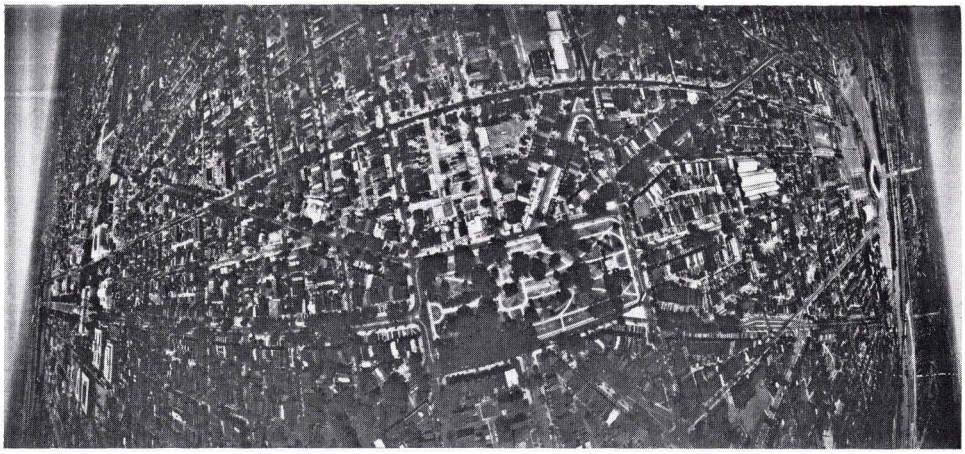

FIG. 1. Vertical panoramic photograph of Washington, D.C. with the Capitol on the left.

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Image Geometry of Vertical & Oblique Panoramic Photography

A transparent grid overlay, like the Canadian Grid, provides a practical means for overcoming the effects of image motion and geometric distortion.

(*Abstract on page 300*)

INTRODUCTION

BY FAR THE MOST COMMON TYPE of camera used in aerial photography is the frame camera, but another type, the panoramic camera, has found increasing usage in recent years. This increased usage stems from the extensive enlargement of the continuous field of view for each photograph, a factor which comes at the cost of distorting the view of the ground space.

DESCRIPTION OF PANORAMIC PHOTOGRAPHY

In its simplest form the panoramic camera consists of a film curved in the shape of a cylinder with the lens on the axis of this cylinder and an exposure slit just in front of the film. The radius of the cylinder is the focal length of the lens. The film is exposed by rotating both lens and slit on the axis of the cylinder, as shown in Figure 3.

It is easy to visualize how this camera achieves a much larger field of view. By rotating the lens the viewing angle is continuously varied for the same piece of film. In one direction the coverage extends to nearly 180°, limited only by the film getting in its own way. For some of the more sophisticated configurations even this limitation is circumvented, with the result that more than 180° of coverage is achieved. This is an important factor, for it allows horizon-to-horizon coverage in vertical aerial photography.

A typical panoramic photograph with horizon-to-horizon coverage is shown in Fig-

* Two previous articles on the same general subject are cited as References 5 and 6 at the end of this article.

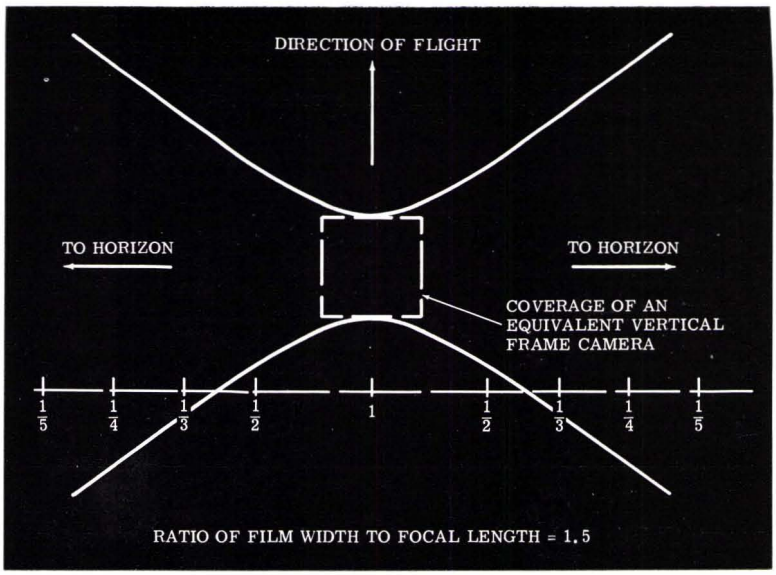


FIG. 2. Top view of the ground coverage for a vertical panoramic photograph. The fractions refer to the ratio of the scale to the scale of a vertical frame photograph.

ure 1. This is a view of Washington, D. C., with the Capitol building appearing on the left. Note that the scale diminishes toward the horizon with a corresponding increase in the area coverage. The actual coverage of a panoramic photograph is drawn in Figure 2. The borders of this coverage are hyperbolas.

Included among the more sophisticated panoramic configurations[1] is a camera with a non-rotating lens and the film stretched between two rollers behind an exposure slit as shown in Figure 3. The panoramic coverage is achieved by rotating a mirror or prism in front of the lens in synchronization with the movement of the film on the rollers. The rotation of the reflector accomplishes the lateral coverage by bending the light rays. Actually a mirror is impractical because it fails to reflect the light rays when viewing directly below. This obstacle is overcome by replacing the mirror with a double dove prism shown in Figure 3, which reflects the light rays in all positions.

Although a trimetrogon array of frame

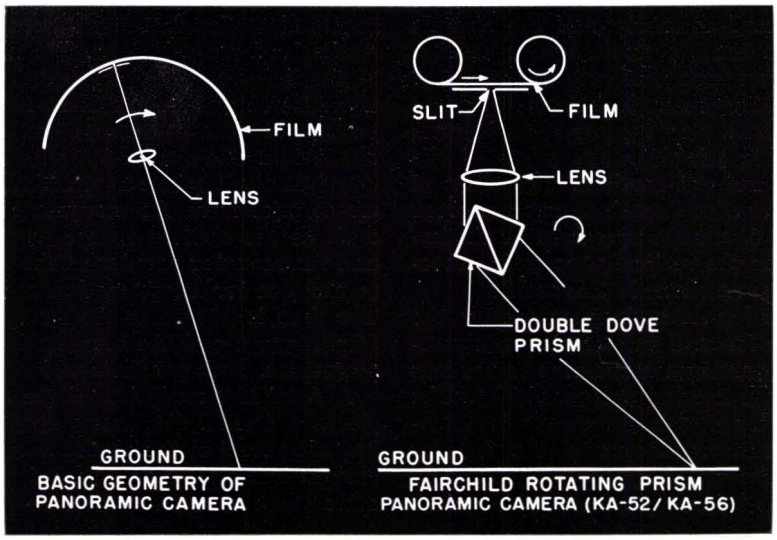


FIG. 3. Simplified diagrams of two types of panoramic cameras.

cameras consisting of a vertical camera and two side oblique cameras is also able to achieve horizon-to-horizon lateral coverage, its imagery is not on a single strip of film and the system requires three cameras. If longer focal length lenses are included in the oblique cameras to achieve higher ground resolutions, abrupt changes in scale result between the vertical and oblique cameras.

Image Motion and Distortion

Two sources of image motion in aerial photography are the forward velocity of the aircraft and the random movements due to varying air densities and atmospheric turbulence. The random motions of the aircraft generally are not a problem except at high altitudes where high resolution is necessary. A stabilized mount may be included to damp out the motions. At very low altitudes fast shutter speeds minimize the effect of the larger pitch and roll motions. The effect of random or rotational motions was examined in Reference 6, but the equations for the image motion on the vertical panoramic photograph are repeated later in this article.

For vertical panoramic photography the effect of the forward velocity is greatly reduced by imparting a movement to the lens or film (image motion compensation). The proper velocity of this movement is determined by a measurement of the angular velocity of the object space, obtained from a velocity-to-height sensor. The required velocity varies with the cosine of the lateral viewing angle. If the sensor and the image motion compensation are fully accurate, the only sources of image motion from the forward movement of the aircraft are the variable scale within the slit (parabolic image motion[2]) and the ground topography.

For the forward oblique position, however, an additional source exists, caused by the variation in magnitude and direction of the image velocity over the film. Complete compensation of this velocity is impossible. Fortunately the image velocity is smaller for the oblique orientation since the scale is less, so that the effect of the residual velocity may be negligible.

The forward movement of the aircraft also causes distortion in the photograph because of the focal plane shutter. The primary distortion in panoramic photography from the true picture of the ground is, of course, the panoramic distortion caused by the curvature of the film. The shutter introduces two additional image displacements. These displacements arise from the movement of the aircraft as the shutter sweeps across the film and the changing position of the lens relative to the film for image motion compensation. The first is known as the sweep positional displacement and the second, the IMC displacement. Both are depicted in Figure 4.

A detailed description of the sweep positional and IMC displacements was included in Reference 5 as the displacements apply to oblique frame photography. It is sufficient to state here that the two displacements do not cancel each other, and there is a residual displacement. However, these displacements are

Abstract: *This is the last of three articles on image motion in aerial photography. The first one, in the January 1965 issue, was a mathematical analysis of the image motion compensation and the image motion from the aircraft's* forward movement *for the oblique frame camera. The second article, in the September issue, analyzed the image motion from random* rotational *motions of the aircraft for the frame and panoramic cameras. To complete the series of studies, the image movement from the* aircraft's velocity *for vertical and oblique panoramic cameras is investigated here. A brief description of the panoramic camera includes the distortions associated with it. Several equations of interest pertain to the panoramic cameras, such as the coordinate transformation and image velocity equations, from which the proper image motion compensation velocity, residual image motion, and the image displacements are derived. An easy method of rapid location of ground points from the panoramic photograph can be applied.*

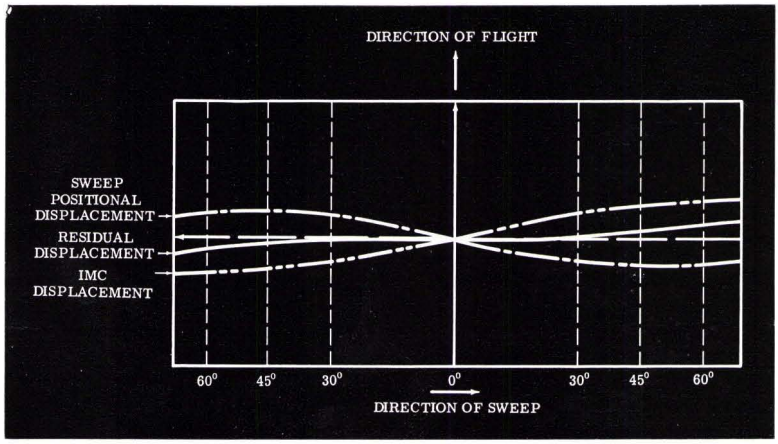


FIG. 4. Displacement of center line due to the sweep positional and IMC distortions.

known and are, therefore, recoverable. For the oblique orientations the tilt of the camera produces another distortion.

These distortions prevent rapid photointerpretation without special equipment. However, there are methods of overcoming this obstacle. One such technique involves a transparent grid overlay which allows the rapid location of image points appearing on the panoramic photograph. This technique is described in more detail later.

Description of the Equations and Method of Derivation

Equations for several parameters of interest are given in this article, as they apply to vertical and forward oblique panoramic photography. These are:

- Transformation equations from photo to ground coordinates.
- The velocity of the image from the forward movement of the aircraft.
- The velocity of image motion compensation.
- The residual image velocity.
- The image velocity from rotational motions of the aircraft.
- The sweep positional displacement.
- The image motion compensation displacement.

The equations for the IMC velocity, the residual velocity, and the sweep positional and IMC displacements are derived from the equation for the image velocity from aircraft movement. The derivation of the latter equation is straightforward for the vertical panoramic, but for the forward oblique orientation the derivation becomes complicated. The actual derivation is given in another article by the author.[3] It consists of resolving the velocity of the ground point into three mutually perpendicular components, one along the line-of-sight which does not contribute to the image velocity and the others in directions parallel to the x and y-directions of the photograph, and multiplying these components by the appropriate scale factors. This procedure is valid because one component contributes only to the velocity in the x-direction and the other to the y-direction.

The formulas for the sweep positional and IMC displacement are derived by integrating the image velocity with respect to time, where time is measured from the moment at which the nadir point is exposed. The difference in the integrations for the two displacements arises in the treatment of the sweep angle; for the sweep positional displacement this angle is treated as a constant and for the IMC displacement it is made a function of time. The reason is that the velocity of an image point depends only upon the sweep angle of the image point, which is a constant, but the velocity of the lens varies with the sweep angle of the lens.

The transformation equations for the oblique camera are derived by treating the panoramic photograph as a series of infinitesimally narrow frame photographs, each at a slightly different roll angle. Then the appropriate substitutions are made in the transformation equations for a tilted frame camera,[4] with the definitions of the pitch and roll found in paragraph (a) of Ref. 5.

The residual velocity is simply the difference between the velocity of the image and the velocity of the film or lens for image motion compensation. It does not take into account the inaccuracies of the IMC drive and the velocity-to-height sensor. However,

it does provide an idea of the minimum image motion, and from this the loss in resolution can be computed.

The equations listed below are valid for most types of panoramic cameras.

Equations for the Vertical Panoramic

1. *Transformation equations.*

$$X = \frac{x'H}{f\cos\theta} \qquad Y = H\tan\theta$$

where $x' = x - \Delta x_{\text{swp}} - \Delta x_{\text{ime}}$.

2. *Image velocity from aircraft velocity:*

$$v_{\text{im}} = \left(\frac{V}{H}\right) f\cos\theta$$

Image motion compensation velocity:

$$v_{\text{ime}} = \left(\frac{V}{H}\right) f\cos\theta$$

Residual velocity: $v_{\text{res}} = 0$.

3. *Image velocity from rotational motions.*

$$\text{Pitch:} \quad v_x = \left(\frac{f^2 + x^2}{f}\right)\cos\theta\cdot\dot{\phi}$$

$$v_y = x\sin\theta\cdot\dot{\phi}$$

$$\text{Roll:} \quad v_{\omega x} = 0$$

$$v_{\omega y} = f\dot{\omega}$$

$$\text{Yaw:} \quad v_{\kappa x} = \left(\frac{f^2 + x^2}{f}\right)\sin\theta\cdot\dot{\kappa}$$

$$v_{\kappa y} = x\dot{\kappa}.$$

4. *Sweep positional and IMC displacements.*

Sweep positional:

$$\Delta x_{\text{swp}} = \frac{Vf}{Hu}\theta\cdot\cos\theta$$

$$\Delta y_{\text{swp}} = \Delta y_{\text{ime}} = 0$$

$$\Delta x_{\text{ime}} = \frac{Vf}{Hu}\sin\theta$$

IMC:

5. *Definition of symbols.*

- x—Film coordinate in direction of flight, positive in direction of vehicle movement on film positive.
- y—Film coordinate in direction of sweep, perpendicular to x and positive when viewing to left.
- θ—Sweep angle, related to y coordinate by the equation $y = f\theta$.
- f—Focal length.
- H—Altitude above ground plane.
- X—Ground coordinate in direction of flight from nadir.
- Y—Ground coordinate to left of direction of flight and perpendicular to X.
- V—Vehicle velocity.
- $\dot{\phi}$—Pitch rate, around a horizontal axis perpendicular to the flight direction.
- $\dot{\omega}$—Roll rate, around the longitudinal axis of the aircraft.
- $\dot{\kappa}$—Yaw rate, around the vertical.
- u—Angular velocity of sweep, positive direction defined as follows: for stationary film panoramic from left to right; for rotating prism panoramic from right to left.

The equations are based upon these conditions: horizontal aircraft, level flight, no yawing or "crabbing" of the vehicle, and all ground points lying in the same ground plane.

Forward Oblique Panoramic Photography

The reasons for pointing a panoramic camera in the forward oblique direction are (1) to photograph the area in the forward direction, (2) to view the ground at an oblique angle, and (3) to view the ground with convergent stereo photography.

An oblique direction of viewing shows the ground relief more clearly and at the same time results in more area coverage. A typical high oblique photograph is shown in Figure 5. This is a view of Manhattan Island from the south at a 20° depression angle. It shows all the area in front of the aircraft from the left horizon to the right horizon, and it gives some indication of the topography of the ground.

Convergent photography is stereo photography with two cameras oppositely tilted in the direction of flight. Its main advantage is that ground points appearing in the areas common to both cameras can be located to a high accuracy, especially in the vertical direction. Panoramic cameras can be used in convergent photography if the panoramic distortion is rectified first.

Equations for the Forward Oblique Camera

1. *Transformation equations.*

$$X = H\left(\frac{x'\cos\phi + f\cos\theta'\sin\phi}{-x'\sin\phi + f\cos\theta'\cos\phi}\right)$$

$$Y = H\left(\frac{f\sin\theta'}{-x'\sin\phi + f\cos\theta'\cos\phi}\right)$$

where $x' = x - \Delta x_{\text{swp}} - \Delta x_{\text{ime}}$
and $\theta' = \theta - (\Delta y_{\text{swp}} + \Delta y_{\text{ime}})/f \approx \theta$.

2. *Image velocity in* x*-direction from aircraft velocity.*

$$v_{\text{imx}} = \frac{Vf}{H}\left(\cos^2\phi\cos\theta - \frac{x}{f}\sin\phi\cos\phi(1 + \cos^2\theta) + \frac{x^2}{f^2}\sin^2\phi\cos\theta\right).$$

FIG. 5. Forward oblique panoramic photograph of Manhattan. (The curvature of the horizon is due to the cylindrical geometry of the panoramic camera, and not to the curvature of the earth.)

3. *Image velocity in y-direction from aircraft velocity.*

$$v_{\text{im},\,y} = \frac{Vf}{H}\sin\phi\sin\theta\left(\cos\phi\cos\theta - \frac{x}{f}\sin\phi\right).$$

Image motion compensation velocity (if compensation is to be included in y direction):

$$v_{\text{imc}\,y} = \left(\frac{Vf}{2H}\sin\phi\cos\phi\right)\sin 2\theta$$

Residual velocity:

$$v_{\text{res}\,y} = v_{\text{im}\,y} \qquad \text{(if no IMC)}$$

$$v_{\text{res}\,y} = \frac{V_x}{H}\sin^2\phi\sin\theta \qquad \text{(with IMC in } y\text{-direction)}$$

4. *Sweep positional and IMC displacements.*

Sweep positional:

$$\Delta x_{\text{swp}} = v_{\text{im}\,x}\cdot\frac{\theta}{u}$$

$$\Delta y_{\text{swp}} = v_{\text{im}\,y}\cdot\frac{\theta}{u}$$

IMC displacement:

$$\Delta x_{\text{imc}} = -\frac{Vf}{Hu}\cos^2\phi\sin\theta$$

$$\Delta y_{\text{imc}} = \frac{Vf}{4Hu}\sin\phi\cos\phi(1-\cos 2\theta)$$

(if IMC is included)

5. *Definition of symbols.*

$x, y, \theta, f, H, X, Y, V, u$—same as for vertical case

ϕ—pitch angle of camera, positive in forward direction from vertical.

The velocity of image motion compensation for both the vertical and oblique panoramic cameras varies with the cosine of the sweep angle, which is not difficult to mechanize. The amplitude of the velocity is proportional to the cosine squared of the pitch angle, so that at the high oblique angles it can be omitted without much loss in resolution. In the sweep or y direction the equations indicate an image velocity which changes direction depending upon which side is being viewed. The compensation of this velocity consists of a sinusoidal oscillation about the panoramic cylinder axis, but it is in general not necessary.

EFFECT OF RESIDUAL IMAGE MOTION FOR FORWARD OBLIQUE CASE

These equations show that there is a residual image velocity in both the x and y-directions for the oblique panoramic camera. It is of interest to examine the relative magnitude of this velocity and its effect on the

photographic resolution. The residual motion exists because the image velocity cannot be completely compensated due to its variation in magnitude and direction over the film.

For a sweep angle of 0°, a pitch offset of 26°, and a film width which is three quarters as large as the focal length, the residual velocity at the front and rear edges of the photograph is approximately 37% of the IMC velocity. This produces a 9% loss in resolution of the focal length of the lens is 6 inches, the static lens-film resolution is 25 lines/mm, the exposure time is 1/500 second and the velocity-to-height ratio is 0.2 (low altitude mapping). The reciprocal square equation is used for the resolution computation and it is assumed that there are no other sources of image motion. For high altitude photography with a V/H ratio of 0.01, a focal length of 24 inches, and film width of $4\frac{1}{2}$ inches, a static resolution of 100 lines/mm, and the same exposure time, the loss in resolution is about 1%.

In the y or scan direction the resolution degradation is computed as 6% for a V/H ratio of 0.2, a focal length of 6 inches, a pitch angle of 26°, a scan angle of 45°, a static resolution of 25 lines/mm and a 1/500 second exposure time.

Use of a Mensuration Grid for Rapid Location

The distortion of panoramic photographs would ordinarily make the rapid location of ground points difficult since a given distance on the ground does not correspond to the same image distance everywhere on the photograph. One method of overcoming the distortions is to rectify the panoramic photograph, i.e., to convert it into an equivalent vertical frame photograph, but this requires complex machinery.

A simpler method of location exists which takes advantage of the fact that a square grid on the ground with line spacing proportional to altitude always photographs in the same pattern for a given camera. If the panoramic image of this square grid is printed on a sheet of transparent material, the sheet can be placed over the photograph (negative or positive) and the location of ground points with respect to the nadir can be readily determined. The procedure consists of measuring the grid coordinates and multiplying them by a scale factor dependent upon the altitude. Obviously this method is limited in accuracy but it is fast and simple and requires little equipment.

A similar method has been applied to oblique frame photography with considerable success. This is, of course, the utilization of the Canadian grid (Ref. 4, p. 355).

From the equations it appears that the sweep positional and IMC displacements vary with the velocity-to-height ratio, which would alter the grid pattern for different ratios. However, it can be shown that if the percentage forward overlap is fixed, the term V/Hu in the formulas is constant, i.e., the cycling rate is proportional to the V/H ratio. The formula for V/Hu is found to be:

$$\frac{V}{Hu} = \frac{(1 - o)w}{2\pi f}$$

where

V/H—velocity-to-height ratio
u—sweep velocity
o—overlap
w—film width
f—focal length

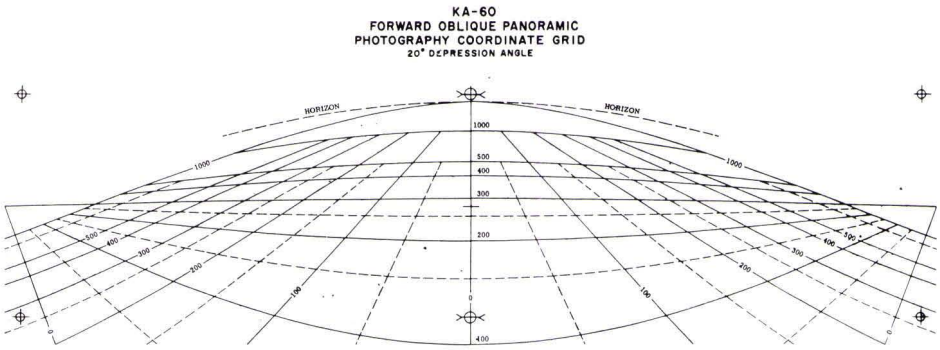
Thus, the sweep positional and IMC displacements at a particular scan angle are dependent only upon the overlap:

$$\Delta x_{\text{swp}} + \Delta x_{\text{imc}} = \frac{Vf}{Hu} \cdot (\theta \cos \theta - \sin \theta)$$
$$= \frac{(1 - o)w}{2\pi} (\theta \cos \theta - \sin \theta)$$

A mensuration grid for a Fairchild panoramic camera is shown in Figure 6b. The grid lines are printed in red to make them stand out against a black and white negative. In using the grid the coordinates as shown are multiplied by the altitude in hundreds of feet to obtain the ground coordinates. The letters and numbers at the top have been included to designate the area sectors.

Figure 7 shows the grid over an actual panoramic photograph. It is a view of an open pit mine in Japan. The border marks are used to align the grid on the photograph, with the horizon lines at the ends aiding the placement of the grid in the lateral direction, thereby minimizing errors from roll excursions of the aircraft.

The same technique of measurement can be applied to the oblique camera, since a square grid pattern is always imaged the same for a fixed pitch angle. An example of this type of grid is shown in Figure 6a. The grid is based upon a depression angle of 20° with the horizon appearing within the field of view. The coordinates as specified on the grid are referenced to the nadir point and are multiplied by the altitude in hundreds of feet. Note that the sweep positional and IMC displacements are not included because they are so small for this angle of obliquity.



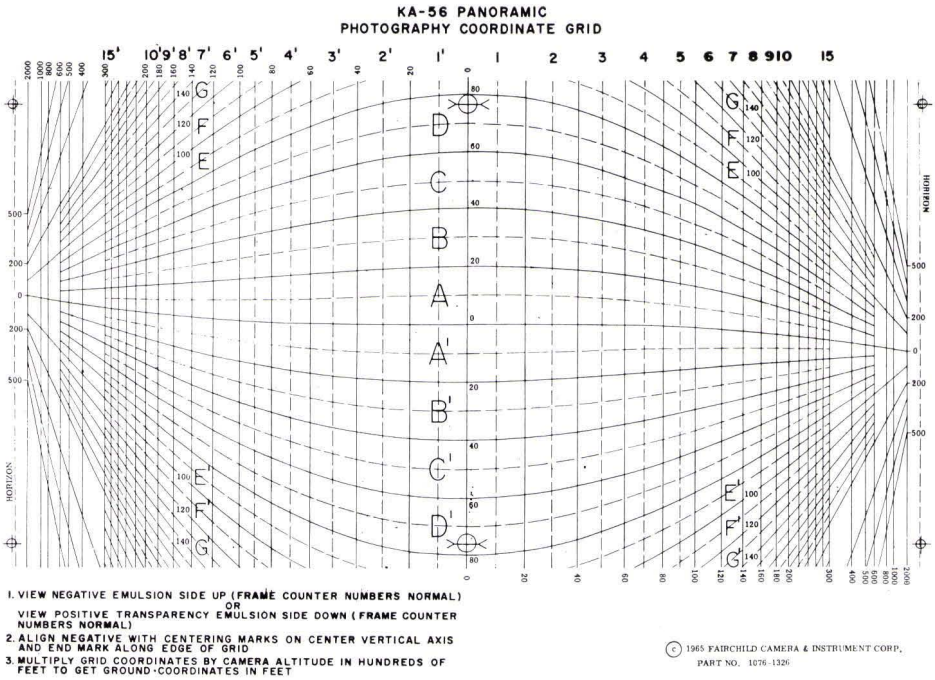


FIG. 6. (a) Grid for a forward oblique panoramic photograph.
(b) Grid for a vertical panoramic photograph.

SUMMARY

Numerous sources of image motion exist in aerial photography. The three primary external sources are the forward movement of the aircraft, the aircraft's random or rotational motions, and vibration. This article and two previous ones[5,6] investigate the image motion from the first two sources for the frame and panoramic cameras in the vertical and oblique orientations.

The optimum method of compensating the image motion for the oblique orientations is also examined. The first article[5] covered the image motion from the aircraft velocity for the frame camera, the second article[6] the image motion from rotational motions for both cameras, and here the image motion and distortion from the aircraft movement for the panoramic.

Due to its curved geometry the panoramic

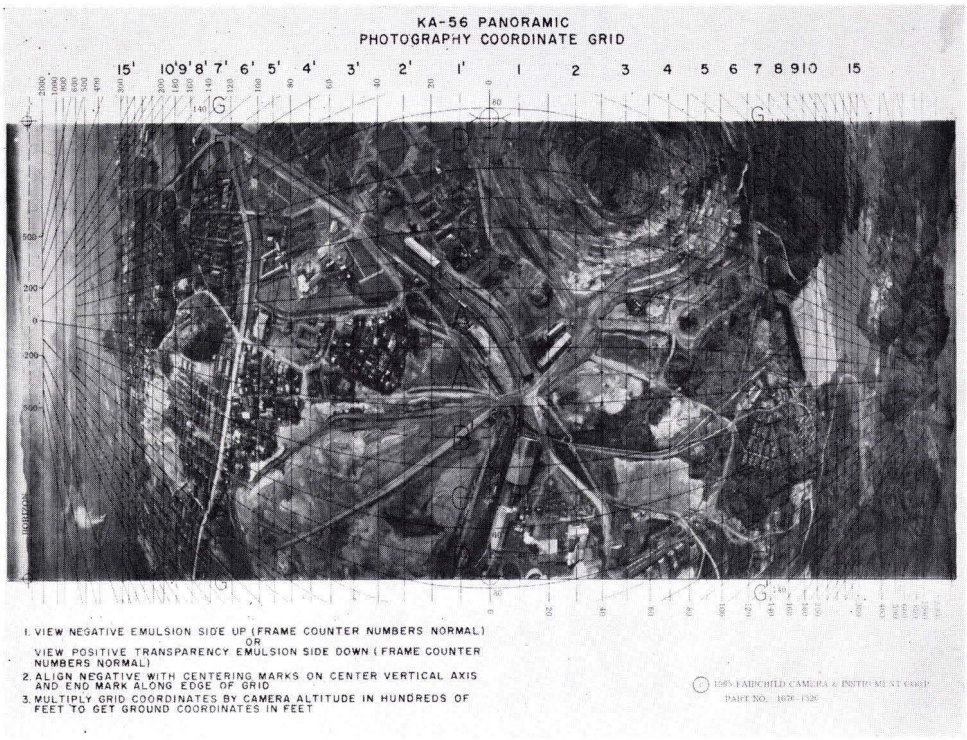


FIG. 7. Vertical panoramic photograph of a area in Japan with the coordinate grid superimposed.

camera presents a distorted picture of the ground. However, this distortion is recoverable, and it is the same for consecutive photographs. The distortions are the panoramic distortion resulting from the cylindrical format, the sweep positional displacement caused by the movement of the aircraft as the lens sweep across the film, and the IMC displacement due to the changing position of the lens (or film) for image motion compensation. Equations for these displacements and the transformation from film to ground coordinates for the vertical and oblique camera orientations are given in this article.

The velocity of image motion compensation varies with the cosine of the sweep angle for the vertical and forward oblique camera positions and the amplitude varies with the square of the cosine of the pitch angle. For the forward oblique orientations there is a residual image velocity due to the fact that the image velocity cannot be completely compensated. The effect of this residual velocity on resolution is found to be small for two sample cases.

As the distortions are the same from photograph to photograph, it is possible to obtain the location of a target directly by use of a transparent grid overlay which contains the panoramic image of a square grid on the ground. Although its accuracy may be limited, its simplicity and ease of use are strong factors in its favor. This method can be applied to both vertical and forward oblique panoramic photography.

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