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All airphotos used in connection with the preparation of this report automatically carry the following credit line: "Photographed for Field Service Branch—PMA—U.S.D.A." Other photographs taken by Joint Highway Research Project staff photographers.

SCALES OF OBLIQUE PHOTOGRAPHS

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I IS not generally realized that the scale at a point on an oblique photograph is not the same in all directions; that is, that the x-scale (scale of lines perpendicular to the principal plane), and the y-scale (scale of lines parallel to the principal plane), and the z-scale (scale of vertical lines) are not the same. This paper will show the derivations of the formulae for these scales in the hope that they will lead to a clearer understanding of the nature of oblique photographs. It is also believed that the formulae may be useful to photograph interpreters in estimating the sizes of buildings and similar small objects from simple measurements on the oblique photograph.

X-scale

Figure 1 shows a view of the principal plane of an oblique photograph mn. L is is the exposure station, n the photograph nadir point, o the principal point, i the isocenter, Lm the trace of the horizon plane, Lo = f is the principal distance, θ is the depression angle of the camera axis. Because the X-scale is measured along a photograph parallel, it is constant throughout the length of the parallel. If a

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FIG. 1. Section of an oblique photograph showing one object point.

is the photograph image of a point A which lies a distance H below the exposure station, then the x-scale

$$S_x = \frac{La}{LA} = \frac{m'a}{LN} = \frac{d\cos\theta}{H} \tag{1}$$

where d = ma, the distance from the true horizon to the point measured in the plane of the oblique.

Special Cases. If the point a is on the principal parallel,

$$d = f \tan \theta \text{ and}$$
$$S_x = \frac{f \sin \theta}{H};$$

If the point *a* is on the isometric parallel,

$$d = f \sec \theta, \text{ and}$$
$$S_x = \frac{f}{H} \cdot$$

Y-SCALE

(a) Points on the Principal Line. Figure 2 is the same as Figure 1, except that two ground points A and B are shown lying in the principal plane. The corresponding photograph images a and b are also on the principal line. The scale of the line ab is obviously

$$S_y = \frac{ab}{AB} \, \cdot \,$$

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FIG. 2. Section of an oblique photograph showing two object points of equal elevation in the principal plane.

Let $ma = d_a$ and $mb = d_b$ and let a' and b' represent the image of A and B on an equivalent vertical photograph. Then

$$AB = \frac{H}{f}a'b' = \frac{H}{f}(ib' - ia').$$

But

$$\frac{ia'}{f - d_a \cos \theta} = \frac{Lm}{d \cos \theta} = \frac{f \sec \theta}{d_a \cos \theta}$$
$$ia' = \frac{f(f - d_a \cos \theta)}{d_a \cos^2 \theta}$$

$$ib' = \frac{f(f - d_b \cos \theta)}{d_b \cos^2 \theta}$$

Therefore,

$$AB = \frac{H}{f} \left[\frac{f^2(d_a - d_b)}{d_a d_b \cos^2 \theta} \right].$$

Inasmuch as $(d_a - d_b) = ab$, therefore

$$S_y = \frac{ab}{AB} = \frac{d_a d_b \cos^2 \theta}{Hf} \,. \tag{2}$$

If the line ab vanishes to a point, $d_b = d_a = d$ and

$$S_y = \frac{d^2 \cos^2 \theta}{Hf} \cdot \tag{2a}$$

Special Cases:

At the principal point $d = f \tan \theta$, and

$$S_{y} = \frac{f}{H} \sin^{2} \theta;$$

if point *a* is at the isocenter, $d = f \sec \theta$, and

$$S_y = \frac{f}{H} \cdot$$

(b) Points Not on the Principal Line. If the line AB (figure 2) lies in a plane parallel to the principal plane, the corresponding photograph points a and b will lie on a ray through the point m, the intersection of the principal line and the trace of the true horizon. In this case, the scale (S_{ab}) of line $(n \text{ can be found from formula (2) or (2a), provided that <math>d_a$ and d_b are measured along this ray. Then the distance

$$AB = \frac{ab}{S_{ab}} \cdot$$

The true ground distance AB can also be found by using in formulae (2) and (2a) the projected lengths of d_a , d_b , and ab on the principal line; that is, by measuring *all three* distances in the direction perpendicular to the photograph parallels.

Measurement by Components

If the line AB is not parallel to the principal plane, its length can be found by obtaining the lengths of its components. The component parallel to the principal plane is obtained as described in paragraph (b) above. The component perpendicular to the principal plane is obtained by projecting the line onto the photograph parallel which passes through one of the line terminals and using the scale (x-scale) of that parallel, as obtained from formula (1). However, it should be noted that, in making this projection on the photograph, the points move along rays to the intersection of the principal line and the horizon, instead of moving normal to the photograph parallels.

Z-SCALE

In Figure 3 the line AB is a vertical line (such as a chimney or the corner of a building) situated at a distance D from the ground nadir point and with its lower terminal at a distance H below the exposure station.

If i' is the conjugate isocenter (that is, the intersection of the oblique plane with an equivalent horizontal photograph), then by analogy with Figure 2,

$$S_z = \frac{e_a e_b \, \sin^2 \theta}{f D}$$

where $e_a = na$ and $e_b = nb$. If line *ab* vanishes to a point,

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FIG. 3. Section of an oblique photograph showing one object point above the other.

$$S_z = \frac{e^2 \sin^2 \theta}{fD} \cdot$$

From similar triangles,

$$\frac{D}{e_a\,\sin\,\theta} = \frac{H}{d_a\,\cos\,\theta}$$

where $d_a = ma$. Therefore

$$S_z = \frac{e_b d_a \sin \theta \cos \theta}{fH} \tag{3}$$

and for a point,

$$S_z = \frac{ed\,\sin\,\theta\,\cos\,\theta}{fH} \,. \tag{3a}$$

Special Cases. At the principal point $d = f \tan \theta$ and $e = f \cot \theta$, and

$$S_z = \frac{f}{H}\sin\,\theta\,\cos\,\theta;$$

At the isocenter $d = f \sec \theta$ and $e = f (\csc \theta \sec \theta) - d$, and

$$S_z = \frac{f}{H} \frac{1 - \sin \theta}{\cos \theta}$$

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Measurements from the Principal Parallel

It is often more convenient to measure from the principal parallel rather than from the horizon or the nadir parallel. If the quantities $(f \tan \theta)$ and $(f \cot \theta)$ are determined as constants for a given oblique photograph, then

$$d = f \tan \theta - y$$
$$e = f \cot \theta + y$$

where y is measured from the principal parallel. If the point is above (that is, toward the horizon the principal parallel) y is plus (+); if below, y is minus (-).

It should also be noted that all the elements of the formulae, except the measured distances, are constants which need be determined only once for a given photograph.

AERIAL PHOTOGRAPHIC INTERPRETATION OF URBAN LAND USE IN MADISON, WISCONSIN*

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THE technique of aerial photographic interpretation has much to contribute to the mapping of land use in urban areas.¹ Nine basic steps for the interpretation of aerial photographs have already been suggested.² The present paper is a description of this technique as applied to the study of urban land use in Madison, Wisconsin. Through the use of this technique, approximately 80% of the mapping of this city was carried out in the office (Figure 1). The task of interpretation was simple. The inventory was made rapidly and efficiently as comparative field checking revealed. By the use of similar techniques, urban studies can be made as simply, efficiently, and rapidly, wherever aerial photographic coverage is available throughout the world.

The field and office work on which this paper is based was carried out in the Madison area from December, 1947 to March, 1948. All identification keys developed for this study, therefore, apply particularly to that city, although they are applicable to many urban areas in the United States, and to some degree to all cities of the western world.³

STAGES IN AERIAL PHOTOGRAPHIC INTER-PRETATION OF MADISON

The seven stages used in this study were a modification of a technique already published.⁴

* Appreciation is expressed to Professor Kirk H. Stone in the Department of Geography at the University of Wisconsin for his helpful suggestions in conducting this study and in the preparation of the present paper.

¹ Aerial photographic interpretation is a research technique by which aerial photographs are studied with the naked eye and with a stereoscope in order to locate and to identify natural and cultural features of the landscape. By this means, recognition characteristics and distribution patterns are determined and identification keys are established.

² Stone, Kirk H., "Aerial Photographic Interpretation of Natural Vegetation in the Anchorage Area, Alaska," *Geographical Review*, July 1948, p. 466.

³ General interpretational keys relating to urban areas in the north-eastern part of the United States have already been published. See Russell, J. A., Foster, F. W., and McMurray, K. C., "Some Applications of Aerial Photographs to Geographic Inventory," *Papers Michigan Acad. of Sci.*, *Arts, and Letters*, Vol. 29, 1943, Ann Arbor, Mich., 1944, pp. 315–341.

⁴ Stone, K. H., op. cit.