An Application of Terrestrial Photogrammetry to Glaciology in Greenland

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ABSTRACT: The change in position of a moving glacial ice-cliff surface was desired. To accomplish this a plan was devised to produce topographic maps by photogrammetric methods. Firstly, a straight base line was established at a desired distance from the ice cliff and nearly parallel to it. Then control targets were established on the ice cliff surface and their positions with respect to the base line were determined by triangulation. Overlapping terrestrial photography was taken of the ice cliff surface from several equally spaced points on the base line, and techniques were developed for using this photography to compile topographic maps of the cliff surface with the Kelsh plotter. Changes in cliff position were then determined by the comparison of two topographic maps based on data obtained at different times.

INTRODUCTION

The material used in this paper is based upon data obtained in Greenland by the author through a research project grant directed by The Ohio State University Research Foundation. During this period the author was a graduate student at The Institute of Geodesy, Photogrammetry and Cartography at The Ohio State University.

This paper explains a method of producing topographic maps of a moving glacial ice cliff surface by terrestrial photogrammetry. The procedures involved in this investigation include: 1.) Field Operations; 2.) Data Computation; and 3.) Map Compilation and Reproduction.

FIELD OPERATIONS

A straight, base-line site about 420 meters in length was selected. Its direction was approximately East-West, and it was located nearly parallel to the ice cliff. The perpendicular distance to the cliff was about 100 meters at the West end of the base line and about 150 meters at the East end. In elevation the West end was about 40 meters higher than the East end. A total of 15 base stakes was established at 30 meter intervals. These stakes were numbered 1 through 15 from West to East. Thin copper strips were nailed to the tops of these stakes for measuring purposes, and a small hole representing the camera station point was pricked approximately in the center of each. Photo control targets were then established so as to give the best distribution in the predetermined stereo model area. Because of the dangerous working conditions on some parts of the ice cliff, all desirable target positions could not be used.

The cliff control targets planned and used for this project consisted of aluminum discs 6 inches in diameter and 1/4 inch thick. Two brackets for bolting the targets to the wooden rods were included in their construction. The side to be observed was painted black with a white center design. Small, light-weight targets were desirable to prevent bending or slipping of the rods to which they were attached, especially under the strong winds prevailing in the field area.

The minimum photogrammetrically usable diameter of these targets was determined by trial photography under the same conditions.

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distance condition as was planned for the field operation. Black targets of 4, 6, 8, 10 and 12 inch diameter were photographed and their image size and clarity noted. The 6 inch diameter disc was selected as the allowable minimum for good image definition at the required distance.

To establish the targets, a hole about 1 inch in diameter was drilled about 6 feet into the ice using a brace, bit and extensions (Stanley and Greenlee Ship Auger types). A wooden rod was then forced into the hole, and the target bolted to the protruding end. After considerable ablation, the targets would sag or fall out. In resetting them the same continuous numbering system was maintained, even though the targets were not always replaced in exactly the same position.

The relative elevations of the base stakes were determined by U.S. Coast and Geodetic Survey Procedures, using a Wild N-2 level and precise level rods. No previously established control was available in the field area, consequently an arbitrary elevation of 100 meters was assigned to base stake no. 1 (West base). The mean difference of elevation for each section was applied to this value in obtaining the relative elevations of the other base stakes.

The base-line measurement was made with a nickel-steel tape (K & E Lovar #9505), calibrated by the manufacturer. Standard base-line measuring techniques were used and the corrected lengths of individual intervals were determined.

Both the base line leveling and measurement conformed to nearly 3rd-order accuracy as specified by U.S. Coast and Geodetic Survey standards.

Horizontal and vertical triangulation observations of the cliff targets were made with the Wild T-2 theodolite set up over the station point at base stake nos. 1, 5, 11 and 15. Three determinations of direct and reverse readings were observed both for the horizontal and for the vertical angles. Each target was observed from two separate base stakes. The three observed values were averaged, and any position varying from the average by more than 5 seconds was re-observed.

Terrestrial photography was obtained using the Wild Phototheodolite T-30 which consists of a camera unit and theodolite. The phototheodolite camera is of the fixed-focus, fixed-aperture type so that the amount of light entering the camera is dependent upon shutter speeds. "Gevaert" topographic rapide ortho, emulsion plates were used in the field operations. It was found that photography taken under bright sunlight conditions between 0930 and 1,300 hours (at that time of year) was the most suitable for plotting purposes. Under these conditions an exposure of ½ second was found to be very satisfactory.

At each of the 15 base-stake stations the phototheodolite was first plumbed over the station point, approximately leveled, and the area to be photographed viewed through the ground glass to insure all of the targets being in place. The tilt mechanism was set on zero at stations 6 through 13 and on 7 grads uptilt at the other stations. The camera axis was made perpendicular to the base line. The instrument was leveled very precisely and the exposure made. The height of the camera was recorded along with the exposure data, tilt, time and date. Two sets of 15 photographs each were taken, the first on July 20, 1955 and the second on August 26, 1955. The triangulation observations of the cliff targets were made as soon as possible following the photography, in order to minimize possible change in the target position during a long time interval.

A photographic dark room was established at the campsite to process the film. Both fine and medium-grain developers were used in processing the plates. The fine-grain developer produces slightly better detail. The medium-grain developer produced a very good negative in less than half the developing time required for the finer-grain developer.

**Data Computation**

The base-stake coordinates were computed with respect to base-stake No. 1 (origin). This was assigned the values: $X=000.000$ m.; $Y=000.000$ m.; $Z=100.000$ m.

The base-line represents the positive $X$ axis of a right-hand rectangular-coordinate system with the $Y$ axis in the direction of the cliff. The $Y$ coordinate of each of the base-stakes is zero. The cliff target coordinates were determined in the base-line coordinate system by plane analytics.

Before photogrammetric map compilation was attempted an evaluation of the characteristics of the camera used was made. These characteristics included: the camera focal length, the position of the
principal point, and the lens distortion. Several determinations of the camera focal-length were made. These determinations were based on the field observed horizontal angles and on comparator distances measured between target images. The maximum deviation of an individual value from the mean was 10 microns. For stereo plotting purposes the calibrated focal length is required. This is obtained by altering the equivalent focal length in such a way as to minimize the distortion. Since the distortion values determined were extremely small, an adjustment was considered unnecessary. Therefore the equivalent focal length was adopted as the proper focal length to be used in the stereo plotter.

The mean difference in position of the computed principal point with respect to the indicated principal point was 0.114 mm. in the $x$ direction and 0.146 mm. in the $z$ direction. These values were found to be too small to be visibly corrected when adjusting the diapositives in the stereo plotter plateholders.

**MAP COMPILATION AND REPRODUCTION**

In order to maintain a one-to-one ratio between horizontal and vertical scale on a stereo plotting instrument, the principal distance of the projection system should be made to equal the calibrated focal length of the camera used for the photography. The camera focal length of 166.09 mm. was outside the range of both the 6 inch (152.40 mm.) and the 8½ inch (209.55 mm.) projectors of the Kelsh plotter used for the compilation. It was necessary, therefore, to use different horizontal and vertical compilation scales.

The use of different horizontal and vertical scales presents no great problem provided both remain constant. If two different focal lengths are used but the same camera separation and parallax conditions are maintained, the resulting horizontal scales will be equal and constant. The shorter focal length, however, will yield a smaller vertical scale. The scale relationships existing between the two different focal lengths are shown in Figure 1.

$$\phi_1 = \phi_2 = \phi \text{ parallax}$$

$$Y_1, Y_2 = \text{distance from perspective center to surfaces 1 and 2 respectively}$$

$$f_1 = \text{original focal length}$$

Fig. 1. Effect of change in focal length on vertical scale.

$$f_2 = \text{reduced focal length}$$

$$B = \text{distance between camera stations}$$

**Formula Derivation:**

$$\phi_1 = \frac{Bf_1}{p}$$

$$Y_1 = \frac{Bf_1}{p}$$

$$Y = \frac{Bf_2}{p} = \frac{KBf_1}{p} = KY_1$$

$$dY_1 = \frac{-Bf_1}{p^2} dp$$

$$Y_2 = \frac{-KBf_1}{p^2} dp$$

$$dY_2 = KdY_1$$

The above formulas shown that the ratio of the vertical scales is equal to the ratio of the focal lengths involved.

The limiting factors of the Kelsh plotter (minimum projector-separation and maximum projection-distance) prevented the use of adjacent photographs for compilation. Therefore, alternate photographs having a ground separation of 60 meters were used.

To obtain maximum plotting accuracy, the maximum vertical and horizontal compilation scales within stereo-plotter limitations were desired. Prior to the compilation, two alternate photographic plates were set up in the plotter and a model formed. Image clarity was found to be satisfactory to a projection distance of 950 mm. which is nearly the maximum permitted by the mechanical construction of the instrument. An approximate maximum vertical-compilation scale, based on this maximum projection distance of 950 mm. and a maximum base line to cliff distance of 160 meters, was determined. This ap-
proximate vertical scale was then used to obtain the horizontal scales corresponding to the 6 inch and the 8½ inch Kelsh projectors. The 6 inch projectors were selected for the compilation because the corresponding horizontal scale was much larger than the horizontal scale obtained for the 8½ inch projectors. Also the instrument base-length required by the 8½ inch projectors would be very close to the minimum possible projector-separation.

An arbitrary projector principal-distance for the 6 inch cones was selected and the compilation data computed as follows:

**Given:**
- Camera Station Separation, \(B\) = 60 meters
- Projector Principal-Distance, \(PD\) = 152.30 mm.
- Camera Focal Length, \(PD'\) = 166.09 mm.
- Minimum Base-Line to Cliff Distance = 93 meters (approximate)
- Maximum Base-Line to Cliff Distance = 160 meters (approximate)

**Computations:**
- Approximate Maximum Vertical Compilation-Scale = \(\frac{0.950}{160} = \frac{1}{168}\)
- Approximate Horizontal Scale (6 inch cones) = \(\frac{166}{152} = \frac{1}{154}\)
- Adopted Horizontal Scales, \(S_h\) = \(\frac{1}{155}\)
- Projector Separation, \(b\) = 60 meter
- Vertical Scale, \(S_v\) = \(\frac{1}{155} = \frac{152.30}{166.09}\)
- Minimum Projection-Distance = 93 mm
- Maximum Projection-Distance = 160 mm

To show the topographic features of a near-vertical ice-cliff surface, a vertical datum plane was required. This introduces some difficulty in the concept of distances which might be scaled from the resulting map. A normal map shows linear distances by their orthographic projection onto a horizontal plane. A distance scaled from a normal map comprises the horizontal \(X\) and \(Y\) components while the vertical \(Z\) component is illustrated by contours. The map produced in this study represents an orthographic projection onto an arbitrary vertical plane. A distance scaled from this map comprises modified \(X\) and unchanged \(Z\) components which are in the plane of the selected vertical datum. The modified \(Y\) component perpendicular to this vertical plane is illustrated by contours.

In this investigation a vertical datum plane was selected which would reduce the amount of contouring required without losing any of the topographic data desired, and would make scaled distances coincide as nearly as possible with normal map distances.

The cliff target \(X\), \(Y\) and \(Z\) coordinates were previously determined in the baseline right-handed rectangular-coordinate system. A coordinate transformation was required to determine the position of the targets in a left-handed coordinate system with respect to the final vertical datum-plane selected. (Refer to Figure 2). These vertical datum plane \(X^*, Y^*\) and \(Z^*\) coordinates were also reduced to the plotting scales to obtain their machine \(x\), \(y\) and \(z\) coordinates for stereo compilation.

Rectangular grid squares and the cliff target \(x\) and \(z\) coordinates were plotted on a base sheet using a beam and compass. Tracing table settings corresponding to a 25 cm. contour interval were computed at the selected vertical plotting scale.

The 10 by 15 cm. glass photographic plates used in the photo theodolite were, of course, too small for the Kelsh plotter plateholders. Diapositives were produced by contact printing the original negative onto the center of a standard 9½ by 9½ photographic plate of 0.06" thickness. The fiducial lines were extended by scribing to the edge of the plate. The plates were then adjusted emulsion side up in the plateholder by superimposing the extended fiducial lines over the plateholder index tabs.

Prior to compilation a check of the plotting instrument calibration was performed and adjustments made where necessary. The ball cam normally required to correct camera lens distortion was removed since the photo theodolite camera lens used was found to be essentially distortion free. In order to obtain the desired ratio between horizontal and vertical scale, the principal distance of each projector was carefully established by a series
of successive approximations using an accurate grid plate. As a final indication of the instrument adjustment, a model deformation test was made using an accurate grid. The test model was slightly deformed, concave upward, with maximum deformation in the central area. The magnitude of the deformation was generally 0.1 mm. or less.

The compilation of a near-vertical surface requires some modification of the standard plotting procedures. The arrangement of the photographic plates in the instrument results in the projection of ice cliffs surface onto the horizontal table of the plotter. The table surface then represents the $X^*Z^*$ datum plane. The position of the tracing table platen establishes the $Y^*$ coordinate for contouring purposes.

Relative orientation of those models formed by photographic plates taken with the camera tilt mechanism set on zero presented very little difficulty. To simplify the orientation procedure the projector separation was made approximately equal to the camera separation at the machine scale. The projector frame was then leveled using an accurate striding level. With the front of the projector nearly parallel to the frame, each projector was leveled individually with the striding level. Then by applying approximately equal Kappa ($K$) rotations to each projector, relative orientation could be nearly accomplished. Refinements were made by slight rotational element changes based on residual $Y$ parallax inspection.

Absolute orientation was performed by scaling and leveling the model with respect to the field control points. It was necessary to tilt the projector frame nearly 6 degrees longitudinally in order to compensate for the datum rotation caused by the coordinate transformation.

Models formed by the projection of two photographic plates, one of which was tilted upward at the time of exposure, were more difficult to orient. The projector frame and the projectors were leveled as before. The approximate Omega ($W$) tilt difference was set into the projectors, an approximate relative orientation made, and the frame tilted about 6 degrees longitudinally. Relative and absolute orientation were performed concurrently by refining the relative orientation and then warping the model to coincide with the control. Proper relative and absolute orientation were attained in this manner by successive approximation. The position and elevation of several pass points obtained from adjacent models were used to check model deformation and control point accuracy.

A few models corresponding to the West end of the cliff area could not be satisfactorily oriented. This was due to the limited extent of the cliff image on the photographic plate, tilt and insufficient control. The cliff image appeared as a longitudinal band covering only about one-fourth of the
photographic plate. An apparent relative orientation could be made, but subsequent efforts to scale and level the model indicated imperfect \( Y \) parallax removal.

Although approximate relative and absolute orientations could be made, sufficient accuracy could not be attained with the amount of control available. For this reason the West end of the cliff area was not compiled.

**Map Analysis**

Map accuracy tests by field methods are obviously impossible for a map of this nature. For this reason every effort was made to minimize errors in each phase of the work. High-precision equipment and methods were utilized during the field control survey. Self-checking computational procedures were used to insure data accuracy. The \( Z \) coordinates of all control points were computed from two different base stations. The average deviation of the computed values from the mean was about 4 mm. The horizontal coordinates may be presumed to have at least the equivalent accuracy.

Camera and stereo-plotter calibration-data were determined and the necessary adjustments made. Stereo plotting was performed with maximum possible precision. Since more than the minimum control was available in each stereo model, the relative and absolute orientation was adjusted to give the best fit to all points. The average deviation from the control was approximately 0.15 mm. at model scale in both horizontal and vertical direction. This represents approximately 2.3 cm. horizontally and 2.5 cm. vertically. A large

![Map of Ice Cliff at Red Rock Lake, Northern Nunatarssuaq, Greenland](image-url)

**Fig. 3.** Ice cliff at Red Rock Lake, Northern Nunatarssuaq, Greenland. Topographic map section based on vertical datum plane—photography July 20, 1955.
number of the contours were drawn twice and the mean contour used. The deviation of the two contours from the mean averaged about 0.3 mm. at model scale, which represents approximately 5 cm. on the ice cliff. Under these conditions it is believed that the topographic maps produced are within standard map-accuracy requirements.

CONCLUSIONS AND RECOMMENDATIONS

This investigation has shown that accurate maps of a moving vertical ice-cliff surface can be produced by adapting standard photogrammetric methods and equipment. The change in position of the ice-cliff surface can be determined at any point by a comparison of the maps obtained. (Refer to topographic maps, Figure 3 and Figure 4.) In comparison to classical ground surveying procedures, photogrammetric methods are especially advantageous in producing maps of this nature since relatively few control points are required.

To keep the required amount of contouring at a minimum when compiling with the Kelsh plotter, the base-line from which the photography is taken should, as nearly as possible, be parallel to the vertical surface to be mapped. Optimum contouring conditions can also be obtained by performing a coordinate transformation as was done in this study. However, the rotation angle corresponding to the horizontal angle between the base line and the datum plane, parallel to the vertical surface to be mapped, should not exceed the maximum permissible longitudinal tilt of the Kelsh projector frame.

Although the Kelsh plotter proved entirely satisfactory in this study, photography obtained using the Wild phototeodolite T-30 is better suited for compilation with a universal instrument which

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**Fig. 4.** Ice cliff at Red Rock Lake, Northern Nunatassuaq, Greenland. Topographic map section based on vertical datum plane—photography August 26, 1955.
can be adjusted to accommodate various focal lengths, plate sizes and camera orientations. Since the principal distance on a universal instrument can be made equal to the camera focal length, equal horizontal and vertical compilation scales can be used. In this case either normal or convergent photography can be plotted accurately. If these values are not equal, as was the case in this investigation, accurate compilation is limited to photography taken normal to the base. The disparity between the instrument principal-distance and the camera focal-length results in a variable vertical scale when convergent photography is used. If a universal instrument is not available, a camera transit or phototheodolite having a focal length within the principal distance range of the appropriate Kelsh projectors is desirable since equal horizontal and vertical scales could then be used.

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References


Project-Glacier*

An Improved Method of Recording a Glacial Advance

James A. Servizi, Univ. of Washington

Lying nearly 110 miles north of Seattle, Mount Baker thrusts its peak above the surrounding Cascade mountain range. On the northwest slope of this majestic mountain an interesting change has been taking place since about 1949, in that Coleman Glacier halted its retreat and began a rapid advance.

Attention was first focused on the glacier's advance by interested students and instructors of the University of Washington.

The data which the party recorded this summer showed an advance of 250 feet from last year's position. Over a period of six years since 1949, the glacier has advanced nearly a thousand feet.

During the first five trips to Coleman Glacier, profiles and significant points on the glacier were located using transit and stadia. Difficulty was encountered in moving about on the glacier because crevasses 20 to 100 feet in depth and varying in width from 1 foot to 10 feet were encountered frequently. Spires and blocks of ice towering approximately 30 feet above

Author's Note: Figure I is from an article which I wrote and contributed, without monetary return, to the student published Washington Engineer; November, 1955. This paper is a re-write in more technical form, of that article.

* This paper was submitted in competition for the Bausch & Lomb Photogrammetric Award, 1955.