Bamberg was ordered to provide office space and quarters for the group and to place the personnel on his salary rolls at the regular German Civil Service rates. The immediate families of the personnel were then removed to Bamberg.

9. Gigas was directed to begin at once a least squares readjustment of the first-order triangulation of the Central European area, covering approximately

306,000 square miles. A block by block adjustment of the same area, just completed by Gigas for German military purposes, was considered inadequate due, among other reasons, to the failure to include in the adjustment any of the numerous first-order bases and Laplace azimuths in the area. A junction method of adjustment, similar to the Bowie Method* previously employed in the United States, was selected as the most practicable means by which the new adjustment could be accomplished in an adequate manner over a two-year period. In the initial months, an exhaustive analysis was made of existing geodetic field and office records over the past 140 years, to assure that the basic data to be used in the adjustment would include the most logical selection of bases, Laplace azimuth, astronomic observations and angle measurements accumulated over this long period of years. Since the greater part of the region's triangulation consisted of areal coverage, it was necessary to select certain arcs and junction figures which should form the network for the Bowie type of adjustment. In these selected arcs, spaced from 75 to 100 miles apart, there were 23 junction figures, 914 triangles, 52 bases, 106 Laplace azimuths, and 199 astronomic stations. The average field triangle closure was 0.52 seconds with a maximum of 3.26 seconds.

10. The original geodetic group of 17 at Bamberg was gradually augmented during the summer of 1945, as German prisoners were released by the Allied Forces, until the rolls carried 60 geodetic engineers, mathematicians and computers. This group continued the adjustment to a successful completion in June, 1947. Condition equations to the number of 1,305 were involved in the adjustment with a resulting probable error of a direction of 0.52 seconds. Following the adjustment of the Central European area, the geodetic position of 714 stations was computed on the International Ellipsoid, all in perfect mathematical relationship with each other. The temporary initial point received its proportional shift in position during the process of reduction for error due to deflection of the vertical. Hence, there is no "Datum Point"; the Central European Datum is a condition of the entire area of 306,000 square miles rather than that of any single station.

11. The installation of the highly qualified group of German geodesists at Bamberg and the launching of the operations on the Central European area immediately raised the question as to the feasibility of extending this adjustment to include the first-order triangulation of all of Europe on a common geodetic datum. Unilateral discussions were held with leading geodesists in the capitals of most of the Western European countries in May and June of 1946, to ascertain if they would consider a cooperative arrangement whereby the desire of European geodesists for many decades might be realized in the near future. The reaction to this proposal to undertake an extension of the Central European adjustment to all of Europe was so favorable that a conference on the subject was called to meet in Paris in August, 1946, concurrently with that of the Permanent Commission of the International Association of Geodesy. The Army Map Service representative proposed at that meeting to have the Bamberg group undertake the extended adjustment under the administrative control of the Chief Engineer of the European Command with the technical supervision resting in an Allied Geodetic Staff consisting of one expert geodesist from each of the interested countries. A special committee was set up in the International Association of Geodesy to study all phases of the problem and to report back to the Secretary of the Central Bureau of the Association. This special committee, named the Commission on the Adjustment of European Triangula-

* The Bowie Method of Triangulation Adjustment, Spec. Pub. No. 159, USC&GS.

tion, consisted of one member from each interested European country; also, by courtesy, one member from the U. S. Coast and Geodetic Survey and one from the Army Map Service. A premise was early recognized to the effect that first-order triangulation data of the various countries, which has been freely published for many years, should be available for any desired adjustment for scientific purposes without the necessity of further authorization.

12. The Commission met again at Paris in October, 1947, with representatives from twelve nations present. Some of the delegates felt that the improved Bowie Junction method of adjustment was not sufficiently refined to serve the scientific purposes intended and that the superior order of observations generally attained in the field work of the European triangulation warranted a more meticulous approach to the adjustment. It was readily conceded that the junction method leaves something to be desired from the scientific angle. It was pointed out, however, that it was the only practical method by which an adjustment could be made within a reasonable time and in an economical manner. Its use in the United States had proved entirely adequate. A proposal was then made and adopted to carry out the adjustment in two phases. The first phase would be done by the junction method in which all geometric conditions would be satisfied and the geodetic positions would be computed on a common datum on the International Ellipsoid, with all distances reduced to the International unit of length. This phase could be completed in 18 months. The second phase would follow at leisure over a period of years and would revise the first phase results by taking into account the discrepancies between the geoid and the spheroid.

13. Having agreed on the two phase approach to the problem, the representatives of those countries that recently had suffered so cruelly at the hands of the Germans declined to permit the Bamberg group to handle the adjustment of the triangulation of their countries even though neither the technical nor the administrative control would be in German hands. Their attitude was entirely understandable. They voted to forego the advantages of the adjustment which they earnestly desired and for which they had assembled in Paris. Immediately following the adjournment, the Army Map Service representative proposed a compromise to the Secretary of the Central Bureau of the International Association of Geodesy whereby the Army Map Service would make the adjustment in Washington on a non-reimbursable contract with the Association. This compromise plan, which was later approved by the Chief of Engineers, was favorably received by the Central Bureau. In the months that followed, the Secretary carried on negotiations with the various countries from most of whom at least tentative concurrence was obtained. In the meantime, the Army Map Service had arranged to sub-let the actual computations to the Division of Geodesy of the U. S. Coast and Geodetic Survey, provided final concurrence of the various nations was forthcoming. That Bureau, which had contributed valuable consultation to the work of the already completed Central European Adjustment, agreed to handle the adjustment of Western Europe for the Army Map Service on a reimbursable basis, and to complete the job within 18 months of receipt of the essential data.

14. The members of the Commission on the Adjustment of European Triangulation met again in August, 1948, during the Eighth General Assembly of the International Union of Geodesy and Geophysics at Oslo, Norway. Further discussions on the subject were held and minor difficulties were ironed out. By the end of the Oslo meeting, assurances of final concurrence had been received from all continental countries in Western and Northern Europe. The triangulation of the British Isles is at present connected to the continent in one figure only and hence is not a mathematical requisite to the adjustment. However, it

is anticipated that the British triangulation will be made available for incorporation into the European Adjustment when a geodetic connection is made between Scotland and Norway.

15. Summarizing, the adjustment of Central Europe has been completed; the adjustment of Western and Northern Europe will be underway shortly and is scheduled for completion by June, 1950. It is logical for Southeastern and Eastern Europe likewise to incorporate their first-order triangulation into the European Datum. Thus, the geodesists' dream of the adjustment of the triangulation of Continental Europe to a true mathematical relationship, on a single geodetic datum and a single spheroid, should be realized within a reasonably short time. The adjustment of the triangulation of Europe to a common datum, the previous adjustment of North America to one datum, and the anticipated adjustments of the remaining continents likewise to common datums, encourage the ultimate of a single geodetic datum for the entire globe based on an improved figure of the earth. Intercontinental connections presently in view through the use of such vehicles as radar, eclipses, star occultations, flare and orthodox ground triangulation, together with the development of electronic computers, give real promise of the attainment of a common world geodetic datum within a decade.

16. The energetic and efficient Secretary of the Central Bureau of the International Association of Geodesy, Monsieur Pierre Tardi, and his official staff in Paris, have labored hard and long to bring about the present status of final agreement on the extension program. Subsequent to the Oslo meeting, they have been assembling the first-order data required from the various countries, including observed angles, bases, Laplace azimuths, and astronomic positions, for transmittal early this year to the Army Map Service. Recommendations from these countries as to the most appropriate use of their data and the verification of junction figures at the numerous international boundaries are a most important contribution to the initial phases of the adjustment.

17. It is apparent that the friendly and wholehearted agreement by the countries of Western and Northern Europe to participate in a scientific achievement of the character described in this paper will serve as a notable example of international goodwill and cooperation. The pattern might well be emulated by nations currently engaged in political deliberations.

18. Photogrammetrists have a mutual interest with geodesists in the effort to systematize and coordinate geodetic control to bring it into proper mathematical relationship over large areas. The term "geodetic control" here includes the vertical although it is not a part of this discussion. It is axiomatic that stereocompilation cannot produce a map to meet standard accuracy requirements without adequate control which in turn depends upon a proper knowledge of the figure of the earth. The mutuality of interest between the photogrammetrist and the geodesist will be confirmed by steadily increasing cooperation and sympathetic study of each others problems where they relate to mapping. The intermingling of engineers of both groups, such as we have here today, as well as dual membership where possible in the corresponding technical societies, will do much to advance this cooperation. Photogrammetric mapping in the United States has made rapid strides in its relatively short span of 30 years. The American Society of Photogrammetry may well be proud of its contribution through the encouragement it has given this important development, without which development the world could not possibly hope to keep pace with the seemingly insatiable demands for increased accomplishment in the mapping field.