*The Method of Least Squares Applied to Multicollimator Camera Calibration**

B. HALLERT,
Div. of Photogrammetry,
Royal Inst. of Technology,
Stockholm, 70, Sweden

A SERIES of tests have been made with photographs taken in the multicollimator at the U. S. Geological Survey, Figures 1, 2, and 3. Least squares adjustments have been made for point combinations in circles around the principal point, Figure 4. Through the adjustment primarily the six ordinary elements of orientation were determined. In addition there was determined affine deformations, standard error of unit weight and the complete error propagation from the basic observations to the

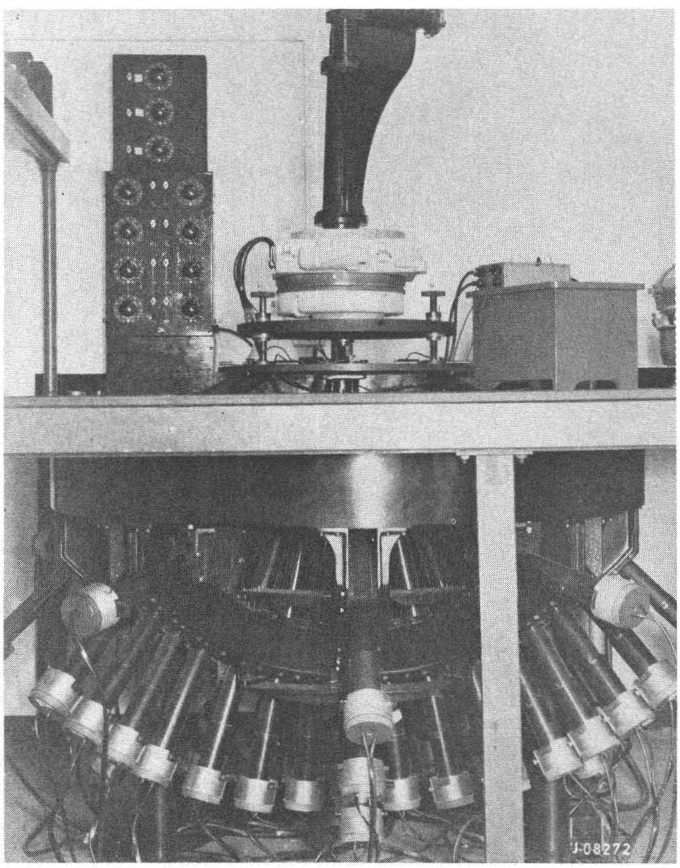


FIG. 1. The U. S. Geological Survey Multicollimator in use. There are 41 collimators, all directed toward the camera lens in angles which are determined with high geometrical quality. Courtesy U. S. Geological Survey.

* Presented at St. Louis ACSM-ASP Convention Panel: Mensuration. Sept. 10, 1962.

AUTHOR'S NOTE: Detailed information on the procedure and results, treated in this brief summary, may be obtained from the following literature: Hallert, B.: Photogrammetry. McGraw-Hill. New York, 1960. Appendix B-6, and Investigations of the Basic Geometrical Quality of Aerial Photographs and Some Related Problems. GIMRADA Research Note Nr 4, 1962.

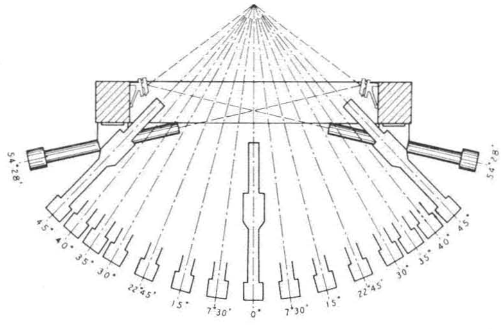


FIG. 2. Angular arrangement of collimators in main bank. The two lateral collimators have recently been added for tests of superwide angle cameras. Courtesy U. S. Geological Survey.

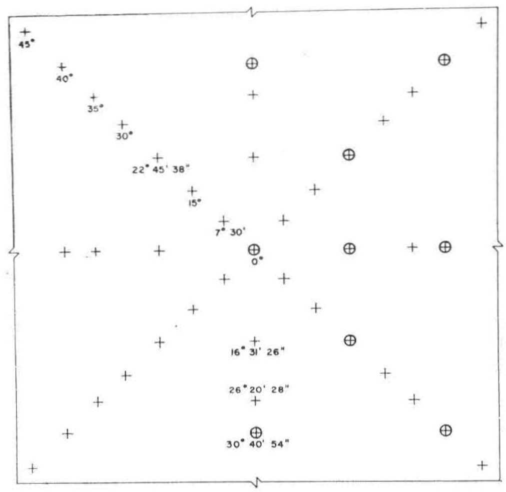


FIG. 3. Target images recorded by a 90° field camera. Courtesy U. S. Geological Survey.

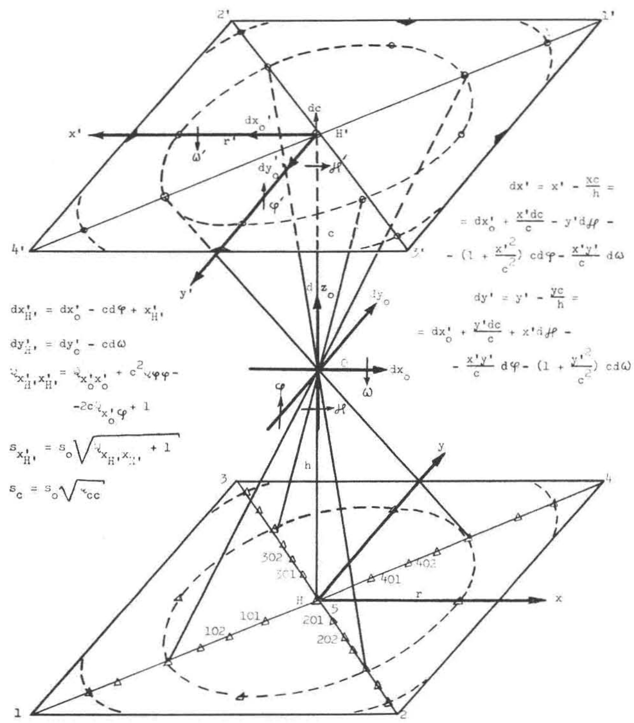


FIG. 4. The principles of tests of central perspectives according to the method of least squares. The discrepancies between measured and given coordinates, according to an ideal central projection, are interpreted as caused by the errors of the elements of orientation, other regular and irregular errors and are adjusted so that the sum of the squares of the residuals becomes a minimum. The adjustment is made separately for each circle combination of points. The determination of the standard errors of the principal point and the principal distance is shown.

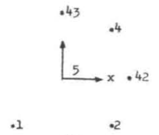
elements of orientation and other functions. The radial distortion was automatically obtained from the scale variations from circle to circle, and the standard error of this regular error was also determined from the basic standard error of unit weight and the weight number of the radial distortion. Convenient forms were used for the computations, Figure 5. A criterion for tangential distortion, founded upon statistical studies of the residuals after the adjustment, was determined. Finally, the residuals in all points were determined and shown graphically. The statistical distribution of the residuals was tested, Figure 6, and the tolerance criterion, according to the *t*-test, was applied.

Through this procedure, the regular errors of the image coordinates become uniquely determined together with their standard errors and the significance of the regular errors can easily be judged according to statistical principles. The geometrical quality of all elements of the interior orientation can be uniquely determined and expressed in terms of standard errors. This is of great importance for several purposes; for instance, the determination of the significance of possible asymmetries in the radial distortion and for the determination of the influence of the errors of the elements of the interior orientation upon the elements of the exterior orientation after single and double-point resection in space.

Particular attention was also paid to the question of the variation of the weights with the position of the points in the image. It was found that there is, already, a very pronounced weight variation of the image coordinates at the calibration of the

VII. 15-62

GRID TESTS OF CENTRAL PERSPECTIVES 9 POINTS



Circle: 4
Radius r = 88 mm
Camera const. c = 152 mm

$\frac{r^2 + c^2}{5r^2} = 0.7967$

$\frac{4r^2 + 9c^2}{40r^2} = 0.7713$

$dr = 0.088 N91 = 95$ microns

$\frac{\sqrt{2}c}{16r} = 0.088\frac{c}{r} = 0.1520$

$\frac{\sqrt{2}}{16r} = \frac{0.088}{r} = 0.00100$

$\frac{c}{40r^2} = 0.000491$

dx = x meas. - x given; dy = y meas. - y given

Point	dx microns	N91 k	N91 kdx	N92 k	N92 kdx	N93 k	N93 kdx	N94 k	N94 kdx	S k	S kdx
5	0										
1	-7	-1	+7	+1	-7	+1	-7	+1	-7	+3	-21
2	-7	+1	-7	+1	-7	+1	-7	-1	+7	+3	-21
3	+1	-1	-1	-1	-1	+1	+1	-1	-1	-1	-1
4	+4	+1	+4	-1	-4	+1	+4	+1	+4	+3	+12
12	-6			+1.4	-8.4					+2.4	-14.4
13	-8	-1.4	+11.2			+2	-16			+1.6	-12.8
42	+6	+1.4	+8.4			+2	+12			+4.4	+26.4
43	+4			-1.4	-5.6					-0.4	-1.6
[dx]	-13	N91x=	+22.6	N92x=	-33.0	N93x=	-13	N94x=	+3	Sx=	-33.4
[dx²]	267										-33.4
	dy										
5	0										
1	-3	-1	+3	-1	+3	+1	-3	+1	-3	+1	-3
2	-8	-1	+8	+1	-8	-1	+8	+1	-8	+1	-8
3	+15	+1	+15	-1	-15	-1	-15	+1	+15	+1	+15
4	+6	+1	+6	+1	+6	+1	+6	+1	+6	+5	+30
12	-14	-1.4	+19.6					+2	-28	+1.6	-22.4
13	+9			-1.4	-12.6					-0.4	-3.6
42	-3			+1.4	-4.2					+2.4	-7.2
43	+24	+1.4	+33.6					+2	+48	+4.4	+105.6
[dy]	+26	N91y=	+85.2	N92y=	-30.8	N93y=	-4	N94y=	+30	Sy=	+106.4 +106.4
[dy²]	1196	N91x N91	+22.6 +107.8	N92x N92	-33.0 -63.8	N93x N93	-13 -17	N94x N94	+3 +33		

Corrections:

$dx_o = -\frac{r^2 + c^2}{5r^2}[dx] + \frac{4r^2 + 9c^2}{40r^2} N93 = -2.8$ micr.

$dy_o = -$ " $[dy] +$ " $N94 = +4.7$ micr.

$dc = -\frac{c\sqrt{2}}{16r} N91 = -16.4$ microns

$d\kappa = -\frac{\sqrt{2}}{16r} N92 = 6.1^{cc}$

$d\varphi = \frac{c}{40r^2}(9N93 - 8[dx]) = -15^{cc}$

$d\omega =$ " $(8[dy] - 9N94) = -28^{cc}$

$[vv] = [dx^2] + [dy^2] - \frac{[dx]^2 + [dy]^2}{9} - \frac{N91^2 + N92^2}{16} - \frac{(9N93 - 8[dx])^2 + (9N94 - 8[dy])^2}{720} = 374$

$S_o = \sqrt{\frac{[vv]}{12}} = 5.6$ microns

FIG. 5. Example of form used for the adjustment computations.

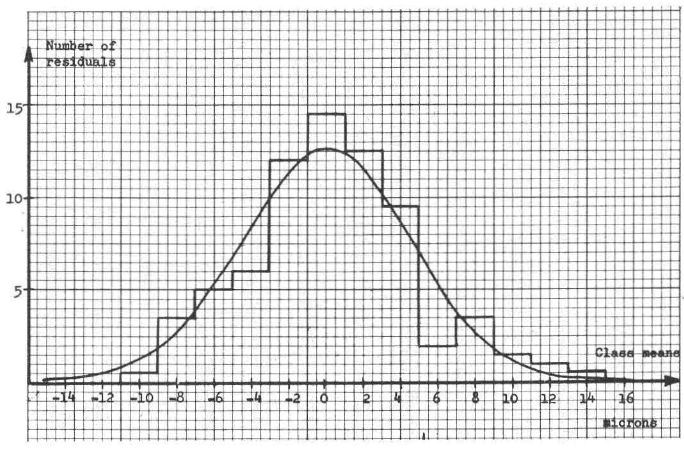


FIG. 6. Normal distribution test of image-coordinate residuals. The differences between the histogram and the normal distribution curve are used for a chi²-test.

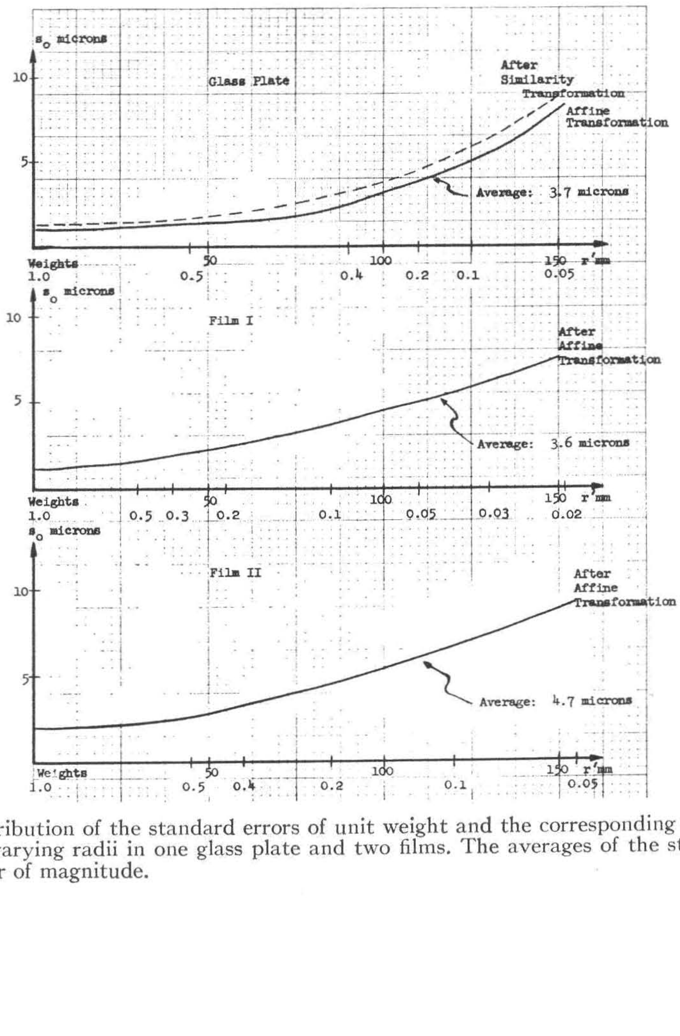


FIG. 7. Distribution of the standard errors of unit weight and the corresponding weights of image-coordinates for varying radii in one glass plate and two films. The averages of the standard errors are of the same order of magnitude.

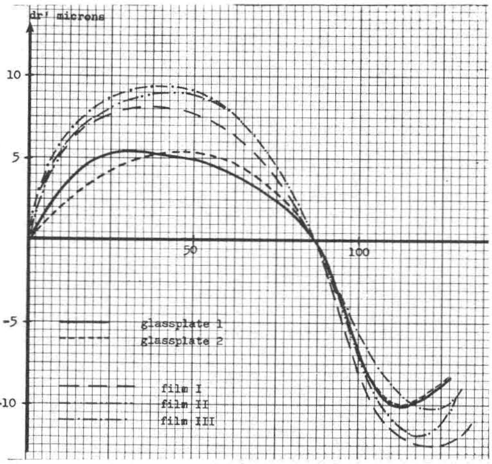


FIG. 8. Radial distortion curves from the same camera but determined from glass plates and films. The difference is significant and may be caused by lacking flatness of the films.

cameras and that this variation confirms the results of previous investigations of aerial photographs taken under operational conditions, Figure 7.

The geometrical quality of glass plate and film negatives was compared. Two types of film were used. In one of them there was a very pronounced affine shrinkage. After correction for the affinity, the averages of the standard errors of unit weight became practically equal, 3 to 4 microns, for the glass plates and the films. Further, a significant difference in the radial distortion curves was found from the glass plate negatives in comparison with the film negatives, Figure 8. This indicates that there are additional sources of this regular error in the film negatives, probably caused by lacking flatness in the supporting back of the magazine.

In summary, the application of the method of least squares to the adjustment of the single-point resection problem in connection with camera calibration in a multicollimator has proved to be of great value. In particular it has proved indispensable for the determination of unique values of the elements of the interior orientation, regular errors of the image coordinates and the standard errors of these data. The weight relation of the image coordinates can also be uniquely determined, which is of basic importance for analytical photogrammetry. The laboratory calibration should always be completed through grid tests under operational conditions.

*Elevations from Parallax Measurements**

CHRISTOS HADJITHEODOROU,
Cornell Univ.,
Ithaca, N. Y.

I. INTRODUCTION

IN THIS paper a survey is made of methods for determining absolute or differential elevations of points on the ground, by use of parallax measurements from a stereoscopic pair of vertical, or nearly vertical, aerial photographic prints. It also includes a proposal for improvement of the calculation of flying height, H, for use in the computation of elevations determined from parallax measurements.

"Parallax" is the term often used to denote displacement of one object with relation to another. In photogrammetry, parallax on aerial photographs is expressed in terms of rectangular coordinates X and Y, with the principal point of the photograph as the origin of the axes, and with the X-axis parallel to the line of flight. "Absolute stereoscopic parallax" then, or simply "parallax" is assumed to mean displacement along or parallel to the line of flight, Y-parallax being displacement at

* This paper was the winner in the Competition for the Bausch and Lomb Undergraduate Award of the American Society of Photogrammetry.