*Some Considerations on the Application of Photogrammetry for Small-Scale Cartography**

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DURING the last few years much attention has been given to the use of photogrammetry for the establishment of topographic maps at scales of 1:50,000 and 1:100,000, due to the urgency of the rapid economic development of vast areas totally lacking in suitable cartographic background material.

The surveying methods which lend themselves to the establishment of such maps must fulfill the following conditions:

1. The maps should be available to the users as rapidly as possible.
2. Their cost should be kept as low as possible
3. Their accuracy must be such as to satisfy, not only the immediate requirements of the development work, but also the subsequent and more exacting ones of general technical mapping. Furthermore these maps must lend themselves to complementing by simple means.

Speed, economy and accuracy, thus are of primary importance, which means that a map system of a given accuracy as to its contents and the planimetric and altimetric position of its points must be completed as rapidly as possible, and with a minimum of financial outlay.

If a review is made of the photogrammetric instruments now available for the making of topographic maps, it is soon seen that the well-known auxiliaries used in extensive cartography, such as lateral photography, Trimetrogon photographs and the relevant methods of compilation, whether they be stereoscopic of some sort of radial triangulation, are apt to produce only unsatisfactory map substitutes for economic development, and that they hardly can yield really trustworthy maps of sufficient altimetric accuracy, as are needed in civil engineering.

On the other hand, the photographs taken with wide-angle lenses, as used for intensive photogrammetry, (1:5,000 to 1:25,000), will give excellent maps in scales of 1:50,000 and 1:100,000 as well. Economy and speed of drafting, however, will be unsatisfactory. The problem often is nearly hopeless, due to the difficulty in obtaining the geodetic data needed for orienting the photographs and to the multitude of models to be measured.

* A study of the further development of WILD-Instruments, Part I.

The author wishes to point out that the discussion of problems in this paper is not to be construed as being a criticism of the application of convergent photography. It is merely an analysis of the available procedures now in use and intended to create a basis for new developments. The publication in PHOTOGRAMMETRIC ENGINEERING is presented after receiving a number of requests from interested photogrammetrists instead of being based solely on the author's initiative. In publishing his ideas in this article, the author endeavours to indicate the direction in which internal development should proceed. This paper was earlier published in *Photogrammetria* in German because of the expressed wishes of the publishing committee of that journal since it represented a very interesting theme for discussion at a suitable time in view of the newly arisen interest in the problem of convergent photography.

Therefore, wherever the scarcity of maps in the scale of 1:100,000 cannot be overcome by extensive or intensive photogrammetry, an attempt is made at developing new instruments and methods which will yield the same economy and speed as the surveys in such small scales as 1:200,000 to 1:300,000, while, at the same time, guaranteeing a degree of accuracy and density of fixed points known up to now only in intensive photogrammetry at a scale of 1:25,000.

The measures recommended in order to achieve such improvements, are the following:

A. FOR THE PHOTOGRAPHY, IN ORDER TO REDUCE THE GEODETIC FIELD WORK:

1. Increase in the picture quality of the known wide-angle lenses, and consequently, greater flying heights.
2. Use of double convergent views with the present-type wide-angle lenses.
3. Increase in the angular field of the cameras through the use of ultra-wide-angle lenses.

B. FOR PLOTTING:

1. Use of aerial triangulation in large blocks of photographs, in order further to reduce the geodetic field work.
2. Use of less expensive plotting instruments for the plotting proper.
   a) on the base of strictly geometric solutions (2nd order instruments)
   b) basing on approximate solutions (3rd order instruments)

I. Increase in the Picture Quality of the Present Type Wide-Angle Lenses

The design of the *Wild "Aviogon"* lens is a first step in this direction (1, 2, 3). It is characterized by an increase in field brightness, an increase in the resolving power, and the absence of distortion, as compared to the former wide-angle lenses. In order better to penetrate the haze when flying at high altitudes. an equivalent infrared photography lens was developed, the *Wild Infragon.* This objective permits *doubling* the flying height while offering the same identification possibilities. Nevertheless, the flying height remains limited in many cases, due to

a) layers of haze and clouds impenetrable to any photographic equipment
b) the ceiling of aircraft
c) the costs involved in high altitude flights
d) the weather

II. Use of Symmetric Convergent Views with Twin Wide-Angle Cameras and Suitable Plotting Instruments to be Developed

In 1950 the use of double convergent photographs taken with Metrogon or Planigon lenses, (wide-angle lenses) mounted in a twin camera was suggested, featuring a convergence angle of twice 20 degrees with respect to the vertical. The purpose of this is to achieve complete overlap in the direction of the flight (Figure 1). This gives the very high base-height ratio of $B:H=1.23$.

The objective pursued is an increase in the base-height ratio and economy as compared to vertical wide-angle photographs. For a 90° wide-angle photograph with 60 per cent overlap, the base-height ratio would be only 0.6. Plotting is to be done from reduced diapositives, by means of new plotting instruments, such as the TWINPLEX and similar constructions.

The advantages which the proposed convergent photographs are said to offer are:

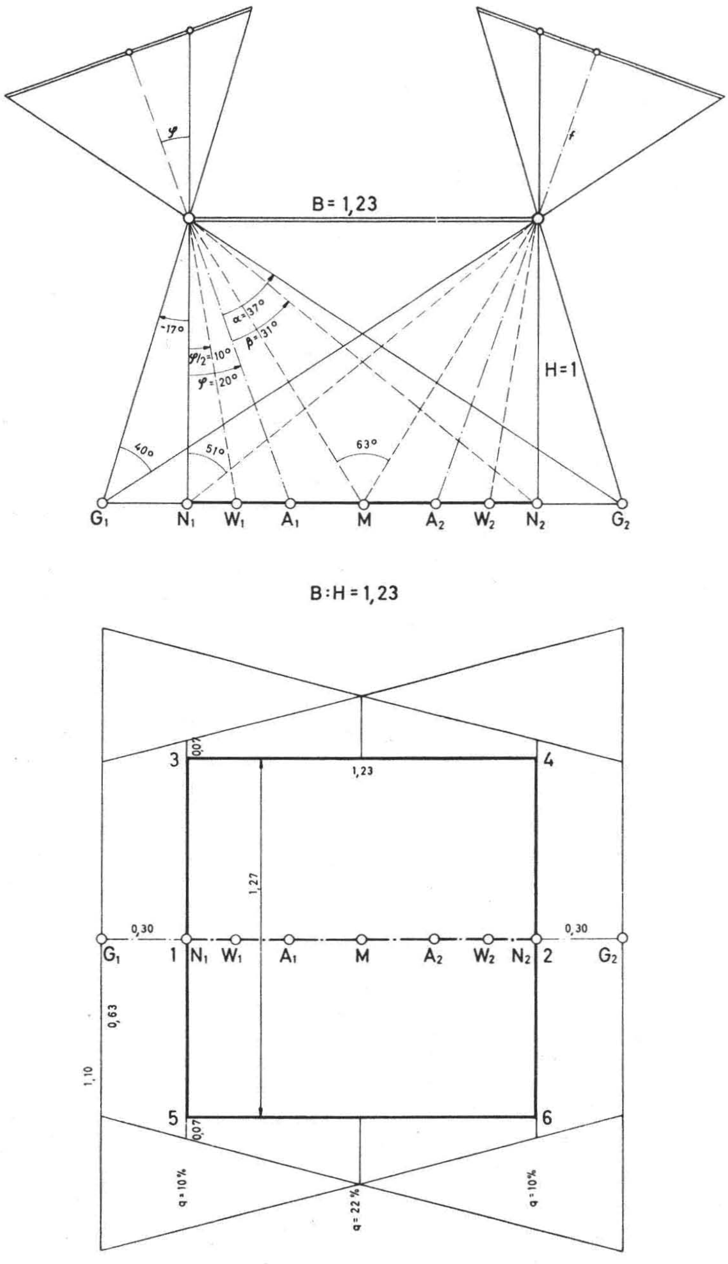

FIG. 1

a) Doubling the base ratio as compared to vertical photographs; this means an increase in the accuracy of height determination thanks to the much wider angle of intersection of the picture rays.
b) The flying heights being the same, the model surface to be plotted is a trifle more than twice that of the one given by vertical views with 60 per cent overlap. Hence a 50 per cent saving in the ground control points.
c) The accuracy requirements being equal, an increase in flying height is

possible, which means that the model surface to be measured is about 3.5 times larger than for verticals.

d) Reduction in the errors of relative orientation.

e) Reduction in the over-all costs, because the gain in height determination accuracy permits the use of greater flying heights with shorter distances (4, 5, 6, 7, 8, 9).

The practical experience acquired up until now and made public in the literature quoted is limited *to one flight* with 73 height test points. The accuracy obtained at a flying height of $H=12{,}000$ ft. $=3{,}650$ m. is ± 2 ft., which corresponds to a mean error, in height, of $\pm .17$ per cent H. The maximum error was 7 ft. $=2.2$ m. Picture scale was 1:24,000, plotting scale 1:10,000, with contour intervals of 20 ft. in hilly country and 10 ft. in flat terrain. Plotting was done on the Twinplex prototype. No details are given as to the nature of the test area.

The drawbacks to be reckoned with, when using convergent photographs with wide-angle lens are:

1. The base ratio is too large. By purely geometric means, it is easy to show an increase in the accuracy of the height readings for the points to be measured (10). However it is often forgotten that stereoscopic vision is greatly reduced as a result of the increased base ratio. For different reasons, an exaggerated stereo-effect should be avoided, because objects like houses, trees etc. do not appear spatially. Thus, there is a tendency for the picture to dissolve into separate pictures, especially so at the borders and in the direction of the base, which might lead to physiological disturbances under green-red observation of the Twinplex plotter.

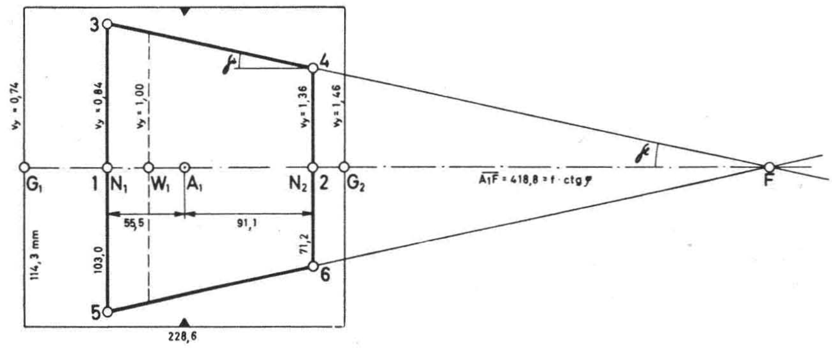

FIG. 2

2. The base ratio of convergent photographs is too rigid. For 100 per cent overlap and at identical flying heights it remains constantly the same. If the character of the terrain varies, there remains too much dead space which cannot be looked into. On the other hand, vertical photographs present the advantage of lending themselves to a more favorable base-ratio that can be chosen by increasing the overlap from 60 to 70 or even to 80 per cent, It is thus possible to adapt the operation to the nature of the terrain.

3. The two single photographs of a model present serious differences in scale within the same picture. In direction Y, which is at right angles to the base, the lengths in the picture must be multiplied by factors varying from $v_y=0.84$ to 1.46, in order to correspond to scale $f:H$ at point W_1 (Figure 2). In direction X (direction of the base) the factors to be con-

tended with are still larger, since they vary between 0.71 and 2.13.

4. Because of the differences in scale, *pairs of contact prints of* aerial negatives *cannot be directly viewed* stereoscopically (Figure 3); for all interpretation purposes, the negatives must first be rectified. This is true also for for the comparison of pictures during plotting.
5. In order to prepare mosaics, the pictures must also be rectified.
6. The model width of 1.27 H, as stated in the suggestion, seems to be somewhat on the optimistic side, for, at the nadir point of both model edges,

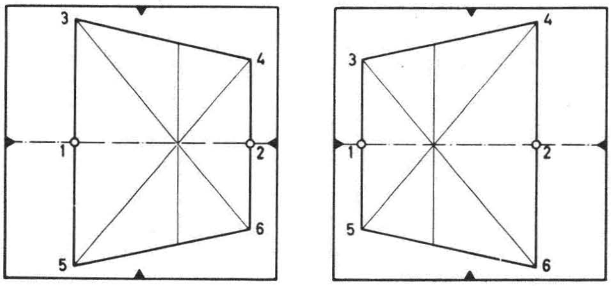

FIG. 3

there is *only 10 per cent overlap* for each picture, equal to twice 0.7 H (Figure 1), which requires too accurate, and therefore too strenuous navigation. In practice, it will be wise to reckon with an overlap of twice that width, setting the effective model width to 1.13, instead of 1.27 H.

7. The negative surface is not used economically. While the total surface of the 9″×9″ picture is 520 cm.^{2}, the effective plotting surface is limited to 255 cm.^{2}, which puts the surface unused to 265 cm.^{2}, or better than the total effective surface.

The Twinplex has been designed for the plotting of symmetric convergent pictures, a Multiplex-type plotting instrument.

The Twinplex is a projection device based on the anaglyph principle. The original pictures are reduced 2.8 times to 11×11 cm. glass diapositives, corresponding to a focal distance of 55 mm. for the projectors. Illumination is by means of an ellipsoidal reflector, designated ER-55. In order to fulfill the Scheimpflug condition, e.g. focusing onto the horizontal table, the projector lens is tilted 2 degrees to the negative. The working height of the Twinplex is 500 mm. If the flying heights are in meters, the scale numbers, on the Twinplex diapositive, are 18 H for the isocenter 16 H and 26 H for the picture edges which are perpendicular to the base and 17 H respectively 25 H for the borders of the model.

The main drawback of the Twinplex is the need for dark-room work and the observation of a poorly defined reflected picture, projected onto the Multiplex plotting table. Furthermore, the large base ratio means an additional psychological strain on the operator, as compared to the Multiplex for vertical views.

It has frequently been suggested that mechanical plotters, for instance, the Wild A7 Autograph, be adapted for plotting double convergent photographs having a convergence angle of twice 20 degrees. This is quite feasible, provided the A7 be equipped with two additional devices.

1. It must feature an equalizing system for the magnification, by which the picture ordinates are magnified at a ratio of:

$$v_y = \cos\phi + \sin\phi \cdot \tan\xi$$

by means of a pancratic system provided in the observation optics, ξ meaning the tilt of the guide rod in direction X, from the vertical.

2. Picture inversion γ must be eliminated (Figure 2) by means of a picture rotation which can be obtained by dove prisms according to the equation

$$\tan \gamma = \sin \phi \cdot \tan \eta$$

in which η means the inclination of the guide rod in direction Y, toward the vertical.

According to principle there are a few important remarks to be made with respect to Twinplex photogrammetry. To prove the practical value of this construction, some rather old comparisons from the history of photogrammetry were used, for the purpose of showing the advantages of convergent photography over vertical photography. For example D. K. Scott refers to publications (9), (11) and (12), dating back to the years 1929 and 1930. With the design of 90 degrees objectives these twin cameras have disappeared completely although their base-height ratio was not unduly large, and the angle of convergence was kept within reasonable physiological limits.

Symmetric convergent photography with wide-angle lenses and the Twinplex system cease to be of any use in photogrammetry, as soon as, instead of 90 degree lenses, ultra wide-angle lenses (120 degrees), with equal optical performance, are used.

Considering the different drawbacks of double convergent photography and in view of the developments in the field of photogrammetry between 1930 and 1940, this conclusion is readily warranted.

III. Widening the Angular Field of a Camera by Using Ultra Wide-Angle Lenses

A. Another method for obtaining a larger surface per photograph from a given flying height

This method consists of widening the angular field. This has already been done twice. A widely known ultra wide-angle lens is the *Pleon*, of Zeiss, Jena, with an angle of 148 degrees (13). In designing this lens, the principal aim was to reduce the so-called "natural" loss of light towards the edges, by introducing a strongly negative (barrel-shaped) distortion

$$\Delta r \approx f(\hat{a} - \tan \alpha)$$

and foregoing the usual perspective picture formation (14), (15). Quite apart from the fact that the assumption according to which a heavy loss of light can only be avoided by introducing a strong negative distortion was incorrect, the Pleon objective yields only distorted pictures, often to such extent as to make the identification of points at the edges of adjacent pictures impossible. The corrected diapositive or paper print is of only limited use, and contains residual distortions inadmissible for reliable height readings. The parts of the pictures beyond 60 degrees of nadir distance are mostly useless. Thus, the development of this lens is of more theoretical interest than any other, and means no practical advance. Further developments along these lines can safely be dispensed with in the future.

B. THE EFFORTS MADE IN THE U.S.S.R.

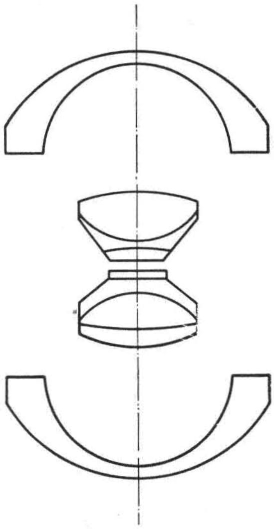

FIG. 4

These efforts are achieving different results. There, several ultra wide-angle lenses have been designed by *M. Russinov*, with an angle of view of 122 degrees, the loss of light of which corresponds to the 90 degrees Metrogon. This means that brightness, in the corners, drops to about 10 per cent of that in the center (16, 17). These lenses are called *Russar*. They are practically free from distortion. Since one of these lenses (probably the Russar 31, $f=7$ cm., picture size 18×18 cm.) (Figure 4) has been used for the photographic work needed for the 1:100,000 map of the Soviet Union, and since this map, which covers the whole of this country's 22 Mill km.[2] territory is now nearly completed (21, 22, 23), this development is worth while mentioning for comparison with the American convergent method.

The Soviet method consists of:

1. Vertical photographs (18×18 cm. picture size) taken with the Russar $f=7$cm., using 60 per cent longitudinal and 30 per cent lateral overlaps (18). Picture scale is $1:14\,H$ if H is in meters.
2. The negatives are reduced to 0.3 of their linear dimensions, on glass diapositives 6×6 cm. for $f=20.9$ mm. This is done by means of a Multiplex reduction printer for a diapositive scale of $1:48\,H$.
3. Aerial triangulation with ultra wide-angle multiplex projectors designed by *N. V. Victorov* (Figures 5 and 6) using the 6-limbed Russar 25, f:10, $f=20$ mm consisting of 6 elements. The usual working distance is 350 mm. The sharp focusing range between 250 and 360 mm. This device is provided with 9 projectors (19, 20).

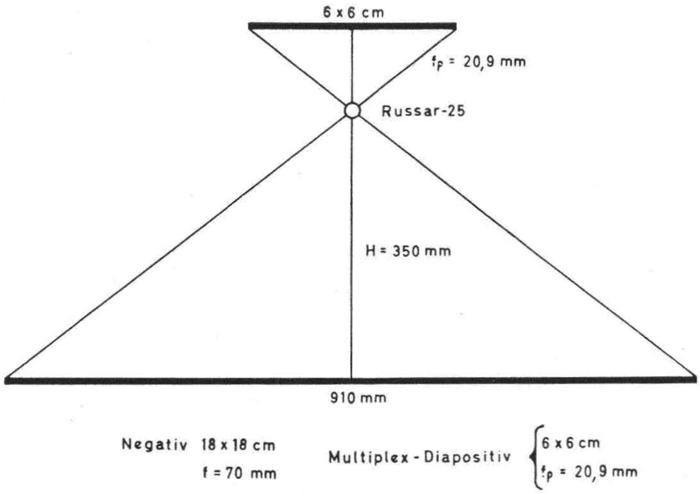

FIG. 5

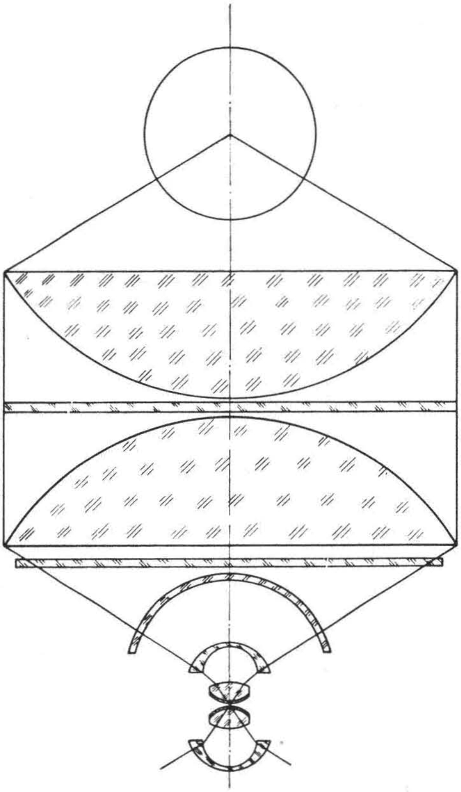

FIG. 6

Allowing for the resolving power of the lens used for photography, the reduction to Multiplex size and the anaglyph observation of the reflected image on the tracing table, the Russians obtained the pictures scales shown on Table I for the relevant map scales.

Plotting is also done by means of 3rd order instruments, designed mostly by Drobyshev (19, 20). These instruments use parallax correction and do not permit geometrically accurate reconstruction of spatial models. Their use will be limited to flat terrain, and requires very small nadir distances.

Judging by some hints to be found in Soviet Literature it would seem that trials are proceeding with the object of using 1st order instruments for aerial triangulation of ultra wide-angle photographs $f=7$ cm. by the method of affinitive plotting at larger plotting scales, for example 10.5 cm. for a picture size of 18×18 cm.

Here a few remarks concerning the Russian method:

1. There is no doubt that, in principle, the idea of using distortion free ultra wide-angle photographs (120 degrees) is more promising than the use of convergent photographs. Since this makes it possible to use 80 per cent longitudinal overlap, one gains, where needed, better insight into the terrain, while it is still possible to work with the very favorable base-ratio of $B:H=0.5$. As a matter of principle, aerial

TABLE I

MAP SCALE HEIGHT ACCURACY	PICTURE SCALE WITH RUSSAR $f=7$ CM., 18×18 CM.		
	Arctic regions and uninhabited territories	Terrain of difficult access and mountainous areas	Cultivated, densely populated areas
1:10,000 $m_h=\pm.5$	——	——	1:15,000 to 17,000 $H=1{,}050$ to 1,200 m.
1:25,000 $m_h=\pm 1.25$	——	1:20,000 to 30,000 $H=1{,}400$ to 2,100 m.	1:20,000 $H=1{,}400$ m.
1:50,000 $m_h=\pm 2.5$	——	1:30,000 to 45,000 $H=2{,}100$ to 3,150 m.	ca. 1:30,000 $H=2{,}100$ m.
1:100,000 $m_h=\pm 5$ m.	ca. 1:75,000 $H=5{,}250$ m.	1:55,000 to 75,000 $H=3{,}850$ to 5,250 m.	1:35,000 to 55,000 $H=2{,}450$ to 3,850 m.

triangulation will be done with a 60 per cent overlap, thus using every third picture.

2. The ultra wide-angle photographs include all advantages of the convergent photography:
   a) large base-height ratio $b:H=1$
   b) $2.6\times$increased surface of photograph, as compared to standard 90 degree wide-angle lenses
   c) reduction of errors in relative orientation of the stereopairs is still more advantageous than with convergent photography.

The drawbacks of convergent photography can be avoided with ultra wide-angle lenses:

a) By changing the overlap according to the terrain character, the rigid base ratio can be avoided, thus obtaining a relatively favorable stereo-effect.
b) There are no differences of scale within the picture or model. Therefore, the contact prints can be used *immediately without being rectified* for mosaics or interpretation.
c) When setting up mosaics, one enjoys the important advantage of an accurately centered perspective displacement of points through height differences.
d) 20 per cent lateral overlap of the strips suffice to render navigation much simpler than with the assumption needed for convergent photography. The overlapping strip of an ultra wide-angle lens is twice .25 H, as compared to the minimum overlap of .07 H in convergent photography.
e) Ultra wide-angle pictures have the further advantage of permitting the immediate bridging, in a plotting device, of a sufficient angular field (or in a stereocomparator), and of also permitting immediate triangulation by means of the slotted templets with simple cutters. Finally, radial triangulation in the radial triangulator is also possible.
f) To cut down the field work and to create a dense network of boundary points, triangulation of very large blocks can be carried out with 60 per cent lateral overlap.

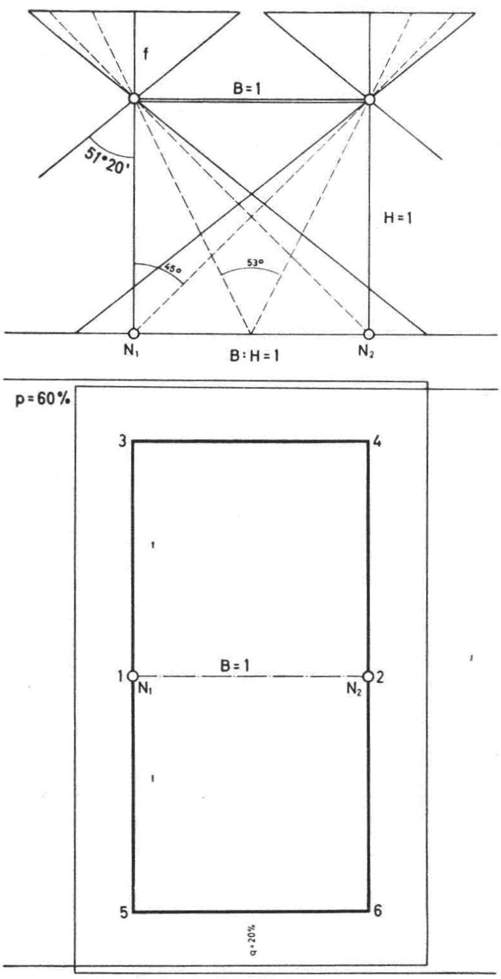

FIG. 7

The Soviet method, however,

also can stand considerable improvement. Its economic value could well be increased, as will be shown in the following section.

C. PROJECT OF AN ULTRA WIDE ANGLE LENS AND COMPARISON WITH CONVERGENT PHOTOGRAPHY

Due to the reduced picture scale, it would seem advisable to use an ultra wide-angle lens (120 degrees) for a picture size of $9''\times 9''$ (23×23 cm.). The focal distance will be around $f=3\frac{1}{2}''$ (92 mm.) for a base-height ratio of $B:H=1$ and 60 per cent longitudinal overlap (Figure 7).

If the flying height be $H=1$ for both convergent and ultra wide-angle photography ($9''\times 9''$), one obtains, according to Figures 1 and 7, the following surfaces to be plotted, for convergent photographs:

$$F_K = 1.23H \times 1.27H = 1.56H^2$$

for an ultra wide-angle lens and $p=60$ per cent, $q=20$ per cent

$$F_W = 2H^2$$

In the first case, the space between the strips is

$$W_K = 1.27H$$

for the ultra wide-angle lens

$$W_W = 2H$$

Because the distance between flight lines in a block is of paramount importance with regard to the cost of flying, while the expenses for field work depend on the number of images and models, according to the *equation* $W_K = W_W = 1.27:2$ the use of an ultra wide-angle lens is considerably more favorable than that of convergent photography.

A square surface $10\ H\times 10\ H$ needs, in the case of the Twinplex method, 8 strips of 8 models each, made up of 16 photographs and 2 superfluous border photographs. This totals 64 models (128+16 photographs).

In the case of the ultra wide-angle lens, only 5 strips of 11 photographs, totaling 50 models, are necessary.

The picture scale, in the Twinplex negative, is somewhere between $1:6\ H$ and $1:9\ H$ in direction Y, and $1:5\ H$ $1:13\ H$, in direction X. At the isocenter it is $1:6.5\ H$. In the case of an ultra wide-angle lens with $f=92$ mm., it remains uniformly 1; 11 H, H to be shown in meters.

The angles formed by the projection rays and the projecting plane are given in Figure 8 for convergent photography, while Figures 9 and 10 give them for ultra wide-angle lenses and a base ratio of $B:H=1$. The dead zones are practically equal. In rough terrain, narrower angles of section of the projection rays are of advantage.

Plotting of rough terrain can be done in different manners. While, with 60 per cent overlap, the area between thick lines of Figure 7 can be fully measured, on the other hand, according to Figure 11 only surfaces 3, 4, 6, 5 or 3′, 4′, 6′, 5′ and parts of 3″, 3′, 5′, 5″ and 4′, 4″, 6″, 6′ can be measured from the base III, IV. In the first case 3″, 3, 5, 5″, must be plotted from base II, III and 4, 4″, 6″, 6 from IV, V. In the second case, those models must be dropped and only the terrain between 3″, 3, 5, 5″ resp. 4, 4″, 6″, 6 will be plotted from I, II and V, VI.

As regards the orientation of the models in aerial triangulation, it can safely be said that the accuracy, both for the lateral and longitudinal tilt, will be *very high*, due to the large angle of view.

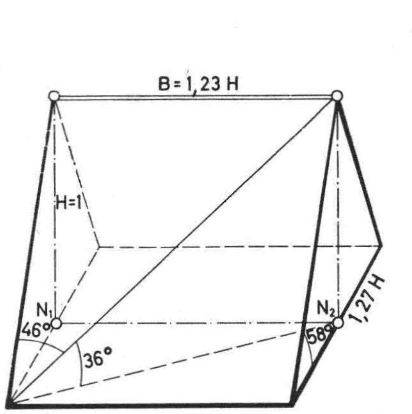

FIG. 8

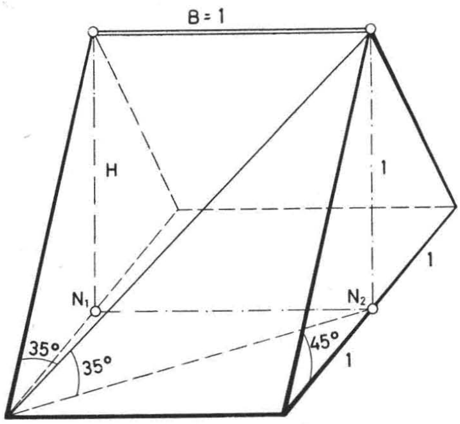

FIG. 9

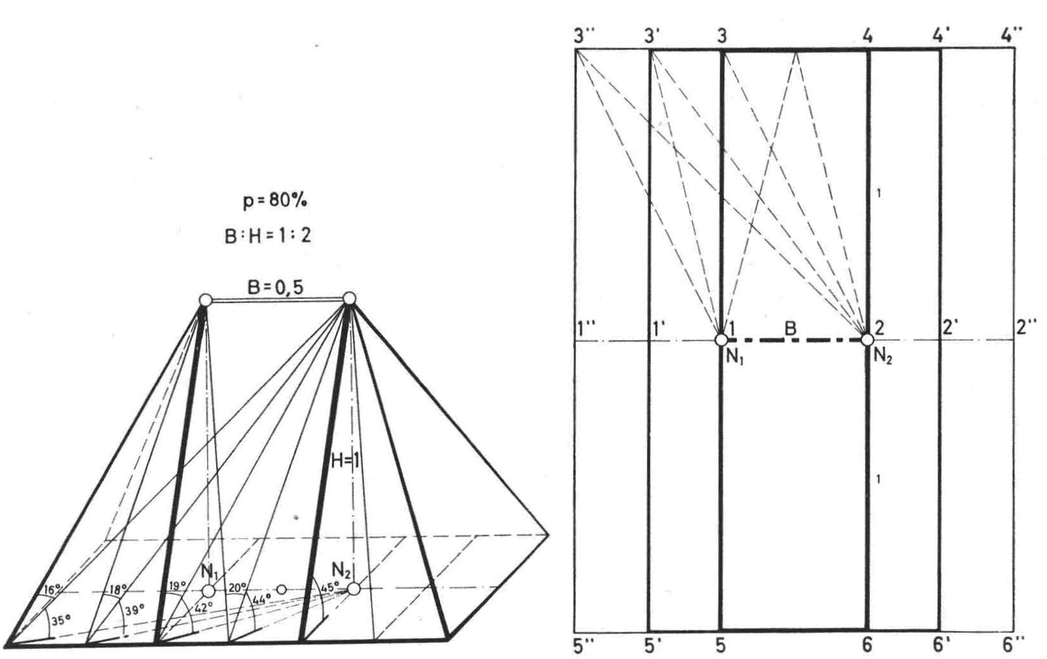

FIG. 10

The factor for over-correction of the ultra wide-angle lens will be

$$n = \frac{1}{2}\left(1 + \frac{f^2}{y^2}\right) = 1$$

which is very favorable as compared to the usual normal and wide-angle lenses.

Another practical example will permit comparing convergent photography with ultra wide-angle photographs taken from the same flying height.

PROBLEM:

One map section 1:100,000 covering 48×70 km., to be photographed with an ultra wide-angle lens, using 23×23 cm. (9″×9″) pictures.

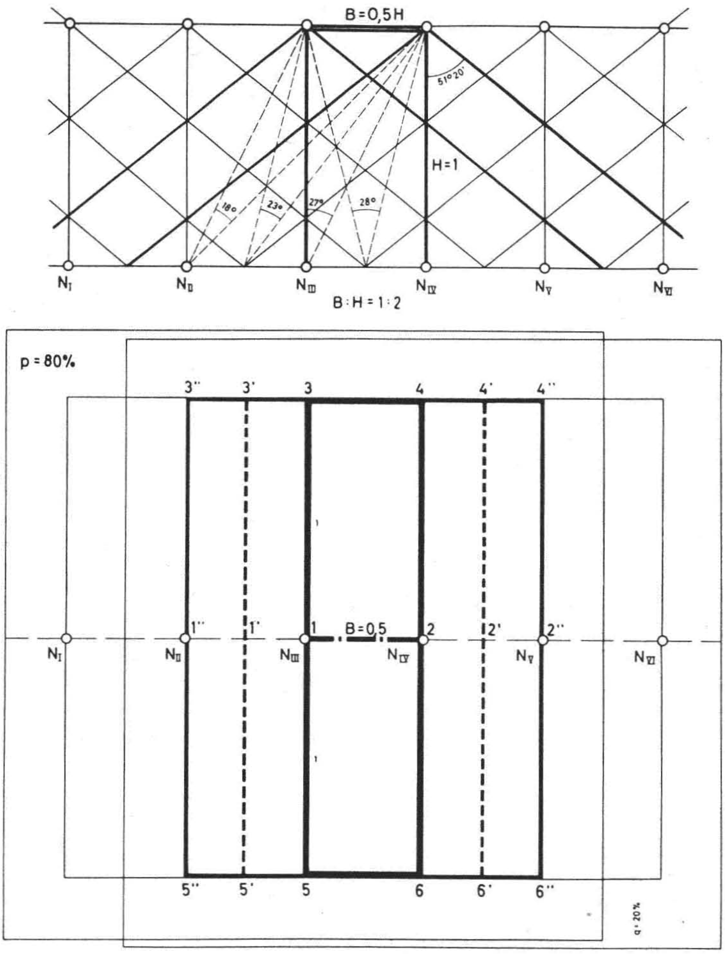

FIG. 11

Focal distances to be considered:

$f=92$ mm. (3.6″)
$f=87$ mm. (3.4″)
$f=82$ mm. (3.2″)

The first of these focal distances corresponds to the base-height ratio of $B:H=1$, the second to an angle of view of 120 degrees taking into account the fiducial marks in the corners of the Wild film cameras, the third to an angle of 120 degrees, the corners not being quite fully used.

Picture scale in all cases 1:70,000.

Overlap to be 60 per cent or, 80 per cent according to terrain.

For this setup, we compare double convergent photographs taken from the same flying height H, with $B=1.23\ H$ and a model width of $1.27\ H$. The results of this comparison may be seen on Table II. The advantages of ultra wide-angle photography, as compared to convergent photography, are obvious.

The availability of a good ultra wide-angle lens reduces to practically nil the usefulness of convergent photography with the usual wide-angle lenses for small-scale cartography, namely 1:25,000 to 1:100,000.

TABLE II. PICTURE SIZE 23×23 CM., PICTURE SCALE 1:70,000, SURFACE 47×70 KM.

	FOCAL DISTANCE f	92 mm.	**87 mm.**	82 mm.
	Flying height	6450 m.	**6100 m.**	5750 m.
	Square area of side length S	16 km.	**16 km.**	16 km.
	Base B at 60%	6.4 km.	**6.4 km.**	6 km.
	$L=70$ km. =	11 $B_{60\%}$	**11 $B_{60\%}$**	11 $B_{60\%}$
	$W_{25\%}$	12 km.	**12 km.**	12 km.
	$Br=48$ km.	4 $W_{25\%}$	**4 $W_{25\%}$**	4 $W_{25\%}$
	Number of strips	4	**4**	4
	Number of photographs with 60% overlap	4 (11+1)=48	**4 (11+1)=48**	4 (11+1)=48
	Additional photographs with 80% overlap	44	**44**	44
	Number of models with 60% overlap	44	**44**	44
Twinplex convergent photographs	Base = 1.23 H Model width = 1.27 H	7.9 km. 8.2 km.	**7.5 km.** **7.75 km.**	7.1 km. 7.3 km.
	Strips	6	**7**	7
	Number of photographs per strip	19×2+2	**10×2+2**	10×2+2
	Number of photographs	108+12	**140+14**	140+14
	Number of models	54	**70**	70
	Useful flying distance	420 km.	**490 km.**	490 km.
	Useful flying distance, with ultra wide angle lens	280 km.	**280 km.**	280 km.

IV. PLOTTING INSTRUMENTS

The Soviet wide-angle Multiplex features the well-known characteristics of the method:

- Heavy reduction to Multiplex diapositive
- Projection principle
- Inclined observation of anaglyphs in dark-room
- Reduced possibility of identification
- Difficulty of comparing with contact prints or enlargements, because of the necessity for dark-room illumination.

Without doubt a triangulation instrument equipped with a plotting device for the optical observation of diapositives gives better results. Due to the inclination of the light rays, an optical system cannot be recommended, as then we have to cope with apparent model tilts when observing the borders of the model. A system with orthogonal observation and a large field of view is to be given preference. Therefore, the mechanical system is the most appropriate. It will however be easier to operate if the pictures are reduced to the half-size of 11.5×11.5 cm. before plotting.

It would also be possible to use the reduced diapositives in a projection instrument of the kind of the ER-55 projector.

The diapositive scale, at this reduced size and for focal lengths of $f=46$ and 41 mm., would be 1.22 H and 1.24 H respectively. It thus always falls somewhere between the variable scales of the 2.8×Twinplex reduction.

The necessary reductions, for various existing or planned instruments, can be seen in Table III.

TABLE III. REDUCTIONS

23×23 cm. $f=15$ cm.	23×23 cm. $f=15$ cm.	18×18 cm. $f=7$ cm.	23×23 cm. $f=9$ cm.
Twinplex 2.8×	Multiplex 5.1×	Soviet Multiplex 3.3×	Autograph with mechanical projection 2×

Aside from the triangulation instrument, it will be advisable to develop a simpler mapping instrument in order to reduce mapping costs. The already existing 3rd-order instruments of Russian, American or European design must still be held unsatisfactory. The preference, in the authors' opinion, should be given to an instrument providing the precise geometric solution or a method which consists of first rectifying, and then mapping, by means of a simplified, yet geometrically accurate instrument.

V. RESULTS OF THE COMPARISON

1. *Photography by means of an ultra wide-angle lens* for 23×23 cm. picture size and approx. $f=9$ cm. focal distance is, in all respects, more favorable than the Twinplex method. It seems advantageous therefore, to design an ultra wide-angle lens of 120 degrees angle of view, based on the Aviogon. The new lens should possess the following properties:
   a) low loss of light
   b) high linear resolving power
   c) a relative aperture, in the middle of the picture, of f:5.6
   d) free from distortion
   e) absence of diaphragm differences
   f) yellow filter within the lens
   g) simultaneous design of an analogous Infra-red lens
2. *Design of ultra wide-angle cone* 9″×9″ for RC5a and RC8.
3. *Design of a triangulation and mapping instrument* for picture size 11.5×11.5 cm. for a range of focal distances from 40–80 mm. and 60 degrees guide rod inclination. Triangulation device, with change from "base inside" to "base outside." As it appears, a simplified A7 system would be most appropriate. It is also recommended to make a correction for earth curvature.
4. *Design of a reduction printer for reduction* to half size of the original photograph.
5. *Additional design of simplified mapping instruments.*

TABLE IV

MAP SCALE	1:50,000		1:100,000	
Picture scale	from 1:30,000	from 1:40,000	from 1:50,000	from 1:80,000
Size of photograph 23×23 cm.	47 km.²	83 km.²	158 km.²	330 km.²
Flying height H for $f=92$ mm. $f=87$ mm. $f=82$ mm.	 2,750 m. 2,600 m. 2,450 m.	 3,700 m. 3,500 m. 3,300 m.	 5,100 m. 4,800 m. 4,500 m.	 7,400 m. 7,000 m. 6,600 m.

TABLE V. COMPARISON BETWEEN CONVERGENT PHOTOGRAPHS TAKEN WITH WIDE-ANGLE LENSES AND ULTRA WIDE-ANGLE PHOTOGRAPHY

COMPARISON UNDER ASSUMPTION OF EQUAL FLYING HEIGHTS H	WIDE-ANGLE CONVERGENT PHOTOGRAPHY	ULTRA WIDE-ANGLE LENS 120°
Image size	9×9″ (23×23 cm.)	9×9″ (23×23 cm.)
Focal length $f=$	6″ (152 mm.)	3,6″ (92 mm.)
Angle of convergence	2×20°	Vertical photograph
Longitudinal overlap $p\%=$	100%	60% (80%)
Lateral overlap $q\%=$	Between 10 and 30%	20% (More advantageous, because constant)
Base-height ratio $B:H=$	1.23	At 60%........1 (At 80%........0.5)
Base $B=$	1.23 H	H
Model width $W=$	1.27 H	2 H
Model area	1.56 H^2	2 H^2 (20% more advantageous)
Number of flight strips for a given area	m.	0.63 m. (Productive flight distance more advantageous by 37%)
Number of models per strip	n.	1.23 n.
Total number of photographs	2 nm+2 m Productive 2 nm	0.63 m (1.23 n+1) =0.78 nm+0.63 m 60% Economy in the number of pictures
Total number of models	nm	0.78 nm (22% more advantageous)
Usable model area per photograph	50%	64% (Therefore more advantageous)

TABLE V (*Continued*)

COMPARISON UNDER ASSUMPTION OF EQUAL FLYING HEIGHTS H	WIDE-ANGLE CONVERGENT PHOTOGRAPHY	ULTRA WIDE-ANGLE LENS 120°
Picture scale	*Variable* over the whole of the picture	*Constant* over the whole of the picture
Direct use of the photographs for stereoscopic observation	Impossible	Possible, in any situation
Direct interpretation	Difficult	Easier
Use for radical triangulation	After previous rectification only	Directly possible
Slotted templet with simple slot cutter	After previous rectification only	Directly possible
Slotted templet with radial-secator	Directly possible	Directly possible
Triangulation on the comparator	After previous rectification	Directly possible
Use of mosaics	After rectification only	Directly possible
Plotting instrument	*Twinplex* Disadvantage: Projection system	*Autograph* for 120°. Advantage: Universal instrument with direct optical observation and large field of view
Preparation of the photographs	Reduction 2.8×	Reduction 2×
Use of the plotter	For 2×20° convergent photographs only	Universally usable for all kinds of wide-angle vertical photographs, triangulation and topographic work
Necessary accessories	Reduction printer 2.8× Rectifier	Reduction printer
Restitution in A 7	With special device for convergency (pancratic system and automatic image erecting)	With affin. command to approx. 1.5 f
Restitution in A 8	Not possible	With affin. command to approx. 1.5 f
Restitution in simple stereoplotters	After rectification only	Directly possible
Stereoscopic observation of the model in the plotter	The exaggerated stereo-effect due to the too large rigid basis *cannot* be corrected	Improvement of the spatial impression by inserting pictures with 80% overlap
	To be used only for flat areas with little detail	Can also be used for mountainous country
Summary:	Many drawbacks, no real advantages	Economy increased, no disadvantages
Conclusion:	Construction not advisable	Construction advisable

The improvement of the camera with regard to larger picture size and higher resolving power on the base of the Avigon lens, and also the design of a high performance plotting instrument with triangulation facilities, will permit using the picture scales in current service up to now for 18×18 cm. (Table I), also for 23×23 cm. picture size, and thus to obtain a 60 per cent increase of measurable surface per model.

The presumable possibilities of use may be seen in Table IV.

A summarizing comparison is also given by Table V.

Convergent photography must be considered as a secondary help in making instruments.

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