VERTICAL EXAGGERATION IN STEREOSCOPIC MODELS*

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Introduction

MANY years ago it was found that two photographs could be taken of the same object from slightly separated points and viewed in a manner to give a three-dimensional model surface. This principle was applied in the parlor stereopticon known to our forefathers. In more recent years, stereoscopic vision has been applied to many fields. Today it is used in such diversified activities as x-ray, topographic mapping, geological and terrain studies, and motion pictures. The old parlor stereopticon has been revitalized in the form of stereoscopic cameras. In each instance, these usages have been to promote increased recognition, interpretation, and understanding of the object being viewed.

The fact that the three-dimensional models created have not been at true scale in all dimensions has been of little concern until recent years. With the increased interpretation uses for photography, the causes and effects of these scale variations have been subject to closer scrutiny. The scale error occurs in the axis parallel to the optical axis of the camera. The relative scale change has been referred to as exaggeration. Diverse opinions have been expressed as to the basic factors involved, with some writers doubtful of the existence of exaggeration in stereo-photography.

The purpose of this paper is to examine quantitatively the primary factors related to the three-dimensional mental image or stereoscopic model. Vertical air photography has been used in the following analyses for two reasons. First it is the type of photography of immediate concern in most of the fields of interpretation. Second, the variable scale of oblique photography is undesirable for a quantitative study of this type. This analysis will present data required for the practical application of visual estimation in dip or slope determination from stereoscopic models.

The author has worked closely with Victor C. Miller[1] and is in accord with the principles set forth in his qualitative evaluation of vertical exaggeration. It is of utmost importance, however, to understand which factors are significant from a quantitative sense.

Concepts of Vertical Exaggeration and Model Distortions

Vertical exaggeration is the change in a model surface created by proportionally raising the apparent height of all points above the base level while retaining the same base scale (Figure 1). This is a constant factor throughout the stereoscopic model. The amount of exaggeration can be recorded as the apparent height of a vertical unit distance divided by the apparent length of an equal horizontal distance or, as Miller expresses it, the ratio of vertical scale to horizontal scale. For values from 0 to 1 the resultant model is negative and for values greater than 1 it is positive.

Model distortions, in contrast, are the effects which are variable within the

* Read at Nineteenth Annual Meeting of the Society, Hotel Shoreham, Washington, D. C., January 14 to 16, 1953.

[1] See pages 592 to 607 for his paper. "Some Factors Causing Vertical Exaggeration and Slope Distortion on Aerial Photographs." This paper was prepared as a part of the report of the Photo Interpretation Committee when Mr. Miller was employed by Geophoto Services. He is now a partner in Miller-McCulloch of Calgary, Alberta.

stereoscopic pair. In some instances, they can be measured systematically but in other cases they are erratic and can be predicated only in general terms.

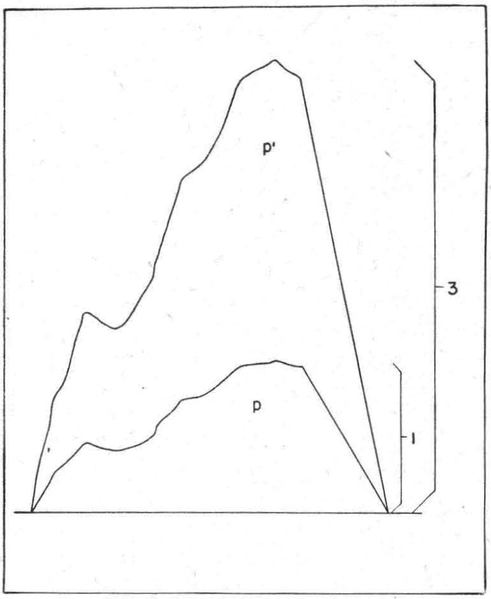

FIG. 1. The cross-section shows a true terrain profile (p) and the effect of three-fold vertical exaggeration (p').

QUANTITATIVE CONSIDERATIONS OF THE FACTORS AFFECTING THE STEREOSCOPIC MODEL

It is essential to know to what degree the stereoscopic model is affected by each of the various factors involved. Inasmuch as there is no instrument capable of measuring the mental impression created by a stereoscopic model, it is necessary to deal with some of these factors in generalities. By empirical testing it has been possible to arrive at certain conclusions about those factors which cannot be determined by analytical processes. The factors affecting the stereoscopic model are classified below with a brief description as to their magnitude. In view of the large number of variables it has been necessary to use constants in many instances. The factors form two major categories, vertical exaggeration and model distortion, which in turn may be subdivided into two main groups, photographic and stereoscopic.

VERTICAL EXAGGERATION

Stereoscopic causes of vertical exaggeration are:

a. *Viewing Distance.* As viewing distance increases the horizontal scale of the stereoscopic model decreases, while the vertical scale remains nearly constant with a resulting increase in vertical exaggeration. For this study, the viewing distance has been taken as a constant, though the author feels certain that values for changes in viewing distance can be established by further investigation.

b. *Separation of Photographs.* With practice it is possible to change the separation of photographs one to two inches and still retain a stereoscopic image. Even the experienced interpreter will notice that the extreme positions are a strain to the eyes and that a normal position is readily obtained for each type of stereoscope used. This separation is the same at all times for the individual. The amount is from five to ten millimeters less than the eye base when using simple lens stereoscopes. This direct relationship with the eye base allows photo separation to be treated as a constant when the proper viewing method is used.

c. *Eye Base.* The eye base is the interpupillary distance. Thirty persons were tested having interpupillary distances from 57 millimeters to 69.5 millimeters. Inasmuch as the majority had interpupillary distances from 60 to 65 millimeters, the extremes are not conclusive for analysis. Two results were established. The stereoscopic model is exaggerated most for persons with the narrowest eye base. This seems contradictory and the only explanation that the author can offer is that these persons are farthest from viewing the two photographs directly over their optical centers. The average interpupillary distance encountered was $62\frac{1}{2}$ millimeters and the average image separation on the photographs viewed was 55 millimeters. A change of 10 per cent in the average vertical exaggeration factors established later in this paper is recommended for every five millimeter change from the average eye base.

d. *Magnification.* Vertical exaggeration varies directly with magnification. The quantitative effect has not been analyzed because the pocket folding type stereoscope with four-inch, fixed focal length, two-power lenses was used in the preponderance of tests. Thus magnification was treated as a constant in these experiments.

Photographic causes of vertical exaggeration are:

a. *Photo Scale.* Photo scale is a function of altitude and focal length. Exaggeration varies directly with scale as the camera height varies, but is constant when scale varies with changing focal length at a given altitude. It is therefore better to consider these variables under the separate headings rather than as a function of photo scale.
b. *Altitude Above the Terrain.* Vertical exaggeration varies inversely with altitude when other factors remain constant.
c. *Air Base.* Vertical exaggeration varies directly with air base when other factors remain constant.

The three factors mentioned above are the primary considerations in vertical exaggeration. Focal length does not affect exaggeration. Inasmuch as scale does not present usable data when analyzed by itself, it is expedient to express it in terms of the altitude and air base. This relationship is commonly known as the base-height ratio. The method of evaluating these data will be discussed in detail later in the paper.

d. *Relief of Terrain.* For each unit of increased elevation within the stereoscopic model the vertical exaggeration increases slightly. Normally no correction factor need be applied. If the differential in elevation within the stereoscopic model exceeds 40 per cent of the flying height the error becomes sufficient that this method of evaluation should not be applied for the higher elevations.

MODEL DISTORTION

Photographic causes of model distortion are:

a. *Tilt.* Tilt in a stereo pair is immediately apparent to the interpreter when there is diverse drainage. If the photographs are twisted in an attempt to compensate for this tilt, local anomalies are created in the parallax values which are more severe than the tilt itself.[2] It is preferable to omit the use of such photographs, but minor amounts of tilt will not affect appreciably the values of slopes in excess of 5 degrees.
b. *Optics.* The camera lenses in use today are of such good quality that the minor distortions found in the shorter focal lengths are insignificant in their relationship to other factors.
c. *Position and Orientation of Image on the Photographs.* The radial displace-

[2] Nowicki, A. L., "Practical Applications of the Stereocomparagraph." MANUAL OF PHOTOGRAMMETRY, pp. 464–499, 1944.

ment of objects from the center of the true vertical photograph is of extreme importance to the interpreter in visual studies of the stereoscopic model. The amount of displacement increases proportionately as elevation above base level increases and also as distance from the photo center increases. This is treated in greater detail following Slope Estimations.

Stereoscopic causes of model distortion are:

a. *Rotation of Photographs.* The proper method of viewing a stereoscopic pair is with the eye base paralleling the azimuth line of the photographs. Improper rotation creates an artificially tilted surface.
b. *Viewing Position.* In examining any portion of the stereoscopic model, the correct method is to move the stereoscope so that the viewing plane is vertical to the horizontal plane of the photographs. The "bowl-like appearance," expressed by some writers, when viewing the stereoscopic models is merely the result of trying to examine the model with an inclined plane of view.

MEASURING VERTICAL EXAGGERATION IN TERMS OF BASE-HEIGHT RATIO

As already mentioned, there is no instrument for measuring the mental impression created by a stereoscopic model. Therefore, the problem has been to find an adequate means of interpreting the mental image. This has been achieved by the analysis of controlled experiments in visual estimation of the slopes or dips of plane surfaces viewed in stereoscopic models.

Every angle of inclination can be represented by its tangent factor (Figure 2a). If the vertical exaggeration in a model surface is 2×, the height is double

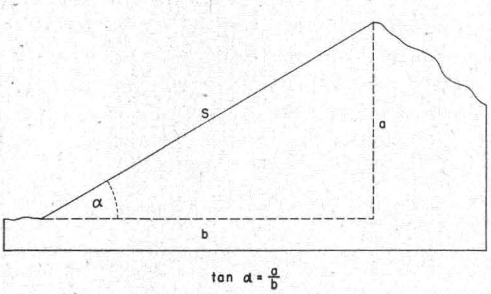

FIG. 2a. A cross section of the terrain, S representing a uniform topographic slope, or rock layer, with α the angle of slope or dip.

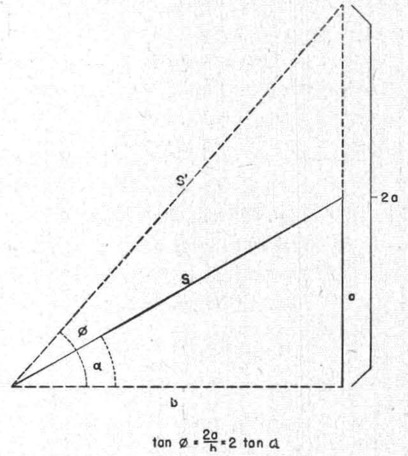

FIG. 2b. Two-fold vertical exaggeration changes slope S to apparent slope S' and angle α to ϕ in the stereoscopic model.

while the base distance remains constant (Figure 2b). The tangent value of the resultant angle of any inclination is doubled. Thus, the new value of any angle can be computed for every given amount of vertical exaggeration (Figure 3). The problem is resolved to determining the exaggeration factor. Victor C. Miller[3] suggests this be done by comparison with the appearance of known angles

[3] Rapid Dip Estimation in Photogeologic Reconnaissance," *Bull. Amer. Assoc. Petroleum Geol.*, Vol. 34, No. 8, August, 1950.

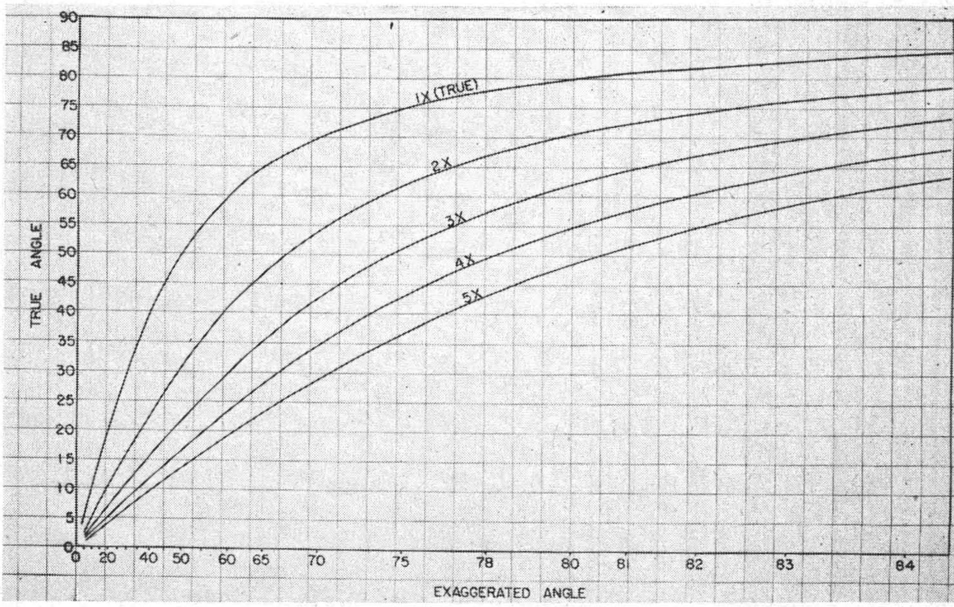

FIG. 3. A graphic representation of the change in values of true angles for different amounts of vertical exaggeration.

in each stereoscopic model. In reconnaissance studies, such as military terrain analysis or structural geologic evaluation, it is often impractical to obtain angles for such comparison. Each change in flying height and overlap demands additional comparison angles. Therefore, a direct method is deemed essential.

METHOD OF ANALYSIS

Studies were undertaken using plaster of paris blocks which were beveled at angles from 2 degrees through 60 degrees. These were photographed in the laboratory under varying conditions of height and base distances, to match normal conditions in air photography. The base-height ratios were computed for each stereoscopic pair. Subsequently, air photography was substituted in a few instances for verification of the data obtained.

A number of trained photo interpreters and untrained personnel estimated the apparent slopes they saw in different stereoscopic models. These models had different base-height ratios, but constant height. The angles estimated by each viewer for a given model were plotted separately against the true dip values (Figure 4). If the estimates gave a consistent exaggeration factor, the factor was plotted against the base-height ratio. Inconsistent factors were quickly traced to poor ability in estimation of angles. When the consistent factors were plotted, their averages formed a straight line function related to the base-height ratio (Figure 5). This experiment was repeated using models of different simulated flying heights with the same result. It, therefore, appears from these data that the individual variables of scale, altitude and air base can all be converted into one simple expression of base-height ratio for purposes of determining vertical exaggeration.

Figure 6 shows the base-height ratio for changes in overlap for different types of photography. For convenience of the interpreter, the data expressed in Figures 5 and 6 are combined in Figure 7. When the size of the print and the focal length of the camera are known, determination of the percentage of over-

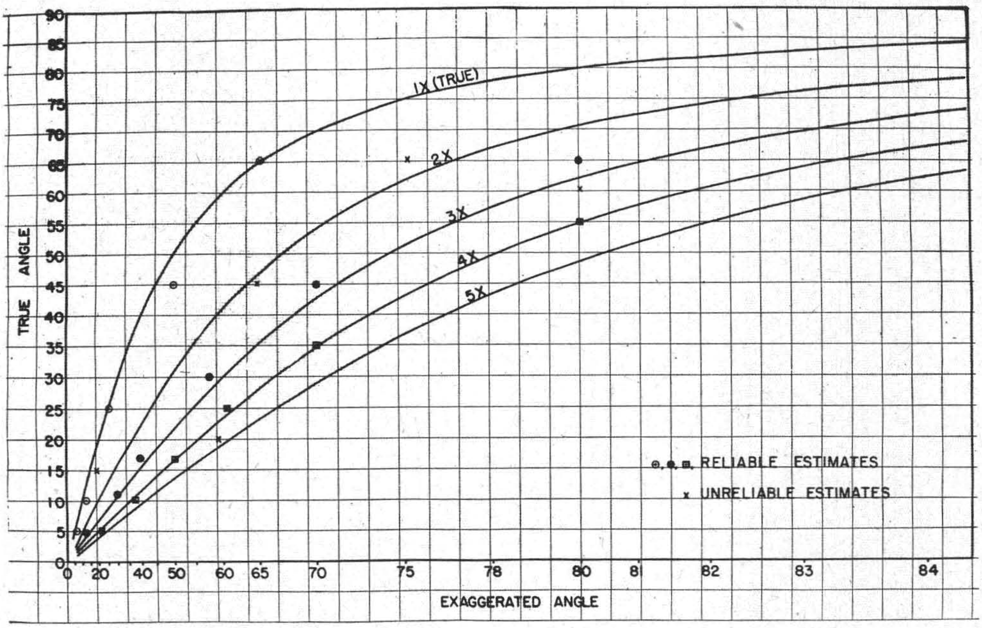

FIG. 4. A graph showing representative sample of testing for exaggeration factors. Each of the four symbol groups represents data from one individual's examination of one stereo pair with the estimated apparent (exaggerated) angle plotted against the measured angle of the plaster of paris model. The reliable estimates have consistent exaggeration factors of 1.05, 2.8 and 4.0.

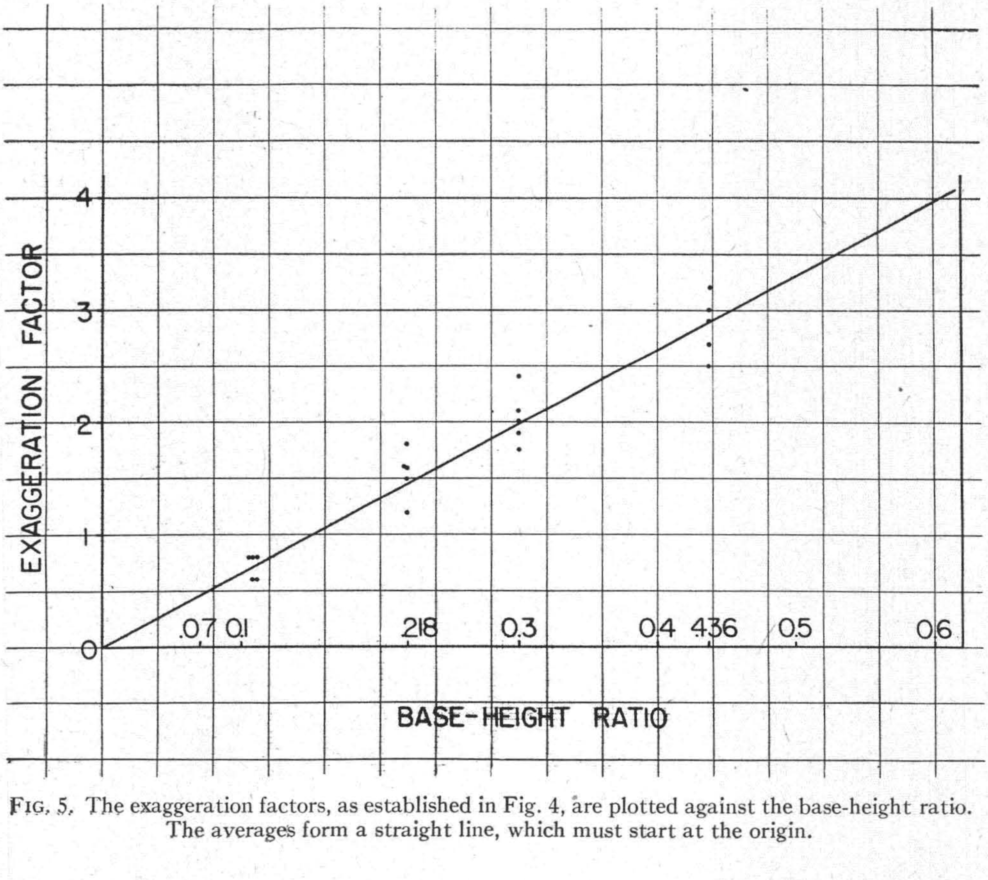

FIG. 5. The exaggeration factors, as established in Fig. 4, are plotted against the base-height ratio. The averages form a straight line, which must start at the origin.

lap allows direct reading of the average exaggeration factor. Applying this factor to Table 1 (a numerical listing of values from Figure 3) estimated slope or dip angles can be converted in the photo recorded angle. Inasmuch as there is a correction for the radial displacement of images on the photographs, the effect of this displacement must be considered before obtaining a true angle.

Without the benefit of the tracing table of stereoscopic plotting instruments, to compensate for changes in elevation, displaced points are viewed in a stereoscopic model in an incorrect position (Figure 9). These points are displaced radially from the midpoint between photo centers. Therefore, if the horizontal trace (strike) of a topographic slope or rock layer is radial from this midpoint,

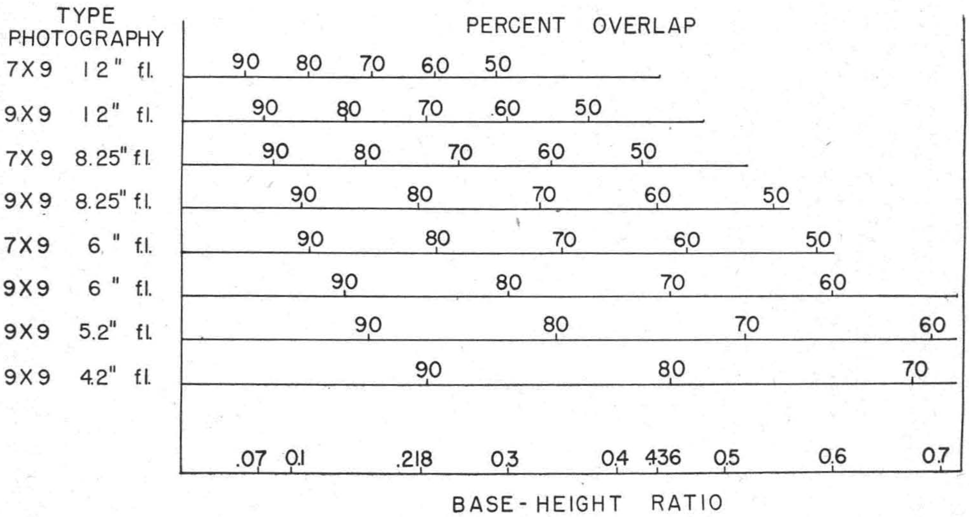

FIG. 6. The relationship of film size, overlap and focal length in terms of base-height ratio shows the wide range in values for "normal overlap photography."

all displacement is represented as skew and the slope angle is not distorted. If the horizontal trace is tangent to a circle centered at the midpoint the maximum error is encountered (Figure 8). The photo recorded angles of slopes toward the center of the photograph are less than the true angle and slopes away from the center are greater than the true angle. The amount of error is dependent upon four variables:

1. angle of the slope or dip
2. distance from the midpoint
3. camera focal length
4. horizontal trace direction relative to the radii.

Table 2 has been computed for an $8\frac{1}{4}''$ focal length camera to show the maximum corrections required based on the recorded angle obtained from Table 1 and the distance from the midpoint of the stereo pair. With shorter focal length cameras these corrections become larger, and conversely with longer focal length cameras the amount of correction decreases. It is important to note that in geologic evaluation, photo-distortion can make a normal sequence of steeply-dipping strata appear overturned.

EVALUATION OF THE METHOD

This method of determining the approximate angle of slopes or dips is dependent on the interpreter's ability to estimate the apparent angle on the stereo-

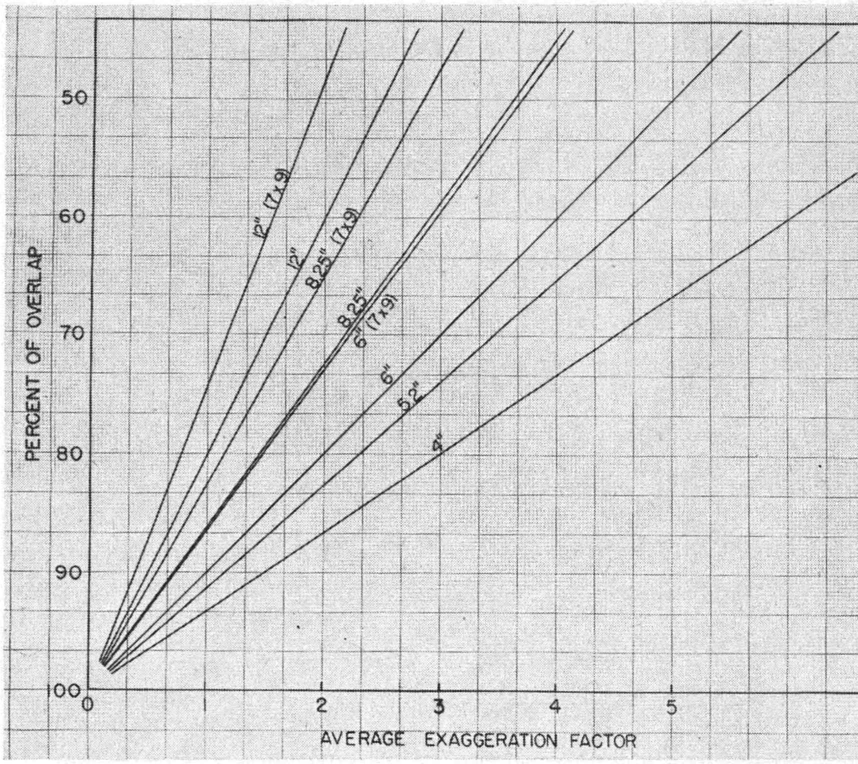

FIG. 7. To determine the exaggeration factor the focal length, print size, and overlap must be known. The factor derived is applied in Table 2. The slope or dip estimated from the stereo-pair is located under the column determined by Fig. 7 and the Recorded Dip is read.

TABLE 1

Recorded Dip	Estimated Dip When Vertical Exaggeration Factor Equals:						
	2	2½	3	3½	4	4½	5
0	0	0	0	0	0	0	0
½	1	1¼	1½	1¾	2	2¼	2½
1	2	2½	3	3½	4	4½	5
2	4	5	6	7	8	9	10
4	8	10	12	14	16	18	19
6	12	15	17	20	23	26	28
8	16	20	23	26	29	32	35
10	19	24	28	32	36	38	41
13	25	30	34	38	42	45	48
15	28	34	39	43	47	50	53
20	36	42	48	52	56	58	61
25	43	49	54	58	62	64	67
30	49	55	60	64	67	69	71
35	55	60	64	68	71	72	74
40	59	64	68	71	74	75	77
45	63	68	71	74	76	78	79
50	67	72	74	76	78	79	81
60	74	77	79	81	82	82	83
70	80	81	83	84	85	85	86
80	85	86	87	87	88	88	88
90	90	90	90	90	90	90	90

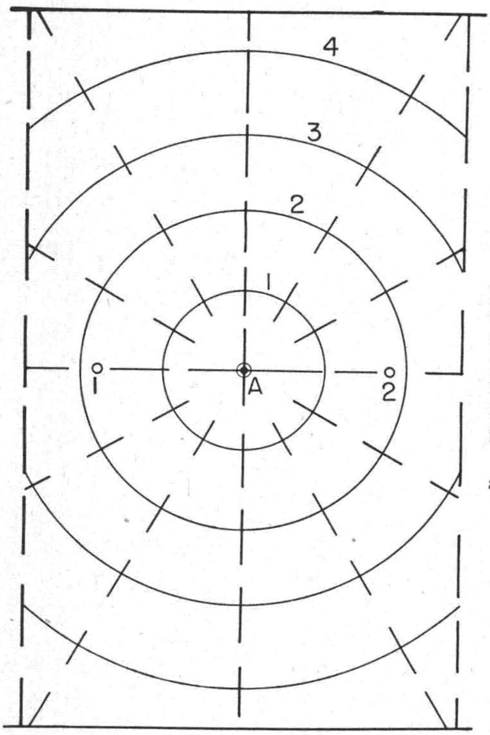

FIG. 8. Points 1 and 2 represent consecutive photo centers, with *A* the midpoint. When the slope or dip is radial from *A* (the horizontal trace or strike being perpendicular to the radial line) the maximum distortion is encountered. When the strike is radial from *A* there is no angle distortion from displacement and the recorded dip is the true dip or slope.

Table 2 indicates the amount of correction to apply for Fig. 8.

TABLE 2. MAXIMUM DISPLACEMENT CORRECTIONS OF PHOTO RECORDED ANGLES[1]

Recorded Dip	Distance from Stereo Center			
	1″	2″	3″	4″
0	0	0	0	0
10	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$
20	$\frac{1}{2}$	1	$1\frac{1}{2}$	2
30	1	2	4	5
40	2	4	6	8
50	3	5	8	11
60	4	7	11	14
70	4	8	13	15
80	5	9	*14*	*17*[2]
90	*5*	*10*	*14*	*18*

[1] *Add* when beds dip toward center. *Subtract* when beds dip away from center. Outcrops with strikes 60 degrees either side of radials from stereo center have an error equal to *one-half* the tabulated values.

[2] *Italicized* figures denote possible reversal of dip direction.

scopic model. Practice is essential in estimating angles from this perspective. The eventual degree of accuracy which might be anticipated is indicated by the breakdown of angles under the column of recorded dip in Table 1. Examination of these data shows that terrain slopes or dips greater than 45 degrees cannot be evaluated with reasonable accuracy. The small range of apparent slope and the amount of photo position error result in less accurate values. Terrain slopes and dips of less than 45 degrees can be determined within small tolerances.

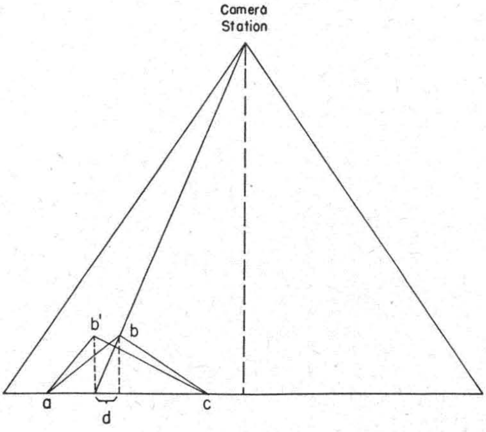

FIG. 9. In this cross-section, *ab* and *bc* represent terrain slopes. When photographed from the camera station *b* is displaced an amount equal to *d*. Viewed stereoptically point *b* is seen vertically exaggerated above *b'*. The photo recorded angles derived from Table 1 are represented in slopes *ab'* and *cb'* when the maximum error (see Fig. 8) is encountered.

CONCLUSIONS

This method of slope or dip determination is currently in use in geological mapping for petroleum exploration. It should have significant application in first phase military terrain studies. An indirect method of determining the heights of objects in a stereo pair is afforded by the use of estimated angles and measured photo base distance in their trigonometric relationship.

Vertical exaggeration can be measured in mathematic values which are systematic for all interpreters. The psychological effects expressed by some writers are now outmoded. The stumbling block that remains is the difficulty for the interpreter to estimate with a high degree of accuracy the apparent angles viewed in a stereoscopic model. A further approach to this problem is opened by the consistency of the results established in these experiments. Parallax varies inversely with height above the terrain and directly with the air base. A simplified method of reading the parallax and apparent slope in the stereoscopic model could conceivably be devised to apply directly with the vertical exaggeration factor.*

* Following the reading of this paper, the following question was asked by Mr. H. Weiner of N.P.I.C.: "Does vertical exaggeration vary in different types of stereoscopes, thereby affecting the estimate of a specific slope?" Mr. Thurrell's reply was "Yes. Magnification and viewing distance (eye to photo) are variable functions affecting exaggeration. V. C. Miller's paper discusses them. In the quantitative studies these were constant, as the two power, four and one-half inch focal length pocket folding stereoscope was used. Referring to Figure 5, the plotted line will always be straight and pass through the origin, but will form different angles with the coordinates as the variable functions change."