Photogrammetric Brief

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An Algorithm for Computing Slope and Aspect from Elevations

ABSTRACT: *An algorithm has been developed for computing slope and aspect for a grid cell from a matrix of surrounding elevation map cells using a specialized formula for regression coefficients. It is incorporated into a system of computer programs called TOPOGO. The system can accept any map scale and cell size, and includes optional code-assignment routines.*

SLOPE AND ASPECT ARE such basic variables that they affect managerial decisions ranging from where to plant trees to setting up a ski site. The degree of slope affects sedimentation rates, as well as choice between mechanized and hand planting; choice of planting sites is affected both by slope and by insolation, which varies by aspect. Accessibility and harvesting methods are largely determined by slope in concert with other factors. Aspect strongly influences species composition of forest stands.

Users and developers of computer-based map systems such as MIADS2[1] find that elevation data can feasibly be coded from topographic maps, although the excessive cost of coding slope and aspect data has prohibited their use. But the joint cost of obtaining all three kinds of data becomes practical through use of an electronic computer. This note describes the mathematics of a system of computer programs for determining slope and aspect. The basic variable, elevation, can be obtained from topographic maps or directly from aerial photographs.

Called TOPOGO, this program computes the slope and aspect of a cell from a matrix of elevations surrounding it. The grid may be oriented in any convenient direction over the map, provided that the program is modified to compensate for deviation of azimuth from north. We compute the regression coefficients of a specialized X-matrix with equal intervals a, b, and mean equal to zero. Consider the cell layout in Figure 1 with east and west sides of the cells, a units in length, north and south sides, b units in length, and elevation of the ith cell equal to Zi. Then to compute the slope and aspect of Z_5, the values of the independent variables are:

$$X' = \begin{bmatrix} a, a, a, & 0, 0, 0, -a, -a, -a, \\ -b, 0, b, -b, 0, b, -b, & 0, \quad b \end{bmatrix}$$

and the value of the dependent variable is

$$Z' = [Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8, Z_9,].$$

The formula for the regression coefficients is

$$B = \begin{bmatrix} (Z_1 + Z_2 + Z_3 - Z_7 - Z_8 - Z_9)/6a \\ (Z_3 + Z_6 + Z_9 - Z_1 - Z_4 - Z_7)/6b \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

If B_1 is positive, the B_1 component slopes to

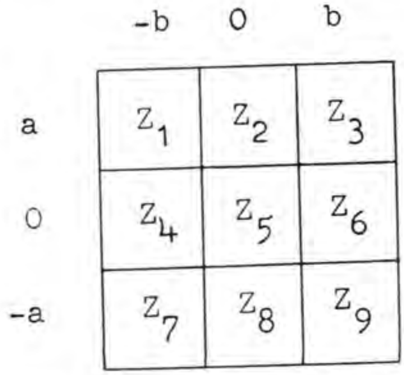

FIG. 1. Layout of cells of elevations.

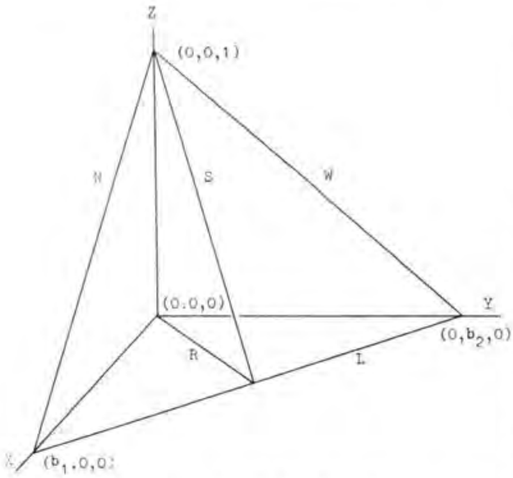

FIG. 2. Geometry of relationships between regression coefficients B_1, B_2 and slope and aspect.

the south; if negative, it slopes to the north. If B_2 is positive, the B_2 component slopes to the west; if negative, it slopes to the east.

Suppose the slope component in the north-south direction has slope α and that in the east-west direction has slope β. In Cartesian coordinates let lines N and W intersect the Z-axis at one unit with slopes α and β respectively (Figure 2). Then if N intersects X at b_1 and W intersects Y at b_2,

$$B_1 = 1/b_1, \quad B_2 = 1/b_2.$$

The slope of the plane segment bounded by N, W, and L is measured along the line S orthogonal to line L. Thus percent slope = 100/R. Then by similar triangles,

$$R/b_1 = b_2/L$$
$$R = b_1 b_2/(b_1^2 + b_2^2)^{1/2}$$

Substituting $1/B_1$ for b_1, $1/B_2$ for b_2 and simplifying,

$$\text{Percent slope} = 1/R = (B_1^2 + B_2^2)^{1/2}.$$

Also, aspect from $\chi = \arctan (B_2/B_1)$, $0° <$ aspect $\leq 90°$.

Aspect is converted to azimuth by determining its quadrant from the signs of the slope coefficients.

TOPOGO consists of six Fortran IV subroutines: (a) SLASCO computes slope and aspect for a given elevation matrix, cell size, and map scale; (b) if it is desired to change percent slope values to integer codes, TABGEN is called once for each run to generate a slope code assignment table; (c) the table is input to SLOINT, whose output is the code for each slope input; (d) if it is desired to change aspect to an integer code, ASPARS is called once for each run to generate parameters for the aspect code system desired; and (e) the parameters are input for ASPINT, whose output is the code for each aspect input. Each user must write a mainline in accordance with his own requirements and data.

Copies of the flow chart,* a listing of the program, a peripheral code assignment routine, and the definitions of the variables can be obtained from the authors or from the Director, Pacific Southwest Forest and Experiment Station, Box 245, Berkeley, California 94701.

REFERENCE

1. Amidon, Elliot L. "MIADS2. . . An Alphanumeric Map Information Assembly and Display System for a Large Computer." *U. S. Forest Service Research Paper* PSW-38. Berkeley, Calif.; Pacific Southwest Forest and Range Experiment Station. 1966.

* Although the flow chart, program listing and definitions of the variables were included in the original manuscript of the paper, the material is not reproduced here for editorial reasons.—*Editor.*