Triangulation with Super-Wide Angle Photographs

Because of the variety of ways in which the same block of photos was triangulated, comparisons could be made between them.

**INTRODUCTION**

ECONOMY, EFFICIENCY AND ACCURACY are factors which govern every surveying operation and, naturally, have a major influence on mapping as well. Therefore, a continual search exists for new improved methods, procedures and instrumentation. The introduction of the Wild Super-Aviogon 1 super-wide angle lens in 1956 and the recent development of the Zeiss RMK A8.5/23 and wide angle photographs were triangulated according to a fully analytical and semi-analytical method and were adjusted by both the sequential and the rigorous simultaneous least squares techniques. All computations were made on an IBM 360/50H computer.

**THE TEST MATERIAL**

The material used for this study was the Swiss Test Block of super-wide angle photographs taken in 1962. There are six strips in this block, with each strip containing 18 photographs. The forward overlap is 80 percent and the sidelp 30 percent. Only the first four strips were included in this investigation, because patches of clouds were in evidence on some photographs of the fifth, and especially of the sixth strip, hindering the relative orientation in the analogical instrument. Furthermore, because of the 80 percent overlap, only the alternate photographs of each strip were utilized, resulting in a block of 32 models.

The area covered by the four strips is approximately 3,000 square kilometers at the foot hills of the Alps with ground elevations ranging from 400 meters to 1,800 meters.
above sea level. The scale of the photography varies accordingly from 1:63,000 to 1:78,000 with an average of 1:70,000, which corresponds to a flying height of 7,300 meters. The Wild RC-9 camera No. 473, fitted with the Super-Aviogon lens No. 33 with a calibrated focal length of 88.24 mm. was employed to take the photographs.

There are about 360 unsignalized ground control points scattered within the test area. Out of these, 110 of them were utilized either as control or check points. These points, once identified, were marked on paper prints but not on the diapositives. All pass-points, however, were drilled in the emulsion of the diapositive with a Wild PUG-3 point transfer device. Figure 1 illustrates the distribution of the control and the check points.

ANALYTICAL AERIAL TRIANGULATION

For this phase of the investigation the measurements were performed in the Wild STK-1 stereo-comparator No. 1121, at the Department of Energy, Mines and Resources in Ottawa, under 20 times magnification. All points were measured only once. Subsequently, the block was assembled and adjusted by a sequential and also by a simultaneous adjustment procedure.

SEQUENTIAL ADJUSTMENT OF ANALYTICAL TRIANGULATION

First the strips were formed following the procedure devised by Schut. All coordinates were corrected for symmetric radial lens distortion and for the influence of atmospheric refraction. However, a correction for the regular dimensional change of the emulsion base was not considered necessary as the shrinkage factor was calculated to be 0.99999764 in both x and y direction. In order to comply with input specifications of the program, the stereocomparator observations were resolved into left and right image-coordinates beforehand. All measurements were treated with equal weights.

Next, the block adjustment was performed according to the polynomial transformation method of Schut. Beforehand, however, four test runs were made to ascertain the best weighting for the ground control points as compared to a weight of unity for the pass points. Weights of 1.0, 1.5, 5 and 15 were considered for both the planimetric and height controls in this connection. A weight of 1.0 resulted in the smallest values for the root-mean-square errors and therefore it was adopted for all subsequent adjustments of the block. This result appears reasonable in view of the fact that all control points were unsignalized natural targets.

Three different control patterns, as illustrated in Figure 2, were selected for the final adjustment and the root-mean-square errors (RMSR) for the control and check points in meters at ground scale are listed in Table 1. Eighty-five check points were used for Control Pattern 1 (see Figure 1), some of which served later as control points in Control Pattern 2 and 3. Fifteen iterations were necessary to reach a satisfactory convergence.

Fig. 1. Perimeter control and distribution of check points.
TRIANGULATION WITH S-W-A PHOTOS

SIMULTANEOUS ADJUSTMENT OF THE
ANALYTICAL TRIANGULATION

A computer program written by Mr. Carlin and named BKAERO\(^2\) was available for the simultaneous least-squares block adjustment. This program is the adaptation of the U.S. Coast and Geodetic Survey analytical block aerotriangulation program\(^6\) to the IBM System 360 Model 50H1 and is written for the Fortran IV G compiler.

BKAERO requires refined image coordinates as an input. Therefore all image coordinates were entered into the image coordinate refinement program of the U.S. Coast and Geodetic Survey\(^5\) consisting of corrections for comparator errors, film shrinkage, systematic radial lens distortion, atmospheric, refraction and earth curvature influences. Afterwards the refined image coordinates of all points on the right half of a photographic plate were then transformed into the coordinate system of the left half of the same plate, thus referring all points in one particular image to a common coordinate system. This operation was performed by the program RTRANS, written in Fortran IV and employing complex number algebra for the least-squares solution of the transformation parameters.\(^9\)

Now the image coordinates were ready for the block adjustment. Control Pattern 1 was only utilized at this time and the RMSE values obtained for the control and check points are listed in Table II.

<table>
<thead>
<tr>
<th>Control Pattern 1</th>
<th>Control Pattern 2</th>
<th>Control Pattern 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_x)</td>
<td>2.37</td>
<td>2.43</td>
</tr>
<tr>
<td>(M_y)</td>
<td>2.47</td>
<td>2.47</td>
</tr>
<tr>
<td>(M_{xy})</td>
<td>3.42</td>
<td>3.54</td>
</tr>
<tr>
<td>(M_x)</td>
<td>1.80</td>
<td>1.77</td>
</tr>
<tr>
<td>(M_{xy})</td>
<td>0.25(^{\mathrm{c}})</td>
<td>0.24(^{\mathrm{c}})</td>
</tr>
</tbody>
</table>

FIG. 2. Schematic diagram of the control patterns.

SEMI-ANALYTICAL AERIAL TRIANGULATION

All control and pass points which were part of the analytical aerial triangulation were now measured in the stereoplottor Kern PG2 No. 1184 of the Department of Surveying Engineering at the University of New Brunswick. Relative orientation was performed according to the empirical method, a nominal model scale of 1:60,000 was introduced and all points were measured only once. Neither the compensation device for the earth curvature nor the lens distortion compensation plate was employed as both of these errors were corrected numerically later on.

SEQUENTIAL ADJUSTMENT OF SEMI-
ANALYTICAL TRIANGULATION

First the observed model coordinates were entered into program MODLRFN, written by Mr. Okuwa.\(^9\) This program transformed the model coordinates into image coordinates, applied a correction for the effect of the symmetric radial lens distortion and of the atmospheric refraction, then transformed all points back into model coordinates. A correction due to earth's curvature was applied directly to the model coordinates.

Schut's method\(^1\) was employed again for the formation of the strips and the block and his method of polynomial transformation was selected for the adjustment as well. Weighting, the degree of the polynomials, and the number of iterations were exactly the same.
TABLE II. RMSE of Analytical Triangulation with Simultaneous Adjustment

<table>
<thead>
<tr>
<th></th>
<th>Control</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_x$</td>
<td>2.01</td>
<td>2.67</td>
</tr>
<tr>
<td>$M_y$</td>
<td>2.02</td>
<td>2.95</td>
</tr>
<tr>
<td>$M_{xy}$</td>
<td>2.86</td>
<td>3.97</td>
</tr>
<tr>
<td>$M_z$</td>
<td>1.28</td>
<td>2.47</td>
</tr>
<tr>
<td>$M_{z,H}$</td>
<td>0.17%</td>
<td>0.34%</td>
</tr>
</tbody>
</table>

as for the sequential analytical aerial triangulation. All three control patterns were tested and the results are given in Table III.

SIMULTANEOUS ADJUSTMENT OF SEMIANALYTICAL TRIANGULATION

As the final phase of this investigation the independent model triangulation data were adjusted in a completely rigorous fashion. A procedure developed by Mr. Maarek at the Department of Surveying Engineering of the University of New Brunswick7 was followed in this connection.

The adjustment was divided into two parts. First the pass points were assigned approximate ground coordinates from the results of the previous polynomial adjustment. These preliminary coordinates together with the refined image coordinates, which were obtained through program MODLRFN, and ground coordinates of the control points were entered into a space resection program.

Approximate values for the rotation elements of each image, approximate coordinates for the exposure stations and an error equation for each control and pass point constituted the output. The final least-squares adjustment of the block was performed by a separate program, written in Matlan language. Unit weights were assigned to all observations.

Unfortunately, at the time of this experiment the latter program was not yet optimized for production and incorporated several statistical analyses of the adjusted parameters. Therefore the capacity of the program was rather limited whereby only the southeastern quarter of the block was included in this study. This quarter-block consisted of two strips, four models each, with 12 horizontal and vertical control points distributed along the perimeter and with 18 check points at the interior. Table IV lists the results obtained.

In order to make a comparison possible between the simultaneous block adjustment of the semi-analytical triangulation and the other procedures employed, the same quarter-block was processed according to the three previously described methods as well. The results of these adjustments also appear in Table IV.

EVALUATION OF THE RESULTS

Due to the variety of ways in which the same block of photographs was triangulated it is possible to draw comparisons between different methods of aerial triangulations. It is apparent from Table IV, in which results by all four triangulation methods are listed, that the independent-model triangulation with polynomial adjustment produced the least favorable results for both planimetry and heights. Therefore, by selecting this method as a reference for comparison, one arrives at the average percentages of improvements gained by the remaining methods as listed in Table V.

It is interesting to note that the accuracy attained by the independent-model aerial triangulation can surpass that of the analytical method with polynomial adjustment if the former is subjected to a simultaneous block adjustment, a fact worth considering by mapping organizations who do not wish to invest in instrumentation required for a fully analytical aerial triangulation. Further tests are recommended to confirm this finding.

<table>
<thead>
<tr>
<th></th>
<th>Control Pattern 1</th>
<th>Control Pattern 2</th>
<th>Control Pattern 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control</td>
<td>Check</td>
<td>Control</td>
</tr>
<tr>
<td>$M_x$</td>
<td>2.93</td>
<td>4.01</td>
<td>2.90</td>
</tr>
<tr>
<td>$M_y$</td>
<td>3.10</td>
<td>4.08</td>
<td>3.12</td>
</tr>
<tr>
<td>$M_{xy}$</td>
<td>4.30</td>
<td>5.75</td>
<td>4.25</td>
</tr>
<tr>
<td>$M_z$</td>
<td>2.53</td>
<td>3.71</td>
<td>2.49</td>
</tr>
<tr>
<td>$M_{z,H}$</td>
<td>0.34%</td>
<td>0.51%</td>
<td>0.33%</td>
</tr>
</tbody>
</table>
Finally it should be mentioned that, unlike that in some tests reported in the literature, the observations were conducted in a production-like manner and all points were measured only once. The reliability of the observations, however, is no way questioned because different operators were employed for the analytical and the independent-model triangulation, and the solutions by the two methods compare favorably.

- In a practical sense, one may utilize aerotriangulation solutions as a model-orientation control for the larger-scale mapping photographs as suggested by Eichert and Eller, or one may perform the mapping directly from the super-wide angle photos as recommended by Gauthier.

An interpretation of the accuracy requirements stated by Eichert and Eller for the control of 1:24,000 mapping suggests that, for 1:50,000 mapping with 50-foot contours, the root-mean-square horizontal error of well defined, point-size map features shall not exceed 50 feet (15 m) and the root-mean-square vertical error at model orientation points shall not exceed 0.25 contour interval, that is, 12.5 feet (3.8 m). All results obtained as part of this investigation satisfy these requirements although the height accuracy of the polynomial adjustments with Control Pat-
terns 1 and 2 is close to the limit of the specification. Therefore these results indicate that the aerotriangulation of super-wide angle photographs covering an area of 3,000 km² at a scale of 1:70,000 to 1:80,000, even with un-signalized perimeter control only for both planimetry and height, can give results that are satisfactory for mapping at a scale of 1:50,000 with 50-foot contours.

Acknowledgement

The author wishes to express his appreciation to Mr. J. Gauthier of the Department of Energy, Mines and Resources who suggested to undertake this investigation, arranged the stereo-comparator measurements and made the material used for this study available. Furthermore, a special thanks is extended to Mr. B. O. Okuwa who performed the observations on the Kern PG-2 and processed all the measurements.

REFERENCES