Analytical Self-Calibration

As a result of the determinacy afforded by convergent photographs, it becomes unnecessary to calibrate a photogrammetric test range.

INTRODUCTION

Thus, in Brown (1964) an extended error model was developed to treat the coefficients of decentering distortion as parameters subject to recovery along with those of symmetric radial distortion and the elements of interior orientation. These developments were further advanced in Brown (1968) wherein in the concept of SMAC (Simultaneous Multi-frame Analytical Calibration) was formulated to admit an indefinite number of frames to a solution for a single set of interior orientation and lens distortion coefficients.

ABSTRACT: A camera calibration technique developed by DBA treats the parameters of the inner cone as variables subject to adjustment or recovery in a simultaneous analytical stereotriangulation solution. The formulation, being completely general, admits virtually any external information (e.g., control points, camera positions, etc.) as appropriately constrained parameters. Moreover, the method is applicable regardless of whether the camera is focussed for infinity or at a finite object distance. Where certain configurations of convergent photographs are employed, a complete calibration may be recovered without recourse to absolute control in the object space. It is in this regard that the process is referred to as analytical self-calibration. The mathematics of the technique applies to the in-flight calibration of the 500-mm Hasselblad camera of APOLLO 14.

covered simultaneously with the orientation of a ballistic camera. Analysis of residuals from numerous subsequent calibrations led Brown to the conclusion that, in many instances, distortions resulting from imperfectly centered elements of an objective were sufficiently large as to produce untenable systematic error in photogrammetric reductions.

* In part, a synopsis of a paper presented in September 1971, San Francisco, Calif., Symposium on Computational Photogrammetry under the title "In-Flight Calibration of the Apollo 14 500-mm Hasselblad Camera." Mr. Kenefick now heads John F. Kenefick Photogrammetric Consultant, Inc., P.O. Box 2602, Satellite Beach, Fla. 32937.
Aside from developments directed toward calibrating cameras focused for infinity, DBA, has also undertaken a multitude of close-range photogrammetric projects (Kenefick (1971)) which heretofore have demanded different means for calibrating cameras necessarily focussed for finite distances. Brown (1971) has reported one close-range range calibration technique which is called the analytical plumb line method. Although extremely simple in its practical application, coordinates of the principal point and the principal distance cannot be recovered. In principle if these parameters are required, Aerial SMAC could be applied to the task. Here again, however, a test range must be established and maintained such that its relative accuracy is well below the random noise level of the photogrammetric triangulation. Typical accuracies required of our close-range work are oftentimes upwards of 1 part in 100,000 of the object's major dimension. Consequently, a calibration range compatible with the photogrammetric accuracy must be at least on the order of 1 part in 300,000 in terms of relative accuracy over its full dimension. From a practical standpoint, we view establishing and maintaining such a test range as being economically infeasible.

**Analytical Self-Calibration**

Our desire to overcome problems arising from significant errors in photogrammetric test ranges as well as for a generalized calibration scheme, applicable whether the camera is focussed for infinity or at a finite distance, led to an in-house development of a calibration program which satisfies these requirements. Development and refinement of the software actually spanned a number of years during which time Brown's error model for the inner cone was extended and implemented much as it had been in the stellar calibration programs (see Introduction). Recent experiments with the fully developed program have demonstrated that a complete calibration of the inner cone may be recovered without absolute control in the object space if certain configurations of convergent photographs are employed.

The mathematical basis for the program is set forth in Brown, Davis, and Johnson (1964) wherein a completely general formulation was developed for the simultaneous reduction of large photogrammetric blocks. The formulation specifically allowed introduction of *a priori* constraints on any of the exterior projective parameters and/or ground points by treating *a priori* knowledge as direct observations of these parameters. Although the general development was designed to admit other parameters (unspecified) in the reduction, the specialization for the accommodation of parameters of the inner cone (*i.e.*, $x_p$, $y_p$, $c$, $k_1$, $k_2$, $k_3$, $p_1$, $p_2$) as variables subject to adjustment or recovery was not explicitly provided. The new program specifically makes provision for these parameters and, therefore, one may view this program as a simultaneous block analytical stereotriangulation program with an error model for the parameters of the inner cone.

**Observational Equations**

The projective or collinearity equations of analytical photogrammetry provide the basic framework for the analytical calibration scheme. Inasmuch as the parameters of the inner cone are to be recovered simultaneously in a block triangulation, the projective equations are augmented with the parameters of the inner cone, as follows:

$$
\begin{align*}
\dot{x}_{ij} - x_p + \dot{x}_{ij}(k_1r_{ij}^2 + k_2r_{ij}^4 + k_3r_{ij}^6) + p_1(r_{ij}^2 + 2\dot{x}_{ij}^2) + 2p_2\dot{x}_{ij}\dot{y}_{ij} \\
= c \cdot \frac{(X_j - X_c)A_i + (Y_j - Y_c)B_i + (Z_j - Z_c)C_i}{(X_j - X_c)D_i + (Y_j - Y_c)E_i + (Z_j - Z_c)F_i} \\
\dot{y}_{ij} - y_p + \dot{y}_{ij}(k_1r_{ij}^2 + k_2r_{ij}^4 + k_3r_{ij}^6) + 2p_1\dot{x}_{ij}\dot{y}_{ij} + p_2(\dot{r}_{ij}^2 + 2\dot{y}_{ij}^2) \\
= c \cdot \frac{(X_j - X_c)A'_i + (Y_j - Y_c)B'_i + (Z_j - Z_c)C'_i}{(X_j - X_c)D_i + (Y_j - Y_c)E_i + (Z_j - Z_c)F_i}
\end{align*}
$$

in which

- $x_{ij}$, $y_{ij}$ are the photographic coordinates of the $j$-th ground point on the $i$-th photograph, referred to the indicated principal point as origin;
- $x_p$, $y_p$ are the photographic coordinates of the principal point of photo-
grammetry, assumed constant over all photographs;
\[ \tilde{x}_{ij} = x_{ij} - x_p; \]
\[ \tilde{y}_{ij} = y_{ij} - y_p; \]
\[ r_{ij} = (\tilde{x}_{ij}^2 + \tilde{y}_{ij}^2)^{1/2}; \]

\( k_1, k_2, k_3 \) are correction coefficients for Gaussian symmetric radial distortion, assumed constant over all photographs;
\( p_1, p_2 \) are correction coefficients for decentering distortion, assumed constant over all photographs;
\( c \) is the Gaussian focal length, assumed constant over all photographs;

\( A, B, C \)
\( \Lambda', B', C' \) = elements of the orthogonal orientation matrix \( T \) of the \( i \)-th photograph, functions of three rotation angles \( \phi_i, \omega_i, \kappa_i; \)

\[ T_i = \begin{bmatrix} A & B & C \\ A' & B' & C' \\ D & E & F \end{bmatrix}_i = \begin{bmatrix} \cos \phi \cos \kappa & \cos \phi \sin \kappa & \sin \phi \\ -\sin \phi \cos \kappa & \sin \phi \sin \kappa & \cos \phi \\ -\sin \kappa & \cos \kappa & 0 \end{bmatrix}_i; \]

\( X_i^e, Y_i^e, Z_i^e \) are the object space coordinates of the \( i \)-th exposure station
\( X_j, Y_j, Z_j \) are the object space coordinates of the \( j \)-th ground point.

Equations 1 are linearized by Taylor's series expansion about the measured quantities \( x_{ij}, y_{ij} \) and initial approximations for the unknown parameters. If all linearized equations are gathered, the collection of equations may be written in matrix notation as:

\[ \mathbf{v} + \mathbf{B} \delta + \mathbf{B} \ddot{\delta} = \varepsilon \]  \hspace{1cm} (2)^1

where
\( \mathbf{v} \) is the vector of photographic measurement residuals;
\( \mathbf{B}, \mathbf{B} \) are matrices of partial derivatives of Equations 1 evaluated with measured quantities and current values of the unknowns;
\( \delta, \ddot{\delta} \) are vectors of corrections to be applied to current values of the parameters;
\( \varepsilon \) is the discrepancy vector resulting from evaluation of Equations ( ) with measured quantities and current values of the unknown parameters;
\( m \) is the total number of photographs and
\( n \) the total number of ground points.

Following the procedure introduced in Brown, Davis and Johnson (op. cit.) we admit supplemental observation equations arising from a priori knowledge regarding any of the parameters carried in Equations 1. For convenience the supplemental equations are grouped according to the subset of parameters involved:

\[ \mathbf{v} - \ddot{\delta} = \varepsilon \] \hspace{1cm} (inner cone and exterior orientations) \hspace{1cm} (3)

\[ \ddot{\mathbf{v}} - \ddot{\delta} = \ddot{\varepsilon} \] \hspace{1cm} (ground points) \hspace{1cm} (4)

^1 In this article dotted matrices are associated with the parameters of the inner cone and elements of exterior orientation, whereas matrices with double dots are associated with the coordinates of the ground points.
where

\[\mathbf{e}, \mathbf{e}\]

are vectors of observational residuals on the parameters and

\[\mathbf{e}, \mathbf{e}\]

are discrepancy vectors, differences between observed values and current values of the parameters.

The entire set of observation Equations 2, 3, and 4 may be merged into a single expression which is conveniently written as:

\[\mathbf{v} + B\mathbf{e} = \mathbf{e}\]  \hspace{1cm} (5)

in which

\[\mathbf{v} = (v, \dot{v}, v)^T, \quad B = \begin{bmatrix} \hat{B} & \hat{B} \\ -I & 0 \\ 0 & -I \end{bmatrix}, \]

\[\mathbf{e} = (\hat{e}, \hat{e})^T, \quad \mathbf{e} = (\hat{e}, \hat{e})^T, \quad \mathbf{k} = 2mn + 8 + 6m + 3n; \quad l = 8 + 6m + 3n.\]

The covariance matrix associated with the merged observation equations is:

\[\mathbf{K} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{\hat{A}} \end{bmatrix}\]

where,

\[\mathbf{A}\]

is the covariance matrix for the measured photographic coordinates,

\[\mathbf{2m, 2m}\]

2×2 block diagonal where independence of image coordinates is assumed,

\[\mathbf{\hat{A}}\]

is the covariance matrix for the parameters of the inner cone and elements of exterior orientation, and

\[\mathbf{\hat{A}}\]

is the covariance matrix for the coordinates of the ground points, 3×3 block diagonal where independence of ground points is assumed.

**GENERAL NORMAL EQUATIONS**

By definition, a least squares adjustment must provide the vectors \(\mathbf{v}\) and \(\mathbf{e}\) that satisfy Equation 5 and at the same time minimize the quadratic \(\mathbf{v}^T \mathbf{A}^{-1} \mathbf{v}\). The normal equations leading to this solution have been shown by Brown (1955) to be:

\[(B^T \mathbf{A}^{-1} B)\mathbf{e} = B^T \mathbf{A}^{-1} \mathbf{v}.\]  \hspace{1cm} (6)

After performing indicated operations and substituting previous notations, we have the general form of the normal equations:

\[\begin{bmatrix} \mathbf{N} + \mathbf{W} \\ \mathbf{N} \end{bmatrix} \begin{bmatrix} \mathbf{\hat{e}} \\ \hat{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} \mathbf{c} - \hat{\mathbf{W}} \mathbf{c} \\ \hat{\mathbf{c}} - \hat{\mathbf{W}} \hat{\mathbf{c}} \end{bmatrix}\]  \hspace{1cm} (7)
in which

\[
\dot{\mathbf{N}} = \dot{\mathbf{B}}^T \dot{\mathbf{W}} \dot{\mathbf{B}}, \quad \dot{\mathbf{c}} = \dot{\mathbf{B}}^T \dot{\mathbf{W}} \mathbf{c}, \quad \ddot{\mathbf{N}} = \ddot{\mathbf{B}}^T \ddot{\mathbf{W}} \ddot{\mathbf{B}},
\]

\[
\begin{align*}
\dot{\mathbf{N}} &= \dot{\mathbf{B}}^T \dot{\mathbf{W}} \dot{\mathbf{B}}, \quad \dot{\mathbf{c}} = \dot{\mathbf{B}}^T \dot{\mathbf{W}} \mathbf{c}, \\
\ddot{\mathbf{N}} &= \ddot{\mathbf{B}}^T \ddot{\mathbf{W}} \ddot{\mathbf{B}}, \quad \dddot{\mathbf{c}} = \ddot{\mathbf{B}}^T \ddot{\mathbf{W}} \mathbf{c}, \\
\mathbf{W} &= \mathbf{X}^{-1}, \quad \dddot{\mathbf{W}} = \dddot{\mathbf{X}}^{-1}.
\end{align*}
\]

Equation 7 is solved with a first-order partitioning scheme by elimination of the ground points so as to form the reduced normal equations as described in Brown, Davis, and Johnson (op. cit.). The practicality of solving such a reduced system is obvious if consideration is given to the sizes of the matrices requiring inversion. The matrices to be inverted are of rank and order \((8 + 6m) \times (8 + 6m)\) and \((3n) \times (3n)\), the latter corresponding to the \((\dot{\mathbf{N}} + \ddot{\mathbf{W}})\) matrix. Now the inversion of \((\dot{\mathbf{N}} + \ddot{\mathbf{W}})\) is extremely simple inasmuch as the computational effort requires nothing more than inverting a series of \(3 \times 3\) sub-matrices. It would appear then, that the bulk of the effort would be associated with inversion of an \((8 + 6m) \times (8 + 6m)\) matrix. However, if convergent photographs are used, as shall be recommended shortly, a typical reduction requires fewer than a dozen frames to recover a precise calibration. Thus, in practice, the actual rank of the \((8 + 6m) \times (8 + 6m)\) matrix does not become too great to be readily inverted by a standard Gaussian elimination.

**USE OF CONVERGENT PHOTOGRAPHS**

Thus far the mathematical development has been entirely general with no consideration being given to the stability of the general solution. In this regard one must consider whether projective compensations will exist between parameters of the inner cone and the exterior projective parameters in a particular photogrammetric net. It is well known, for example, that for vertical photographs, near perfect compensations exist between \(x_p, y_p, \mathbf{c}\) and \(X^2, Y^2, Z^2\) if relief in the object space is small relative to the camera-to-object distance. This means then, if \(x_p, y_p, \mathbf{c}\) are to be recovered, we must have explicit \textit{a priori} knowledge of the exposure station coordinates and statistically enforce these coordinates in the reduction. Although provision has indeed made for such \textit{a priori} knowledge in the foregoing derivation, it is the very desirability of eliminating the need for such knowledge (lest a restriction of practical significance be imposed on the technique) that has led us to seek other means to override the mechanisms of projective compensation.

To this end we have found that the use of highly convergent photographs will counteract primary projective compensations. Moreover, it also turns out that the overwhelming geometric strength and redundancy of a photogrammetric net composed of highly convergent photographs requires no external information whatever to maintain determinacy! It is necessary only to fix one exposure station in any arbitrary position and attitude and to allow all other projective bundles to adjust freely to this fixed bundle. It is in this context that the method is referred to as analytical self-calibration.

As a result of the determinacy afforded by convergent photographs, it becomes unnecessary to calibrate a photogrammetric test range, even nominally; a completely uncalibrated range will suffice. In fact, in the absence of a field of targeted points, an array of artificial targets (PUG points stereoscopically transferred among frames) may be used instead, as shall be demonstrated by the example given in the following section. Thus, having completely eliminated a need for externally established exposure station or coordinates point in the object space, we realize a method of calibration that is both powerful and operationally demanding. Equally important is the fact that the method applies to cameras focussed either for infinity or at finite distances.\(^2\)

It is important to note here that convergence alone will not permit a complete recovery of all parameters of the inner cone. If the primary mechanisms of projective compensation are suppressed, secondary compensations between the elements of interior orientation and the angular elements of

\(^2\) A modification has been incorporated to account for variations in distortion with object distance . . . a factor which may assume significance when lenses of moderate distortion are used for very precise close range work. See Brown (1971) for a derivation of the functions.
exterior orientation begin to assume significance and they too must be counteracted. As an example consider the differentials:

\[
\frac{\partial x_{ij}}{\partial \phi_i} = 1 \\
\frac{\partial x_{ij}}{\partial \phi_j} = -\frac{\phi_j^2 + \phi_i^2 \cos \kappa_i + \phi_i \phi_j}{\cos \kappa_i} \sin \kappa_i.
\]

Note that Equation 9 does not depend upon convergence (i.e., \( \phi \) and \( \omega \)). In the situation where \( \kappa_i = 0^\circ \), Equation 9 reduces to:

\[
\frac{\partial x_{ij}}{\partial \phi_i} = c \left[ 1 + \left( \frac{x_{ij}}{e} \right)^2 \right].
\]

As the cone angle of the camera decreases, the range of values assumed by Equation 10 also decreases, resulting in an increasing degree of projective compensation between \( x_p \) and \( \phi_i \) as their respective partial derivatives approach a constant relationship. On the other hand, if \( \kappa_i = 90^\circ \), \( y_p \) is not as strongly coupled with the angular elements and is more amenable to calibration. However, if \( \kappa_i = 0^\circ \), the entire situation is reversed: \( y_p \) is coupled with \( \phi_i \) but \( x_p \) is fairly independent.

Thus, to circumvent these secondary mechanisms of projective compensation and recover both \( x_p \) and \( y_p \) with equal precision, it is necessary to incorporate exposures having nominally orthogonal \( \kappa \) angles in the same reduction.

A similar \( \kappa \) dependent coupling operates between \( x_p, y_p \) and the coefficients of decentering distortion \( p_1, p_2 \). Accordingly, the exercise of frames with nominally orthogonal \( \kappa \)'s is also essential to the sharp recovery of \( p_1, p_2 \).

**AN APPLICATION**

Original applications of the method of analytical self-calibration were devoted to in-house calibration of DBA close-range metric cameras. To be more precise, the mathematics presented earlier are the backbone of DBA's Photogrammetric Structural Measurement Software. With the error model for the inner cone incorporated into this software, the programs serve a dual function. In the first capacity the programs serve the function of calibration (and not necessarily self-calibration) whereas in the second mode of operation the parameters of the inner cone are recovered as nuisance parameters within a reduction in which coordinates in the object space are of primary interest. (This latter mode of operation eliminates a need for pre or post calibration where the same camera is used for all exposures.)

Recently the self-calibration technique has been applied to two other calibration tasks. In one application reported by Brown, Kenefick, and Harp (1971), two Hasselblad cameras were calibrated in support of an investigation into the cause for the failure of the first OAO satellite to achieve orbit after a successful launch. Photographs of explosive bolts on the canopy of the launch vehicle were analyzed photogrammetrically after the cameras were calibrated. The second application required an in-flight calibration of a 500-mm Hasselblad camera flown on the Apollo 14 mission. This particular application has been selected for presentation here because of the nature of the background leading up to the requirement for the calibration, the interesting results obtained, and the ultimate usefulness of the calibration data.

**DESCRIPTION OF THE PROBLEM**

Apollo 14 was the first lunar mission to secure successfully metric photographs of the lunar surface. The primary function of the Lunar Topographic Camera (described briefly by Doyle (1970)) was to secure photographs for mapping future Apollo landing zones. Before photographs were secured of the Apollo 16 site near Descartes, however, the camera failed, leaving only backup photographs obtained with a 500-mm Hasselblad for compilation of the charts. Because it was not anticipated that the Hasselblad photographs would be used for mapping, a rigorous preflight calibration of the camera had not been obtained. Moreover, an immediate postflight calibration was hampered by normal quarantine restrictions. Faced with pressing compilation schedules, NASA contracted DBA to perform a calibration using the actual photographs which had been secured with the camera while in lunar orbit. Fortunately, the photographs had been deliberately exposed in a convergent fashion to increase the precision of height measurements. This very convergence was of pivotal importance for extracting a calibration of the camera as well.

From the convergent photographs secured during revolutions 27, 28, and 30, three frames from each of the three passes were selected for the calibration work. As illustrated in Figure 1, the selected frames within any one orbit span an arc of approximately 6° which, coupled with a spacecraft altitude of 108.5 km, provided an approximate angle of
convergence of 80° between end photographs. Passpoints were marked on all 9 frames with the aid of a Wild PUG 4 by transferring points from the middle frames to the frames at the ends of the arcs. Areas not common to all 9 exposures were marked with supplemental passpoints to obtain a wider distribution of data points over the 55×55-mm format of each frame. In all, there were approximately 170 passpoints. The corners of the imaged format were used as fiducial points for lack of a better reference system.

RESULTS OF CALIBRATION

As it turned out, two reductions were actually performed. The first solution was one of pure self-calibration wherein one exposure station was fixed in an arbitrary attitude and position and all other bundles were allowed to adjust freely to this fixed bundle. In the second reduction, the exposure station coordinates, as determined through correlations in time of the midpoint of shutter pulses and tracking data from earth based stations, were introduced into the solution as constrained unknowns. As all computations in this second solution were performed in a rectangular, moon-centered coordinate system, a covariance matrix to be attached to individual exposure stations was computed according to the technique developed by Gyer (1970). In substance, the approach involves transforming estimated positional standard deviations, expressed in terms of orbital in-track, cross-track and radial components, to a full covariance matrix in the moon-centered system. After several experimental reductions with varying levels of constraints on the exposure stations, it was found that the exposures in revolutions 27 and 28 could be constrained to 50 and 10 meters, respectively, without contaminating

4 Tracking data were available only for REV 27 and 28. The midpoint of shutter timing was lost on REV 30.
the photogrammetric closures; a mean error of 16.2 μm was achieved as compared to a mean error of 15.8 μm for the pure self-calibration solution.

In both reductions the parameters of the inner cone which were recovered included \( c, k_1, p_1 \) and \( p_2 \). Higher-order symmetric radial distortion terms were constrained to zero because a one-term Gaussian function completely describes the symmetric radial distortion of the Zeiss Tele-Tessar lens used in the camera. The terms \( x \) and \( y \) were also constrained to zero because incomplete geometry of the photogrammetric net and the narrow field (8° across diagonals) of the camera would not permit recovery of these parameters (see earlier discussion).

Data of major significance resulting from the two solutions are the focal lengths and symmetric radial distortion functions which are summarized in Table 1. All data in this table have been balanced such that the symmetric radial distortion \( \delta_r \) is zero at a radial distance \( r \) of 30 mm—a criterion adopted simply to facilitate comparison with other calibration data in the next section. The term \( c' \) is the balanced or calibrated focal length corresponding to its balanced radial distortion function. As is evident from the

<table>
<thead>
<tr>
<th>( r ) (mm)</th>
<th>( \delta_r ) (μm)</th>
<th>( \sigma_{\delta_r} ) (μm)</th>
<th>( \delta_r' ) (μm)</th>
<th>( \sigma_{\delta_r'} ) (μm)</th>
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<td>+378</td>
<td>11</td>
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</table>

\( c' = 502.630 \text{mm ± 2.060}; \quad c'' = 504.056 \text{mm ± 1.503}. \)

Table, exercising the positional constraints has had virtually no effect upon the recovered calibration, with the exception of a modest improvement in the standard deviation of the focal length. Although the two focal lengths differ by approximately 1.4 mm, this is not a significant disparity considering that the standard deviations of the focal lengths range from 1.5 to 2.0 mm.

**COMPARISON OF CALIBRATIONS**

After the initial work on the in-flight calibration had been completed, the camera was released from quarantine. NASA immediately performed a calibration of the camera using the Stellar SMAC programs developed by DBA (Brown (1968), Gyer, Haag, and Llewellyn (1970)). Results of the stellar calibrations are plotted in Figure 2 along with the results from the in-flight calibrations (Table 1). In addition, nominal average lens distortion data provided by the manufacturer are also plotted. The correspondence of all three curves is extremely close considering that no attempt has been made to match the curves per se; the only normalization applied has been simply to pass all functions through zero at a radial distance of 30 mm. As plotted,

![Figure 2. Comparison of symmetric radial distortion functions and calibrated focal lengths. All curves are balanced such that \( \delta_r' = 0 \) at 30 mm but no additional effort has been made to match the curves.](image-url)
the SMAC and self-calibration curves lie well within their combined one-sigma error bounds except in the region between 5 and 20 mm radial distance where the difference between the curves is 5 to 6 \( \mu m \). However, a very slight rebalancing would have the two curves lie completely within their respective one sigma error bounds.

The real item of interest in comparing the in-flight calibrations to the SMAC calibration lies in the corresponding calibrated focal lengths, for it is here that the differences cannot be satisfactorily reconciled on the basis of their respective standard deviations or by rebalancing the distortion functions. Indeed, a full two-sigma adjustment of the focal length from either in-flight calibration is required for agreement with the values from the SMAC calibration. This result naturally leads to the supposition that the spacecraft rendezvous window, through which the convergent photographs were exposed, may have acted as a very weak negative lens element in the photographic system. By approximate calculation, an effective focal length of 84 meters for the spacecraft window would account for the difference in focal length.

NASA has confirmed that the focal length shift as determined from the in-flight calibrations is indeed significant. In their attempt to establish mapping control in the Descartes region, 9 convergent frames from the 500-mm Hasselblad were included in a simultaneous block adjustment with one pass of vertical 80-mm Hasselblad photographs. The 80-mm frames served as a bridge between landmark control points which had been established on either side of the Descartes

![Fig. 3. Simulated photographic sequence for pure self-calibration. The orbits are consecutive and of low inclination; a spacecraft pitch maneuver allows an 80° angle of convergence between the end photographs. The arrows indicate that the camera is rotated 90° between individual exposures.](image)

**Table 2. Results of Simulated Self-Calibration for the Mapping Camera Sub-system**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard Deviation (( \mu m ))</th>
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</thead>
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</tr>
<tr>
<td>( y_p )</td>
<td>2</td>
</tr>
<tr>
<td>( c' )</td>
<td>3</td>
</tr>
<tr>
<td>( g' )</td>
<td>6</td>
</tr>
<tr>
<td>( p_0' )</td>
<td>1</td>
</tr>
</tbody>
</table>

* At corner of format.

area by means of sextant sightings from the spacecraft. In original trial reductions, the focal length from the Stellar SMAC calibration was employed, with very discouraging results. Upon convergence of these reductions a mean error for the photographic measurement residuals was typically 250 to 300 \( \mu m \). When the focal length from the self-calibration with positional constraints solution was introduced, however, the mean error dropped immediately to 60 \( \mu m \)!

At the time of preparing this manuscript NASA had not conducted additional experimental reductions. However, it is the opinion of the authors that further experimentation with variations of several hundreds of micrometers in the focal length may well improve the photogrammetric closures by yet another factor of 2 to 3.

**Potential for In-Orbit Self-Calibration**

Although the results of the Hasselblad calibration reported above are quite satisfying, the Apollo 14 data do not give a complete picture of the power of pure self-calibration because of the narrow cone angle of the camera and the fact that all frames were exposed with the same \( \kappa \) angle. To conclude this article we wish to demonstrate the full potential of pure self-calibration by presenting the results of one of several simulations which have been performed to depict the precision with which a full calibration of the inner cone can be recovered given typical Apollo orbits, specific exposure configurations, and cone angles.

The specific simulation selected for presentation here is one for the metric 3-inch focal length, wide angle Mapping Camera Subsystem (MCS) which was successfully flown on the Apollo 15 mission and which is scheduled for the two remaining missions as well. In this simulation a photographic scheme nearly identical to that encountered with the 500-mm Hasselblad was adopted; i.e. three low

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4 Personal discussions with Robert Hill, NASA Mapping Sciences Laboratory and Ronald Davis, Lockheed Electronics Company.
inclusion of lunar orbits, spacecraft altitude of 100.5 km, three frames per orbit, 80° convergence between end photographs of each pass, and an array of approximately 200 passpoints. Inasmuch as the MSC incorporates film flattening, IMC, and a focal plane reseau, a 5-μm photogrammetric error budget was adopted. In addition, to round out the ideal photographic geometry for self-calibration, it was also assumed that the photographs were exposed with 90° increments in $\kappa$ as illustrated in Figure 3.

Results of the simulation are presented in Table 2 in terms of the posteriori standard deviations of the recovered parameters. For the symmetric radial and decentering profile functions, standard deviations of the functions at the corners of the 4.5 X 4.5-inch format have been tabulated for ease of interpretation. The very magnitude of these results give ample testimony to the power of analytical self-calibration. Indeed, numerous applications of the technique to the calibration of our close range cameras have consistently provided standard deviations on the order of one half of those given in Table 2.

REFERENCES


Notice to Contributors

1. Manuscripts should be typed, double-spaced on 8 1/2 X 11 or 8 X 10 1/2 white bond, on one side only. References, footnotes, captions—everything should be double-spaced. Margins should be 1 1/2 inches.

2. Ordinarily two copies of the manuscript and two sets of illustrations should be submitted where the second set of illustrations need not be prime quality; EXCEPT that five copies of papers on Remote Sensing and Photointerpretation are needed, all with prime quality illustrations to facilitate the review process.

3. Each article should include an abstract, which is a digest of the article. An abstract should be 100 to 150 words in length.

4. Tables should be designed to fit into a width no more than five inches.

5. Illustrations should not be more than twice the final print size: glossy prints of photos should be submitted. Lettering should be neat, and designed for the reduction anticipated. Please include a separate list of captions.

6. Formulas should be expressed as simply as possible, keeping in mind the difficulties and limitations encountered in setting type.