*Atmospheric Refraction and Its Distortion of Aerial Photographs**

ARTHUR H. FAULDS,† *Associate Professor*
and ROBERT H. BROCK, JR., *Assistant Professor,*
Civil Engineering Dept.,
Syracuse University, Syracuse, New York

ABSTRACT: *A method has been developed for computing the effects of atmospheric refraction and earth curvature on aerial photographs which allows the use of in flight atmospheric sampling data. Tables have been computed on an IBM 650 for altitudes ranging from 2,000 to 150,000 feet using a model atmosphere.*

A procedure for determining the associated photographic coordinate corrections is included which utilizes the Church angles of tilt and swing.

WHEN an optical ray travels from a point on the surface of the earth to a point in space, the ray is continuously refracted as it passes through an atmosphere of continuously decreasing density. Atmospheric refraction thus "bends" optical rays to curvilinear form and this effect must be considered in aerial photogrammetry when the utmost accuracy is desired. As shown in Figure 1, an optical ray emanating from point M on the ground would appear at point m on a vertical aerial negative if the density, and therefore the index of refraction, of the atmosphere was constant. Actually, however, the optical ray follows a curved path with the result that the image of point M is formed at m' rather than at m. Unless a correction is applied for the displacement $m'm$ on the photograph, photogrammetric procedures will incorrectly establish the position of point M as being at M'. The magnitude of this error will depend upon atmospheric conditions, the angle between the optical ray and the vertical line through the exposure station, and the elevations of the ground point and the exposure station.

The relationship between atmospheric density and index of refraction is given by the expression

$$n = 1 + (n_0 - 1)\rho/\rho_0 \tag{1}$$

where

n = index of refraction at altitude in question,
n_0 = index of refraction at sea level,
ρ_0 = air density at sea level,
ρ = air density at altitude in question.

In analyzing the effect of atmospheric refraction on an optical ray, it is convenient to consider that the atmosphere is comprised of a series of distinct layers whose boundaries form a set of concentric spheres about the center of a spherical earth, and that the optical ray is refracted only as it passes from one layer to the next. Then, according to Snell's Law:

$$n \sin i = n' \sin i' \tag{2}$$

where

n = index of refraction of the first layer of atmosphere,
n' = index of refraction of the second layer of atmosphere,
i = angle between the incident optical ray and the normal to the boundary between the two layers,
i' = angle between the refracted ray and the normal to the boundary between the two layers.

If the atmospheric layers are assumed to be shallow, the difference in index of refraction between layers will be small and $(n+dn)$ may be substituted for n' and $(i+di)$ for i' in Equation (2). Thus,

$$n \sin i = (n + dn) \sin (i + di). \tag{3}$$

But, by Taylor's Theorem,

$$\sin (i + di) = \sin i + \cos i \, di - \cdots . \tag{4}$$

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† Presently Chief Photogrammetrist for Fairchild Camera and Instrument Corporation, No. 5 Aerial Way, Syosset, Long Island, New York.

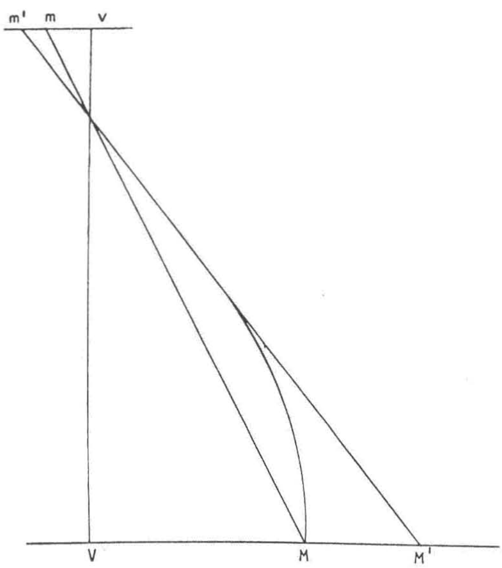

FIG. 1. Diagram showing the effect of atmospheric refraction.

Under the assumed condition of shallow atmospheric layers, the numerical value of di will be small and therefore higher order terms of this series may be neglected without significant error.

Substituting (4) in (3),

$$n \sin i = (n + dn)(\sin i + \cos i\, di)$$

which reduces to

$$n \cos i\, di + dn \sin i + dn \cos i\, di = 0.$$

For the conditions assumed, the numerical values of dn and di will be so small that the term $(dn \cos i\ di)$ can be omitted without noticeable error, and

$$n \cos i\, di + dn \sin i = 0$$

or

$$di = \frac{-dn}{n} \tan i. \qquad (5)$$

If the index of refraction were constant within the atmospheric layers, Equation 5 would be sufficient for determining the angle of refraction, di. However, even within a shallow layer of atmosphere the index of refraction varies continuously with altitude and this variation must be taken into account. Accordingly, it is necessary to determine the total angle of refraction over the interval between the altitude limits of the atmospheric layer in which refraction is considered to take place. This may be accomplished by integrating Equation 5. Thus,

$$i = \tan i(\ln n_1 - \ln n_2) + c. \qquad (6)$$

The constant of integration, c, is zero in this case. Furthermore, since the angle i on the left side of the equation is actually the angle of refraction, the symbol r is substituted to avoid possible confusion with the angle of incidence of the optical ray. The value of the angle of refraction may be determined directly in seconds of arc by multiplying the right-hand side of the equation by the factor 206,264.81. Equation 6, then, becomes

$$r'' = \tan i(\ln n_1 - \ln n_2)\,(206,264.81) \qquad (7)$$

where

r'' = angle of refraction in seconds of arc,
i = the angle between the incident optical ray and the normal to the boundary of the refracting layer of atmosphere at the point where refraction occurs,
n_1 = index of refraction of the first boundary of the refracting layer of the atmosphere,
n_2 = index of refraction of second boundary of the refracting layer of the atmosphere.

The angle between the refracted ray and the normal to the atmospheric boundary at which refraction is considered to occur is determined from the relation:

$$i' = i + r. \qquad (8)$$

The problem of determining the effect of atmospheric refraction over the full path of an optical ray as it passes through the assumed atmospheric layers is illustrated in Figure 2. The exposure station of the aerial camera, L, is situated at an altitude H above sea level and an optical ray emanating from the ground point M, whose elevation is H_M, is refracted as it passes through the spherical atmospheric boundaries located at H_1, and H_2, and H_3 above sea level. Since these boundaries are assumed to form a series of concentric spheres about the center of the earth, the radii of curvature of these surfaces and the radius of curvature of a spherical surface through ground point M may be found from

$$h_1 = H_1 + R$$
$$h_2 = H_2 + R$$
$$\cdots\cdots\cdots$$
$$h_M = H_M + R$$

where R is the radius of curvature of the earth. The refracted ray enters the camera lens at an angle α with respect to the plumb line through the exposure station. If the ray were not refracted, it would enter the lens at an angle designated as β in Figure 2. The

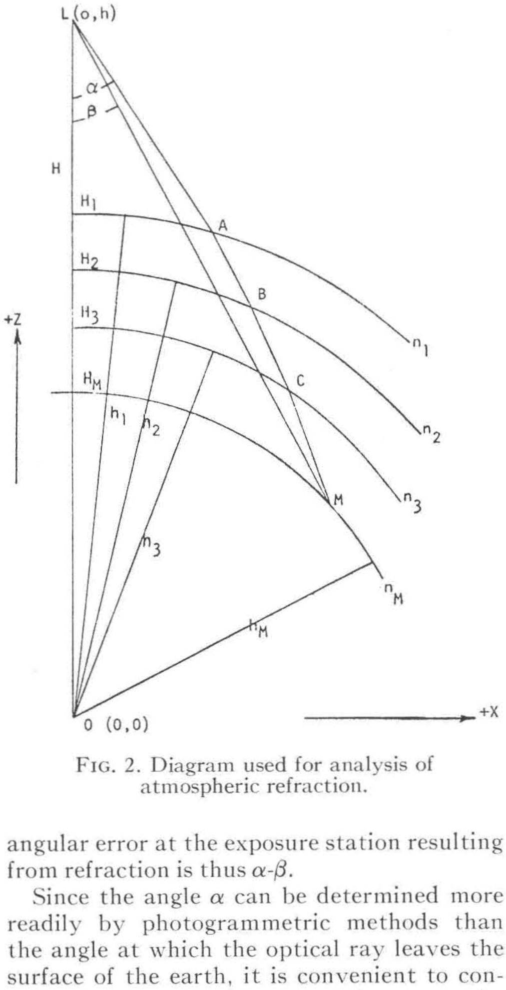

FIG. 2. Diagram used for analysis of atmospheric refraction.

angular error at the exposure station resulting from refraction is thus α-β.

Since the angle α can be determined more readily by photogrammetric methods than the angle at which the optical ray leaves the surface of the earth, it is convenient to consider that the optical ray originates at the exposure station and strikes the ground at M. No error is introduced here, because a ray from L to M would follow exactly the same path through the atmosphere as would a ray from M to L.

If a rectangular coordinate system is now established, as shown in Figure 2, with the origin at the center of the earth and with the plumbline through the exposure station as the Z-axis, the path of the optical ray can easily be traced as it passes from one atmospheric layer to the next. Under the assumption that refraction occurs only at the boundary of two atmospheric layers, the optical path within any layer will be a straight line. Accordingly, for the segment of the optical path from L to A,

$$Z_A = (X_L - X_A)\,\mathrm{ctn}\,\alpha + Z_L \tag{9}$$

where the angle α is considered to be in the first quadrant. In the XZ-plane of Figure 2, which contains the optical ray, the equation for the atmospheric boundary through point A is

$$X_A{}^2 + Z_A{}^2 = h_1{}^2 \tag{10}$$

where, as before,

$$h_1 = H_1 + R.$$

The coordinates of point A may be determined from the simultaneous solution of the Equations 9 and 10. Thus, the value of X_A may be found from

$$X_A = \left[(Z_L + X_L\,\mathrm{ctn}\,\alpha)\,\mathrm{ctn}\,\alpha - h_1{}^2(1 + \mathrm{ctn}^2\alpha - (Z_L + X_L\,\mathrm{ctn}\,\alpha)^2\right]/(1 + \mathrm{ctn}^2\,\alpha) \tag{11}$$

Once X_A is known, Z_A may be readily established from either Equation 9 or 10.

At point A, the optical ray is refracted, but the angle of incidence of the ray with respect to the normal to the atmospheric boundary through point A must be established before the magnitude of refraction can be determined. The problem is illustrated in Figure 3. It is evident in the figure that

$$i = \alpha + \gamma \tag{12}$$

where γ is the angle between the layer boundary and a horizontal line through A. The tangent of this angle is equal to the slope of the layer boundary at point A and may be determined by differentiating Equation 10 and substituting the numerical values for X_A and Z_A. Thus,

$$\gamma = \mathrm{arc\ tan}\,\frac{X_A}{Z_A}\,\cdot \tag{13}$$

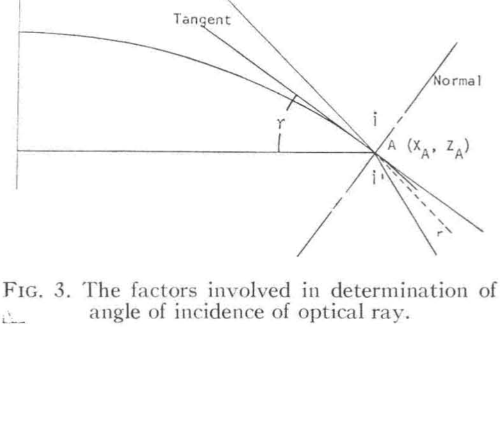

FIG. 3. The factors involved in determination of angle of incidence of optical ray.

TABLE 1

COMPUTED REFRACTION AND EARTH CURVATURE CORRECTIONS IN SECONDS AS OBSERVED FROM THE AIR STATION

Altitude (Feet above sea level)	$\alpha=80°$*	*75°*	*70°*	*65°*	*60°*	*55°*	*50°*	*45°*	*40°*	*35°*	*30°*	*25°*	*20°*	*15°*	*10°*	*5°*
2,000	7	5	3	3	2	2	1	1	1	1	1	1	0	0	0	0
5,000	20	13	10	7	6	5	4	3	3	2	2	2	1	1	1	0
10,000	37	24	81	14	11	9	8	7	6	5	4	3	2	2	1	1
15,000	54	35	26	20	16	13	11	9	8	7	5	4	3	3	2	1
20,000	62	40	30	23	19	15	13	11	9	8	6	5	4	3	2	1
30,000	75	49	36	28	22	18	15	13	11	9	7	6	5	3	3	1
40,000	94	61	44	34	28	23	19	16	13	11	9	7	6	4	3	1
50,000	109	70	51	40	32	26	22	18	16	13	11	9	7	5	3	2
60,000	104	66	48	37	30	25	21	17	15	12	10	8	6	5	3	2
75,000	102	64	46	36	29	24	20	16	14	12	9	8	6	4	3	1
100,000	82	49	35	27	22	18	15	12	10	9	7	6	5	3	2	1
150,000	65	35	24	18	15	12	10	8	7	6	5	4	3	2	1	1

* Degress from the vertical line with the exposure station as the vertex.

Substituting (13 in (12),

$$i = \alpha + \text{arc tan}\,\frac{X_A}{Z_A}\cdot \tag{14}$$

The value for the angle of incidence, i, determined from Equation 14 is then substituted in (7), together with appropriate values for the index of refraction at the surfaces of the next layer boundaries, to establish the magnitude of the angle of refraction introduced at the point A. This angle is added algebraically to the angle α in the previous layer to obtain the angle, α_1, with respect to the plumb line through the exposure station at which the optical ray travels in the second layer.

Equations 9, 10, and 11 are then modified to read

$$Z_B = (X_A - X_B)\,\text{ctn}\,\alpha_1 + Z_A$$

$$X_B^2 + Z_B^2 = h_2^2$$

$$X_B = \left[(Z_A + X_A\,\text{ctn}\,\alpha_1)\,\text{ctn}\,\alpha_1 - h_2^2(1 + \text{ctn}^2\,\alpha_1) - (Z_A + X_A\,\text{ctn}\,\alpha_1)^2\right]/(1 + \text{ctn}^2\,\alpha_1)$$

and solved, as before, for the location of the point B, the point where the optical ray meets the next atmospheric layer boundary, as shown in Figure 2.

Equation 14 is similarly modified to obtain the angle of incidence of the optical ray with respect to this new boundary, and again, Equation 7 is used to obtain the angle of refraction.

This process is repeated from one atmospheric boundary to the next until, finally, the optical ray reaches the ground point M in Figure 2. Computation of the last segment of the optical ray furnishes rectangular coordinates of the ground point, M. With these coordinates and those of the exposure station the angle designated as β in Figure 2 is calculated. This is the angle at which the optical ray would enter the camera lens if refraction did not occur. Thus,

$$\beta = \text{arc tan}\,\left[X_M/(Z_L - Z_M)\right] \tag{15}$$

and the total angular error at the exposure station resulting from refraction is equal to the difference between angles α and β.

The foregoing procedure has been programmed on the IBM 650 and the results are listed in Table 1. A family of refraction curves based on the computational results are given Figure 4. The atmospheric parameters were assumed in accordance with the ARDC Model Atmosphere, 1959 (1) which was based in part on rocket and satellite data.

The computed refraction tables will vary according to the physical condition and layer break-down of the atmosphere.

The refraction correction is the last correction to be applied to the photographic coordinates, and must necessarily be delayed until the approximate camera position is determined from a preliminary solution.

If the preliminary solution shows the photograph to be vertical, the radial displacement, d, of the image on the photograph may be determined from the relation

$$d = f(\tan\alpha - \tan\beta) \tag{16}$$

where f is the focal length of the lens, α is the angle computed from corrected photo coordinates, and β is determined by tracing the ray from the air station to ground. (See Figure 5(a).) Also note that d could be determined by using Table 1 along with α and the approximate flying height. If the radial distance from the principal point to the image is R, then

$$R' = R - d$$

and the x_1 and y_1 photographic coordinates now also corrected for refraction would be

$$x_1 = \sin nR' \tag{17}$$

$$y_1 = \cos nR' \tag{18}$$

According to Figure 5(a):

$$n = \text{arc tan } x/y$$

where x and y are the photographic coordinates corrected for all known systematic errors but refraction.

The refraction corrections to be applied to photographic coordinates in cases of tilted photography involve a considerable amount of computation.

Assuming that the approximate orientation of the photograph is known refer to Figure 5(b). The angle α must be determined in order to apply the tabular correction for refraction. This is accomplished as follows:

$$\overline{op'} = \left[(x)^2 + (y)^2\right]^{1/2}$$

$$\overline{ov} = f \tan t.$$

In order to compute the position of p' with respect to the nadir point v it is first necessary to rotate the plate coordinates x,y through the angle swing

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -\cos S + \sin S \\ -\sin S - \cos S \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}. \tag{19}$$

A translation along the y' axis is then necessary to bring the origin to the nadir point v.

$$x'' = x'$$

$$y'' = y' + f \tan t. \tag{20}$$

Also,

$$\overline{vp'} = \left[(x'')^2 + (y'')^2\right]^{1/2}$$

$$\overline{Lv} = f \sec t$$

$$\lambda = \text{arc tan } \frac{op'}{f}$$

$$\overline{Lp'} = f \sec \lambda$$

where x and y are the photographic coordinates corrected for all known systematic errors but atmospheric refraction. The law of cosines can now be applied and

$$\alpha = \text{arc cos } \frac{\overline{Lv}^2 + \overline{Lp'}^2 - \overline{vp'}^2}{2 \cdot Lv \cdot Lp'}. \tag{21}$$

Given α the refraction error, $d\alpha$, can be determined from ray tracing or the previously computed tables. Thus α_1 is known. It is now possible to determine the true position

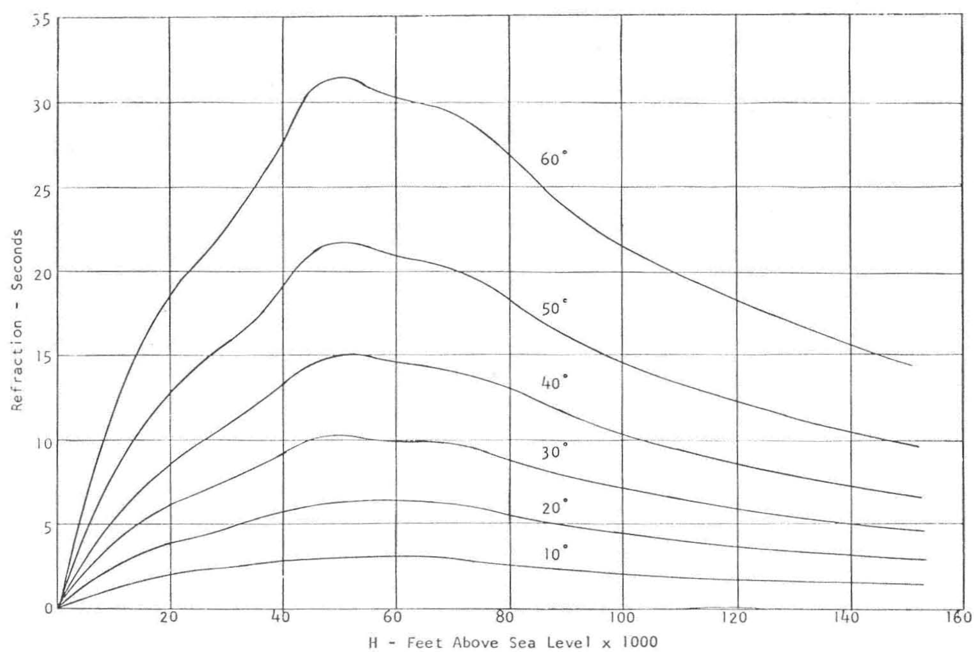

FIG. 4. Refraction curves for varying degrees from the vertical.

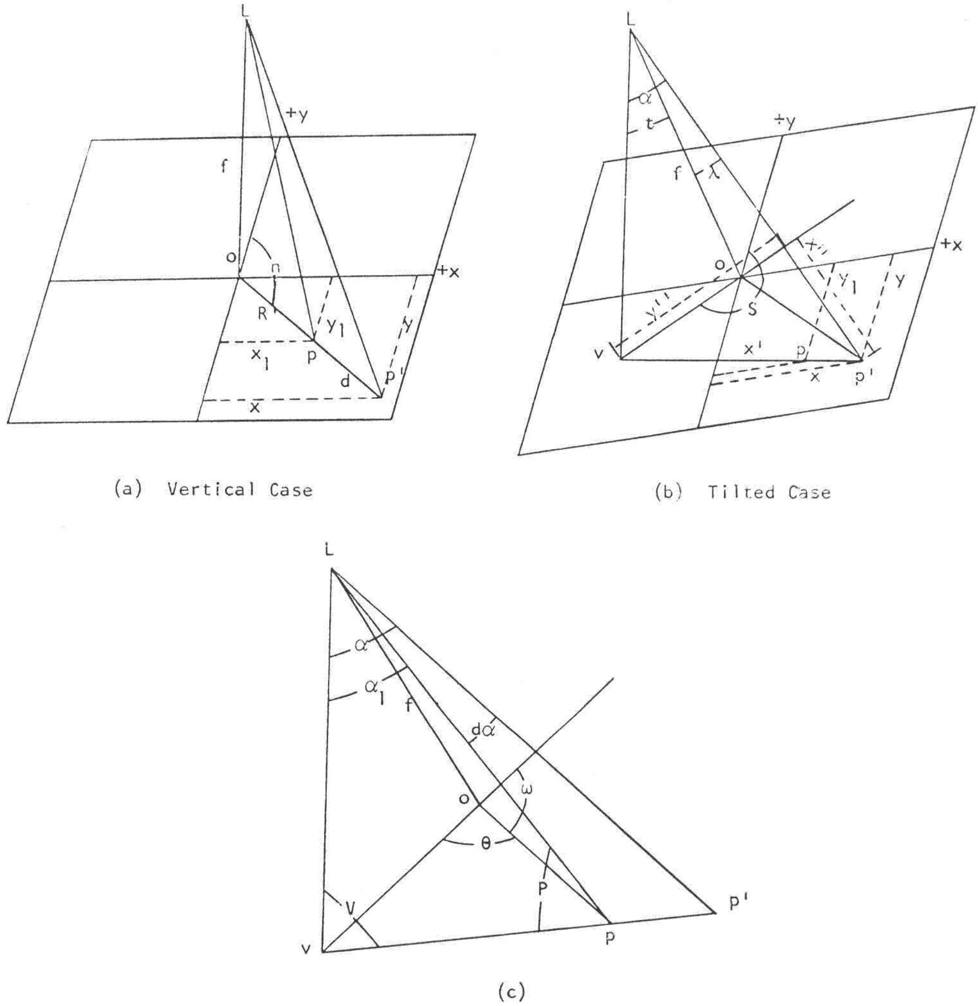

FIG. 5. Geometry for refraction corrections.

p, of the image, p' on the photograph. See Figure 5(c).

According to the law of sines,

$$V = \arc\sin \frac{\overline{Lp'} \sin \alpha}{vp}$$

Thus,

$$\text{Angle } P = 180° - V - \alpha_1.$$

Also,

$$\overline{vp} = \frac{\sin \alpha_1 \cdot Lv}{\sin P}$$

and

$$\overline{Lp} = \frac{\sin V}{\sin \alpha_1} \overline{vp}.$$

It is now possible to compute distance $\overline{op}$. See Figure 5(c).

$$\overline{op} = [(\overline{Lp})^2 - (f)^2]^{1/2}$$

Now then, $\overline{op}$ is not equal to op' and therefore the previously computed x' and y' values have changed.

From the law of cosines

$$\theta = \arccos \frac{\overline{ov}^2 + \overline{op}^2 - \overline{vp}^2}{2 \cdot \overline{ov} \cdot \overline{op}}$$

$$180° - \theta = \omega$$

$$x' = \sin \omega \cdot \overline{op} \tag{22}$$

$$y' = \cos \omega \cdot \overline{op}. \tag{23}$$

The inverse solution of equation 19 substituting in the x' and y' values of Equations 22 and 23 give the completely corrected photographic coordinates x_1 and y_1 as follows:

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\cos S - \sin S \\ +\sin S - \cos S \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} \quad (24)$$

or

$$x_1 = -\cos S \cdot \sin \omega \cdot op - \sin S \cdot \cos \omega \cdot op$$
$$x_1 = -op \sin(\omega + S) \quad (25)$$
$$y_1 = +\sin S \cdot \sin \omega \cdot op - \cos S \cdot \cos \omega \cdot op$$
$$y_1 = -op \cos(\omega + S). \quad (23)$$

These are the x_1 and y_1 coordinates for the image at p as shown in Figure 5(b).

The computation must be carried out for each image point on the photograph that is used. The amount of adjustment to the photographic coordinate for a given camera, and camera position, depends on the distance op and the angle ω.

In conclusion, it might be well to note that the only significant error in the above procedure is the assumption that the atmosphere is comprised of a series of distinct layers. This assumption results in a more or less approximate determination of refraction, depending upon the thickness assumed for the atmospheric layers. When shallow layers are assumed, however, errors introduced by this procedure should be no greater than those resulting from our inability to determine the exact atmospheric conditions which exist at the time of photography. This method of computing refraction effects does however provide a means of easily inserting "in flight" atmospheric sampling data.

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For information write to

Mr. Rupert B. Southard, Jr., 229 Burke Road, Fairfax, Va.

Abstract of next article

Model Flatness—A Guide for Stereo-Operators

By G. P. Katibah

Abstract: *Stereomodel deformations which are directly attributable to the optical characteristics of any part of a lens system are notably of a systematic nature. This paper deals with methods for isolating those deformations which are attributable to the plotter-camera combination without the influence of operational variables. The methods discussed are available to the stereo-operator, and are intended to serve as a guide to him for checking the potential flatness of the stereomodel when using different types of nominal 6-inch aerial photographs in a Kelsh-type plotter.*