# SOME FACTORS CAUSING VERTICAL EXAGGERATION AND SLOPE DISTORTION ON AERIAL PHOTOGRAPHS* 

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#### Abstract

When two overlapping aerial photographs are viewed stereoscopically, the vertical scale of the terrain model seen is rarely the same as the horizontal scale; the model is vertically exaggerated, usually with the vertical scale exceeding the horizontal. In addition, all topographic features are not mentally displaced vertically alone, but also in a horizontal direction, depending on the position of the stereoscope. This is termed model "distortion."

Vertical exaggeration is considered the product of the photographic variablesfocal length, camera height, air base, relief, tilt, and optical imperfections; vertical exaggeration also varies with the stereoscopic factors-viewing distance, photograph separation, eye base, and photograph rotation. Image distortion is largely dependent upon the position of the stereoscope. The entire paper treats the qualitative aspects of these variables. Though not substantiated by quantitative data, it is felt that the general magnitude of vertical exaggeration is largely dependent on the photographic variables and that the stereoscopic variables introduce minor differences in exaggeration.

All accompanying illustrations are diagrammatic. At the end of the paper the author presents several suggestions for improved stereoscopic examination of aerial photographs. He also includes a new graph relating vertical exaggeration, true ground slope, apparent (stereoscopic model) slope; this graph is intended primarily for the photogeologist who must estimate dips seen on aerial photographs.


## Introduction

IN A recent paper (Miller, 1950) the author presented a method for obtaining an accurate yet rapid estimate of slope or dip, through the estimation of apparent slope or dip, as seen stereoscopically on aerial photographs, and the application of a correction scale. The amount of vertical exaggeration, assumed to be constant for a given pair or set of aerial photographs, determined the amount of correction necessary in a given case.

Since the time of that publication considerable thought has been given to the factors which influence vertical exaggeration. The results of this later consideration are presented in this paper.

Slope distortion is closely related to, but in some ways independent of, vertical exaggeration. If all image exaggerations were vertical only, there would be a simple tangent relation between slope and exaggeration. This, essentially, is the assumption on which the author's previous
paper was based, but the variables considered below show that no such simple relation exists. Many photogeologists, or at least those geologists who only on occasion look at aerial photographs, may not be aware of the errors introduced by the improper (though common) use of the stereoscope.

## What is Vertical Exaggeration?

When one looks at overlapping vertical aerial photographs through a refraction stereoscope, he sees a three-dimensional model whose vertical scale exceeds the horizontal. Such a model, or mental image, is said to be vertically exaggerated. Quantitatively, vertical exaggeration may be defined as the ratio of vertical scale to horizontal scale.

Vertical exaggeration has been discussed or mentioned in many articles and books on photogrammetry and photogeology. When illustrated graphically, the subject is usually depicted by diagrams

[^0]similar to Figure 1. As Figures 1a, b, c and d, stand, they are geometrically sound, but one point must be made; in Figures 1b, c and d, the eyes of the viewer are pictured as being directly above the principal points (centers) of the photographs. If this were the case in actual practice, problems of

vertical exaggeration would be greatly simplified.

Under such viewing conditions, exaggeration would simply be a function of eye-to-photograph distance, and if this distance be equal to the focal length of the camera that took the photographs, vertical

Fig. 1. An illustration of the commonly used method of depicting overlapping aerial photography and various geometric relations associated with stereoscopic examination of aerial photographs.
A) Two cameras, located at $L_{1}$ and $L_{2}$, are shown photographing a 100 -foot pole and a 100 -foot ground distance.
B) The photographs taken in Figure 1A are viewed stereoscopically from eye positions $E_{1}$ and $E_{2}$. Vertical exaggeration is 1.50 .
C) The photographs taken in Figure 1A are viewed stereoscopically from eye positions $E_{1}$ and $E_{2}$. Vertical exaggeration is 0.65 .
D) The photographs are shown here again being viewed from the same distance as in $C$, but now the separation of the photographs is greater. Vertical exaggeration is still 0.65 .
scale and horizontal scale would be the same; the solid model seen would have true three-dimensional scale. By moving the eyes farther from the photographs, a positively exaggerated image of the model would be obtained. (Figure 1b; Vertical Exaggeration 1.50), and by decreasing eye-to-photograph distance, a negative exaggeration would be produced. (Figure 1c; Vertical Exaggeration 0.65.)

Under these conditions the separation of the photographs would have no bearing on vertical exaggeration. Figure 1d shows the geometric relations existing when eye-to-photograph distance is the same as in Figure 1c, but with greater photograph separation. It will be noted that vertical exaggeration is the same in both instances ( 0.65 ). This is true because in both 1 c and 1d all angular relations are the same, and all corresponding triangles involved are similar.

It has been the author's observation that in photogeologic study the refraction, or lens, stereoscope is the type most commonly used. It is unfortunate that the above relations, invitingly simple though they may be, do not apply when the refraction stereoscope is employed. Two aerial photographs (usual dimensions: 9 $\times 9$ inches or $9 \times 7$ inches) cannot be viewed through a refraction stereoscope in such a way that the eyepieces stand directly over the photograph centers.

However, by the use of a mirror stereoscope, it is possible to look directly down on the centers of the photographs, but in practice the photographs or the instrument are generally placed so that the observation point is at some chosen portion of the photographs, regardless of image position relative to the centers. Therefore, even when a mirror stereoscope is used, a photograph is rarely viewed from a position over the photo centers, and as a result the geometry of Figures 1 b, c and d, again does not apply.

The proper method of viewing a stereo pair with a refraction stereoscope consists of setting the lenses of the stereoscope at a proper separation (determined by measuring one's eye base) and placing the instrument over the photographs, which are spaced at an optimum separation. The photographs are not side by side, but partially overlapping, with identical images separated about 2 inches. This separation may not be optimum for all persons or for
all stereoscopes, but many measurements made by the author, using the Fairchild Model C-2 stereoscope, vary inconsiderably from the 2 -inch figure.

Figure 2 shows the relation between camera lenses at the time of photography and eye positions at the time of stereoscopic examination. (Note should be made of the fact that in this figure, and all that follow, actual conditions or dimensions of stereoscopy or photography are not attempted, but merely the various geometric principles involved.) The height $A B$, of the object photographed, is the same as the ground distance $A C$. Point $X$ is another ground point on the level of points $A$ and


Fig. 2. An illustration of the usual relation between camera lenses and stereoscope position. In this figure, vertical exaggeration is 2.65 .
C. These points are photographed from lens positions $L_{1}$, and $L_{2}$, and appear on the resulting prints at points $a_{1}, b_{1}, c_{1}, x_{1}$, and $a_{2}, b_{2}, c_{2}$, and $x_{2}$, respectively. When these photographs are arranged for stereoscopic study, the instrument is placed over them so that the viewer's effective eye positions are at $E_{1}$, and $E_{2}$. The left eye, at $E_{1}$, sees images $a_{1}, b_{1}, c_{1}$, and $x_{1}$, while the right eye, at $E_{2}$, sees images $a_{2}, b_{2}, c_{2}$, and $x_{2}$. The projected model, identified by points $A_{1}, B_{1}, C_{1}$, and $X_{1}$, is seen.
The unnatural vertical elongation of the object, $A_{1} B_{1}$, is striking. To determine vertical exaggeration $A B$ is not compared with $A_{1} B_{1}$, but $A_{1} B_{1}$ is compared with
$A_{1} C_{1}$. These two distances were originally equal, and vertical exaggeration, as described above, is the ratio of vertical scale to horizontal scale. Therefore, $A_{1} B_{1}$ and $A_{1} C_{1}$, representing identical distances, may be compared directly to determine vertical exaggeration. $A B$ measures 2.65 times as large as $A_{1} C_{1}$, therefore vertical exaggeration in this case is 2.65 .

The point $X$ was photographed from the left from $L_{1}$, and from the right from $L_{2}$. With the stereoscope in the position shown, however, $x_{1}$ and $x_{2}$ are both seen from the left hand side. The author believes this fact to be highly significant because the terrain at and around $X$ cannot be correctly restituted in the stereo image if one of the viewer's eyes looks at it from a direction opposite to that from which it was photographed. This incorrect viewing position is the cause of distortion, as distinct from vertical exaggeration. Actually the vertical exaggeration at and around $X$ would be the same as that in the vicinity of the object $A B$, because the purely vertical element is a function of parallax displacement, but a stereoscopic model also depends on the manner in which the photographs are viewed. While $E_{1}$ and $E_{2}$ roughly approximate directionally $L_{1}$ and $L_{2}$ with respect to the photograph images of $A, B$ and $C$, they are out of relationship with respect to $X$. Therefore, if $X$ were to be correctly viewed it would be necessary to move the stereoscope to the right, and thus duplicate the original directional relations from $L_{1}$ and $L_{2}$ to $X$.

## Variables

Aerial photographs are subject to many variable factors which directly influence the three-dimensional impression when the terrain is studied stereoscopically. Some of these variables involve the geometry existing at the time the photographs were exposed; others are concerned with the manner in which the photographs are viewed. These variables may be called, respectively, "Photographic" and "Stereoscopic."

## Photographic variables include:

1) Scale; this in turn is a function of
a) Camera focal length, and
b) Camera height, at the time of photography.
2) Air Base; the horizontal distance be-
tween two consecutive photograph exposures.
3) Relief; both over-all relief and differences in relief from one area to another.
4) Tilt; a complex and complicating factor which, due to the limited scope of this paper, is omitted in the following discussion.
5) Optics; optical imperfections of modern lenses are so minor that their effect on vertical exaggeration would be too slight to be given consideration here.

## Stereoscopic variables include:

1) Eye-to-photograph distance; the actual distance, if the photographs are viewed without a stereoscope; the effective distance if lenses are used.
2) Photograph separation; the photographs may be moved closer together or farther apart.
3) Photograph rotation; the rotation of one or both photographs ayay from proper alignment.
4) Movement of the stereoscope; a change in the position of the stereoscope, or one's eyes, in a plane parallel to that of the photographs.
5) Eye base; the separation of one's eyes. (This of course is constant for any one individual, but different for different persòns.)

## Discussion of Variables

Let us consider these various factors individually, and attempt to picture diagrammatically how stereoscopy is affected by each.
To be understood fully, scale must be considered to be a function of focal length and camera height, both of which are variable. Figure 3 ( a and b) shows how photograph scale may be kept constant though both focal length ( $f_{1}$ and $f_{2}$ ) and camera height ( $H_{1}$ and $H_{2}$ ) are changed. Air bases, $L_{1} L_{2}$ and $L_{3} L_{4}$, are equal, and the pyramid photographed, $A B C$, is the same in both cases. The sides of the pyramid slope 30 degrees. It will be noted that $a_{1} c_{1}$, the image of the pyramid base as photographed from camera position $L_{1}$, is equal to $a_{3} c_{3}$, that taken from $L_{3}$. Images $a_{2} c_{2}$ and $a_{4} c_{4}$ are also of equal size. However, $a_{1} b_{1}, b_{1} c_{1}$, $a_{2} b_{2}$, and $b_{2} c_{2}$ are not equal in size to the corresponding images, $a_{3} b_{3}, b_{3} c_{3}, a_{4} b_{4}$ and $b_{4} c_{4}$. In other words, there is a difference in the


Fig. 3. A) Pyramid $A B C$ is photographed from camera positions $L_{1}$ and $L_{2}$.
B) The same pyramid is photographed from camera positions $L_{3}$ and $L_{4}$. Compare with Figure 3A.
parallax displacements on the two sets of photographs, and as a result they will not give the same three-dimensional image when viewed beneath the stereoscope.

Figure 4 (a and b) shows how vertical exaggeration is influenced by the above noted differences in the two sets of photographs, both having the same scale. For a proper comparison, the conditions of stereoscopy are kept constant for both stereo pairs; eye-to-photograph distance, eye base, and photograph separation are the same. Model A.B.C. (V.E. 1.79) appears considerably more exaggerated vertically than does Model $A B C$ (V.E. 1.39). It is therefore apparent that if scale be kept constant, by the maintenance of a constant $f / H$ ratio, vertical exaggeration will vary as an inverse function of camera height and focal length.

A change in camera height, or focal length, if not accompanied by a corresponding change in the other, will of course cause a change in photograph scale. Figure 5 illustrates photography in which the focal length, $f$, is the same as that shown in Figure 3a, while the camera height, $H$, is equal to that shown in Figure 3b. The scale of the photographs in Figure 5 is smaller than that of Figures 3a and 3b. The geometry of Figure 5, however, is similar to Figure 3b; the ratio of $a_{5} b_{5}$ to $b_{5} c_{5}$, for example, is the same as the ratio of $a_{3} b_{3}$ to $b_{3} c_{3}$. A change in scale due to a change in focal length, therefore, produces the same
results as does photographic enlargement or reduction of a single negative, whereas a change in scale due to a change in camera height produces different relative parallax displacements which cannot be duplicated by projection in the photographic laboratory.

When the photographs taken in Figure 5 are placed beneath the stereoscope they appear as shown in Figure 6; again the conditions of stereoscopy are the same as before. If Figure 6 (V.E. 1.66) is compared with Figure 4b (V.E. 1.39), it will be noted that, other factors being equal, vertical exaggeration is greater when photograph scale, due to smaller focal length, is smaller. Vertical exaggeration, then, varies inversely with photograph scale, when scale is a function of focal length.

Though there is but a slight difference in the vertical exaggerations of Figures 6 (V.E. 1.66) and 4a (V.E. 1.79), it should be noted that if a greater difference in camera height had been chosen for illustration, it would be much more evident that with a decrease in scale, due to an increase in camera height, there is a corresponding decrease in vertical exaggeration. In other words, vertical exaggeration varies directly with scale when scale depends on the camera height. This is true because with increased camera height there is a decrease of relative parallax displacement caused by relief.
Figure 7 shows the conditions of photog-


Fig. 4. A) The photographs taken from camera positions $L_{1}$ and $L_{2}$ in Figure 3A are viewed stereoscopically from eye positions $E_{1}$ and $E_{2}$. Vertical exaggeration is 1.79. This figure should be compared with Figure 4B.
B) The photographs taken from camera positions $L_{3}$ and $L_{4}$ in Figure 3B are viewed stereoscopically from eye positions $E_{3}$ and $E_{4}$. Vertical exaggeration is 1.39 .
raphy when there is a smaller air base than that shown in the previous figures. This will result in greater overlap, when the same photographic equipment is used. This figure may be compared with Figure 3a, because both have the same focal length and camera height, and hence the same scale. Both cameras, $L_{7}$ and $L_{8}$, in Figure 7 "look down" more directly on the pyramid than did the cameras shown in Figure 3a. When these new photographs are viewed, again under the same conditions of stereoscopy as before, the model seen is much less exaggerated vertically (Figure 8). Compare Figures 8 (V.E. 1.22) and 4a (V.E. 1.79). It then follows that vertical exaggeration varies as a direct function of air base. (Or inversely with overlap.)
R. F. Thurrell, Jr. ${ }^{2}$ is at present carrying on a quantitative investigation of the interrelation of base-height ratio and vertical exaggeration. In his work, Thurrell
considers the actual air base, focal length, camera height, and overlap encountered in aerial photography. His conclusions, being of a quantitative nature, are expected to supplement the qualitative aspect of the present paper. ${ }^{3}$

The relative amount of relief in an area is also a factor that must be considered. In Figure 7, points $X$ and $Y$ stand at equal vertical intervals above point $B$. These intervals are equal to the height of $B$ above the base of the pyramid.
In Figure 8 the stereoscopic images of points $X$ and $Y\left(X_{4}, Y_{4}\right)$ appear to be almost equally spaced, but actually the vertical image $B_{4}-X_{4}$ is shorter than the height of the pyramid and longer than the image $X_{4}-Y_{4}$.
It might be concluded from the above that the higher elevations shown on any given stereo-pair will produce a slightly smaller vertical exaggeration than will

[^1]${ }^{3}$ See page 579.


Fig. 5. The photographs shown here are being taken with a camera whose focal length is $f_{1}$ (the same as in Figure 3A), from a height of $\mathrm{H}_{2}$ (the same as in Figure 3B).

Fig. 6. The photographs exposed in Figure 5 are shown being viewed from eye positions $E_{5}$ and $E_{6}$ under stereoscopic conditions identical to those shown in Figure 4. Vertical exaggeration is 1.66.


FIG. 7. In this figure air base is smaller than that shown in Figure 3 A ; focal length and camera height are the same. Points $X$ and $Y$ are equally spaced vertically above point $B$; the vertical intervals $B X$ and $X Y$ are equal to the height of the pyramid.


Fig. 8. The photographs taken in Figure 7 are shown being viewed from eye positions $E_{7}$ and $E_{8}$. Vertical exaggeration of the pyramid is 1.22 . The image height of the pyramid is greater than the image interval $B_{4} X_{4}$, which in turn is greater than $X_{4} Y_{4}$.


Fig. 9. Photograph images $X_{7}, Y_{7}, X_{8}$, and $Y_{8}$ are shown being viewed from eye positions $E_{7}$ and $E_{8}$, as in Figure 8. Image separation $X_{7}-X_{8}$ is the same as $a_{7}-a_{8}$ in Figure 8.
those points at lower elevations. This will be true if both high and low topographic points are studied stereoscopically with the photo separation being the same at all times.

Figure 9 shows the viewing of points $X$ and $Y$ with the photos separated to a position in which the interval from $x_{7}$ to $x_{8}$ is equal to the original distance $a_{7}-a_{8}$ shown in Figure 8. Since other conditions of stereoscopy have not been changed between Figures 8 and 9, a comparison between the stereoscopic image of the height of the pyramid in Figure 8 and image $X_{4}-Y_{4}$ in Figure 9 will show the relative vertical exaggerations produced when high and low topographic points are individually studied with identical image separation.

It will be noted that the stereoscopic image distance $X_{4}-Y_{4}$ in Figure 9 is almost twice the height of the image of the pyramid in Figure 8. It may therefore be concluded from this illustration that the vertical exaggeration of high topographic points is greater than the vertical exagger-
ation of low topographic points provided that the high and low areas studied stereoscopically are each viewed with identical photo-image separation.

The eye base (distance between $E_{9}$ and $E_{10}$ ) in Figure 10 is greater than the eye base in Figure 8, even though all other conditions of stereoscopy, including the photographs beneath the stereoscope, are the same. The vertical exaggeration ( 0.86 ) of the model seen in the case of Figure 10, is considerably less than that shown in Figure 8 (V.E. 1.22), where the eye base is smaller. To some this comparison may not seem sound, as it is difficult for one individual to comprehend the optical perception of another individual. However, the geometry of the comparison should be obvious. An individual with a relatively small eye base does not see a solid object in exactly the same way as another, with a larger eye base, In this instance the two individuals see images from two different sets of perspective points, and it would be expected that their mental images or stereoscopic models should not be identical. Recent quantitative tests, carried on by the author, substantiate the above statements; anaglyphs were used in the testing of relative vertical exaggerations seen by


FIg. 10. The photographs illustrated in Figure 8 are shown here being viewed by a person with a greater eye base. Other stereoscopic conditions are the same. Vertical exaggeration is 0.86 .
persons with different eye bases. Eye bases of from 57 mm . to 69.5 mm . were tested, and results were very satisfying.

Focal length, camera height, air base and relief, are all variables, but at the time a photograph is exposed, the results of these factors are "frozen" in the photographic images. After exposure, they are no longer variable. Eye base of course is constant for any one individual.

There now remain to be considered those factors which may be varied by the observer, with or without his conscious knowledge. When a refraction stereoscope is used a constant eye-to-photograph distance is maintained. However, when different stereoscopes are used or the photographs are viewed without a stereoscope, as is frequently the practice in the field, the distance from the eyes to the photographs may be increased or decreased while a stereoscopic model is still maintained in the mind.

There is some disagreement as to how the vertical dimension of the model changes when this viewing distance is changed. Some state that with increased viewing distance there is an increase in vertical exaggeration. On the other hand there are those who maintain that changes in viewing distance have no such effect. Salzman (1950), for example, has taken this latter view. The author believes that there is very definitely a direct relation between viewing distance and vertical exaggeration. With increased distance both vertical and horizontal model dimensions change, and the ratio of vertical scale to horizontal scale, which is the expression of vertical exaggeration, is increased. Compare Figure 11 with Figure 8. Both show the same eye base, photo separation, and the same photo images. The stereoscopic model produced in Figure 11, when viewing distance is greater, is much more exaggerated vertically. (Vertical exaggeration in Figure 11 is 1.87 ; in Figure 8 it is 1.22.)
Photograph separation has been mentioned several times in this paper. How does this factor influence the stereoscopic model? In Figure 12 (V.E. 1.39) the photographs of Figure 11 (V.E. 1.87) are shown being viewed with a smaller photograph separation.
It will be noted that the greater the separation, the greater the vertical exaggeration. (Figures 1c and 1d, which are not representative of actual viewing conditions,
would give the false impression that changes in separation are of no consequence.)
This particular fact bears strongly on the practice of estimating slope and dip without resorting to photogrammetric measurements. Since photograph separation affects the viewer's impression of vertical exaggeration, it is important to keep separation constant in order to maintain constant exaggeration. With practice, it can be ascertained that there is a definite photograph separation that "fits" the viewer's eyes best. At this separation, the stereoscopic model has a minimum, or lack of, eye strain. Once this stage has been reached a few times, it will become almost automatic to arrange the photographs at this separation. For the beginner, it might be advisable to devise some sort of template or convenient scale that might easily be slipped over the photographs to check on the consistency of separation.

Another closely associated type of photograph movement is that of rotation. Many times it is possible to see a stereoscopic model even when one or both photographs are slightly rotated out of proper alignment. Most texts on photogeology and many on elementary photogrammetric methods treat the proper method of mounting photographs for stereoscopic study, so a discussion of that particular subject will not be included here. What is significant in this respect is that, in addition to the actual change in orientation, there is a variance in photograph separation from point to point, and therefore a change in vertical exaggeration within the same stereo pair. This change may not be very great, but is nevertheless worthy of mention.

The final variable, and it is of prime importance, is the position of the stereoscope over the photographs., When a stereo pair is studied the viewer "looks down" on the terrain of the three dimensional model. It is common practice to move the stereoscope over the photographs and to view each locality from a position directly overhead, rather than leaving the stereoscope in one position and seeing the entire area of overlap by oblique viewing.

The direction in which a certain feature is viewed determines the form of the model which is seen. By moving the stereoscope from side to side, after obtaining a stereoscopic model, it is possible to note the


FIG. 11. This figure should be compared with Figure 8; both are identical except for the factor of viewing distance. Vertical exaggeration is 1.87 .
changes which take place in the model; slopes facing the direction of movement will become steeper, while those facing away from the direction of movement will become more elongate and gentle. Hill tops and ridge crests will "follow" the stereoscope as it is moved.

Similar distortion occurs when the stereoscope is moved toward or away from the observer.

Compare Figure 13 with Figure 12. They are identical in that they represent an observer with a given eye base, at a constant distance from the photographs, looking at the same photographs which in both cases have the same separation. The difference is that in Figure 13 the eyes had been moved to the right. Whereas Figure


FIG. 12. This figure should be compared with Figure 11; the only difference between them is one of photograph separation. Vertical exaggeration is 1.39 .

12 produced a symmetric model form, Figure 13 makes the hill quite asymmetric.

The great majority of objects photographed must appear in positions laterally marginal from the central portion of the overlap zone, toward the far or near edges of the photographs. Figure 14 illustrates the photography of the pyramid by a plane which is flying not parallel to the page, but normal to it, toward or away from the reader. Therefore two consecutive camera positions, $L_{21}$, and $L_{22}$, are designated by the same point, and the two prints, converging rays, etc., are all indicated by the single diagram.

Figure 15 diagrammatically shows the position of such images on overlapping


Fig. 13. The photographs depicted in Figure 12 are here shown being viewed from different eye positions (Eyes have been moved to the right).
prints. When such areas are being viewed, it is not possible to merely put the stereoscope over this part of the photographs and "look down," and still expect to get a realistic mental model of the terrain. The camera was "looking out" at this area, and therefore the viewer must move his stereoscope (or his eyes) a little nearer the central portion of the photograph; he must "back off" to get a more nearly true mental picture, and look toward the marginal zone.

Figure 16a shows the effect of looking straight down on the images, and Figure 16b shows how the model is improved by looking at this part of the photograph from an oblique position. The vertical exaggeration mentioned earlier will of course not be eliminated by moving the eyes or stereoscope about in this manner,
but at least the severe distortion of slopes that may be superimposed on exaggeration may thereby be minimized or completely removed.

## Summary of the Effect of the Independent Variables

The foregoing illustrations and comparisons have brought out the following general relationships:

## Vertical exaggeration varies;

a) inversely with camera height, when scale is constant
b) inversely with focal length, when scale is constant
c) inversely with scale, when focal length varies
d) directly with scale, when camera height varies


FIg. 14. The air base in this figure is normal to the page. Note that the pyramid is near the margin of photographic coverage.
e) directly with air base
f) inversely with eye base
g) inversely with ground elevation, when constant photo separation is maintained; directly with ground elevation, when constant image separation is maintained.
h) directly with viewing distance
i) directly with photograph separation


Suggestions for Better Stereoscopic Study

The author suggests the following stereoscope practices:

1. By whatever method the individual finds best for his work, he should maintain a constant image separation.
2. He should prevent photograph rota-


Fig. 15. The photographs taken in Figure 14 are here shown arranged for stereoscopic examination.


Fig. 16. A) The photographs taken in Figure 14 are shown viewed stereoscopically, with the observer's eyes directly over the images of the pyramid. Compare with Figure 16B.
B) The same photographs are viewed from a position farther to the left, that is nearer the centers of the photographs.
tion by following sound standard methods of photograph alignment.
3. The use of a single stereoscope removes the possibility of a change in viewing distance. If he uses different instruments, or works part time without a stereoscope, he should be well aware of the effect of changing his viewing distance.
4. When viewing ground objects which were apparently midway between the two camera stations, he should place the stereoscope so that each eye "looks in," about equally, toward the photographic images.
5. When viewing ground objects which appear near the center of, say, the left hand photograph, and extreme left marginal zone of the right hand photograph, he should place his stereoscope so that his left eye "looks down," while the right eye "looks in" at a definite angle. This may not be most comfortable. but it will help to eliminate slope distortion. If exact slope or dip determination is not a prime requisite, he may not go too far wrong in looking down from the most convenient position to study the topographic or geologic details in this area.
6. When studying ground objects which
appear on laterally marginal portions of both photographs, he should keep his stereoscope a bit nearer the central strip of the photographs, and look obliquely at these marginal areas. During this same time, he should note whether or not those objects are midway between photograph centers, or more nearly opposite the one or the other, and then modify his direction of viewing as in $\# 5$ above.

These practices should effectively cut down much of one's "self-inflicted" distortions of slopes and topography. As pointed out above, vertical exaggeration cannot in this way be eliminated, but if distortion is done away with, essentially, then the desired tangent relation between vertical exaggeration and slope and dip exaggeration will more nearly be brought about, and the principle proposed in the author's previous paper will be much more applicable.

An improvement has been made on the original graph presented in that earlier paper. The new graph, Figure 17 is more legible and much more workable. It is here presented in the hope that it will be of some use to those who are not in a position to apply photogrammetry in their


Fig. 17. This graph is essentially similar to the one published in the November 1950 AAPG Bulletin. The author feels, however, that it is much more useful and accurate, especially in the low dip (or slope) range.

Vertical exaggeration is shown by the concentric arcs, which represent exaggerations of 1.0 , $1.5,2.0,2.5$, and 3.0. The apparent slope or dip, that seen through the stereoscope, is shown on the uniformly divided inner scale. The true dips or slopes appear as the curved lines which intersect the arcs representing exaggeration.

The graph may be read in two ways:

1) If the amount of vertical exaggeration is known to be 2.5 , for example, and a certain slope appears to be $60^{\circ}$, the radial line from the $60^{\circ}$ point on the inner scale is traced outward until it intersects the 2.5 exaggeration line. The position of this point of intersection is then noted with respect to the curved true slope lines. In this case, a value of a little over $34^{\circ}$ is found to be the true slope.
2) If a known slope of $26^{\circ}$ appears as one of say $44^{\circ}$ on the photographs, the radial $44^{\circ}$ line is traced to its intersection with the curved $26^{\circ}$ line. The position of the point determined by this intersection is noted with respect to the arcs representing exaggeration, in this case 2.0 .
geological study of aerial photographs, but who wish a rapid, yet simple, way to rectify slope and dip estimation.
In conclusion, it is well to repeat that the illustrations accompanying this paper are intended to depict the principles, and not actual conditions of photography and stereoscopy; also, the magnitude of variation in vertical exaggeration and distortion produced by each variable discussed may or may not be significant. Each variable
must be subjected to an intensive quantitative test before its relative importance can be known.

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# AN EQUATION FOR APPROXIMATING THE VERTICAL EXAGGERATION RATIO OF A STEREOSCOPIC VIEW 

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$A^{T}$T THE VII International Congress of Photogrammetry, attention was called to the interesting subject of the stereoscopic estimation of dip-slope angles by the photogeologist. One of the important factors involved in estimating dip angles is the amount any given stereoscope exaggerates the vertical scale over the horizontal.
This vertical exaggeration of the stereoscopic view need only be determined approximately. However, up to the present time, the author has not seen any mathematical expression of the vertical exaggeration ratio, although several articles on the subject have appeared recently in Photogrammetric Engineering.
The purpose of this paper is to offer such an equation and, at the same time, to substantiate it with sufficient proofs to at least invite its further discussion.
The geometry in this paper is limited to that concerned with truly vertical air photographs; also, for simplicity and convenience, ground relationships are represented in the positive counterpart of the lens-negative plane.

For use of the equation, the only information needed is the focal length of the air-camera lens. All other terms can be measured on a stereo-pair of contact prints. The eye base and viewing distance will correspond to the individual observer and the stereoscope used, and can easily be measured.

## Definitions

At this point some of the terms to be
used in the discussion will be defined. No claims are made for these definitions of terms other than to clarify their meaning within the text of this paper. Photogrammetric terms of general usage have the same definitions as given in chapter XIX of the Manual of Photogrammetry.

## 1) Vertical Exaggeration

Exaggerated impression of depth over image size when viewed stereoscopically.

## 2) Photo Base

The distance measured between the center points of two photographs of a stereo-pair, when the images of all common ground points which lie at the datum elevation are in coincidence.

## 3) Photographic Datum Plane

Theoretical plane at the elevation of any selected photographic image point (in this paper a low point of the stereo-view), from which image displacements are measured, and to which the measurement of the photo-base is referred.

## 4) Object Ray

Ray which has passed through the air camera lens and by which the photographic impression of a ground object has been made.

## 5) Image Ray

Ray reflected from the photographic image of a ground object to the eye or to the stereoscope eyepiece.


[^0]:    * Paper read at Nineteenth Annual Meeting of the Society, Hotel Shoreham, Washington D. C., January 14 to 16, 1953. It was a part of the Report of the Photo Interpretation Committee.

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[^1]:    ${ }^{2}$ Geophoto Services, Denver, Colorado.

