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A Plotting Instrument for Close-Range Photogrammetry

The design, calibration, and operation of a stereo plotter employing Balplex projectors and suitable for close-range stereo and mirror photography is described.

Abstract: *A Biomedical Laboratory was established at the University of Washington with a grant from the National Science Foundation. During research conducted in this laboratory, there was a need for graphical plotting for facial studies and for amputees. To provide the most economical means for such mapping, a new plotter was designed and fabricated from available parts of a Balplex plotter. The instrument is an affine restitution plotter. The calibration process and the achievable relative accuracy are described. The instrument is capable of mapping, using close-range stereo-photographs with fixed orientation elements as well as variable orientation elements. Mirror photographs also can be accommodated. Thus a complete three-dimensional mapping of the subject can be accomplished.*

Introduction

Many articles have been published in professional literature which report the development and application of close-range photogrammetry to the bio-medical field. In 1974 a Bio-photogrammetric Laboratory was established at the University of Washington as a result of a grant given by the National Science Foundation for bio-medical research. In this Bio photogrammetric Laboratory, the camera system has been established. The description and the application of such a system has already been reported (Veress and Tiwari, 1976).

In this article one of the instruments constructed at the University of Washington for bio-medical purposes will be described. The camera system consists of three metric MK 70 mm Hasselblad cameras, two of which have been factory matched, having the same focal length and nearly the same characteristics of distortion.

There are different types of instruments available at the University of Washington which can be used for measurement and plotting of photographs taken by these mentioned cameras. These instruments are the OMIAP/C analytical plotter, the Zeiss 1818 stereocomparator, a Kelsh plotter, several Balplex plotters, etc. Of these, the AP/C analytical plotter and Zeiss 1818 stereocomparator are used basically for bio-medical purposes when high accuracy in measurement is required. These instruments, of course, can be used only in connection with analytical photogrammetric methods and they provide graphical or numerical data output showing one or more views of the object photographed, but they take more time and cost more than the classical photogrammetric method. When lower accuracy is acceptable, as is often the case, the time and cost can be considerably reduced by employing the stereo-plotting method which uses another group of instru-

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ments, such as the Balplex plotter. There are many such applications in the bio-medical field for graphical plotting. Among others are the mapping of the human stump for below-the-knee amputees (Verces, Tiwari, and Hatzopoulos, 1976) and a number of facial studies when the required accuracy need not be more than ± 2 mm. These graphical data were necessary to evaluate the effect of the shape of the stump for physiological support of the prosthesis or for the various types of facial studies which will be presented later in this article.

In so many cases the object has to be mapped, not only from one view but also in all of the three-dimensional views. Therefore, a three-dimensional map of the stump or the human face has to be compiled. For this reason, a system of mirror photography was designed and used in connection with the modified stereo-plotter to be described.

The laboratory could not afford the purchase of a complex camera and plotter system that would accommodate all of these tasks. It was, therefore, necessary to design a plotter around the available material and to fabricate it, using the minimum amount of resources available, in order to be able to undertake the studies of motions of the human body and to solve the problems already mentioned.

Considerations for the Design

The Bio-photogrammetric Laboratory has to accommodate and thereby produce photographs from various sizes of objects, ranging from 2 or 3 inches to the full length of the human body. Provisions therefore were made for the taking of stereo photographs. The base length of these stereo cameras also varied. However, the one most used was the one which has 0.6 meter length. The focal length of the cameras is 61.34 mm. However, when they are focused for the finite distance, the principal distance is 63.89 mm for each camera.

The following are the design criteria used for the reconstruction of the photographs taken by these cameras:

- Make the plotting for the Biophotogrammetric Laboratory as fast as possible by using fixed orientation elements.
- Assure an accuracy between ± 2 mm.
- Perform the plotting from mirror photography without making changes in the orientation of the projectors.
- Use optical projection systems with Balplex equipment which is available at the University of Washington. This equipment is composed of two Balplex projectors of 760 mm, a diapositive centering device, and a metallic bar for the accommodation of the base of the projectors. The Balplex projectors were preferred because they have a principal distance close to the cameras' focal length. They also have efficient magnification and depth of field, thus giving the least amount of affinity.

If one wants to construct the spatial model of the object to be photographed by these cameras in a Balplex projector, the construction of such a model can be only an affine restitution because the principal distance of the instrument (f_i) is different from that of the camera focal length (f). Therefore, the ratio is

$$a = \frac{f}{f_i} \tag{1}$$

The affine factor, therefore, is

$$a = \frac{63.89}{55} = 1.16 \tag{2}$$

As was mentioned earlier, a criterion for the new design was that it should be able to reconstruct mirror photography. The principle of mirror photography was first developed by Dr. Kratky of the National Research Council of Canada and was used in connection with bio-medical photogrammetry (Kratky, 1975). He used an analytical transformation and an analytical approach to solve the bio-medical problems. In this article, a graphical transformation will be used to obtain the mapping from the mirror photographs.

Geometry of the Mirror Photography and Plotter Arrangement

The general geometry of the mirrors is shown in Figure 1. The optimum object space is about 0.32 meter by 0.15 meter. The angle between each mirror and the respective camera axes as shown in this figure is 53°. Due to the mirror images, there are three objects as shown in the figure. The first is the middle object which is the actual subject. The second one is regarded as the L-object and is a mirror reflection of the object and the third is regarded as the R-object and is also a mirror reflection of the object. The expression of M-, L-, and R-object used in this paper will stand, furthermore, for simplification of these mirror objects or, more precisely, mirror images. In the case of the instrument, similar expressions will be used in connection with the established stereo model of M, L, and R. The mirror coverage of the actual object is shown in Figure 2. It is apparent from these figures that

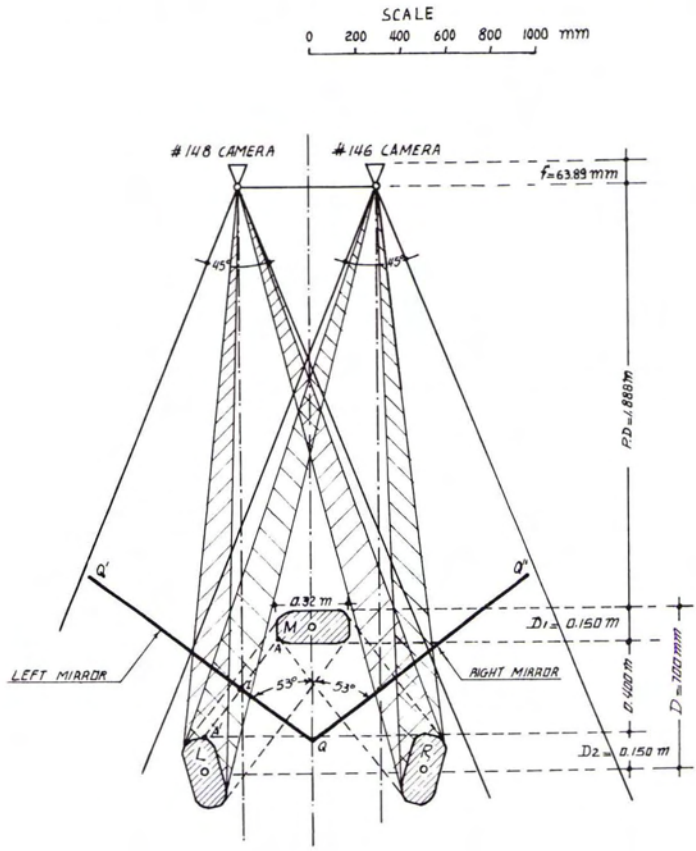

FIG. 1. Geometry of mirror photography.

a target on the double overlap area gives two images on the same photograph. This concept is useful for the coordinate transformation of L- and R-models to the M-model. This coordinate transformation can be done in two ways. One, for instance, given by Dr. Kratky, is an analytical method and the second is a graphical or instrumental transformation (Hatzopoulos, 1976). Furthermore, Figure 1 is a section of the instrument with a vertical plane, which means that M-, L-, and R-models in this figure are cross sections of the actual plotting. The actual plotting is an orthogonal projection of the three submodels to a horizontal plane. (See E_1 or E_2 in Figure 3 and the plotting in Figure 6.) Due to the fact that Figure 6 is a three-dimensional map, it is always feasible to

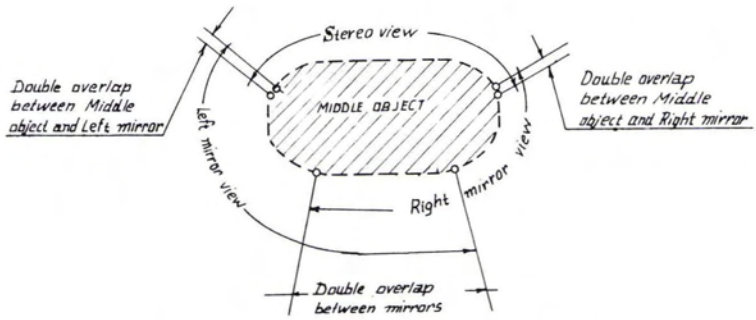

FIG. 2. Photographic coverage of the object.

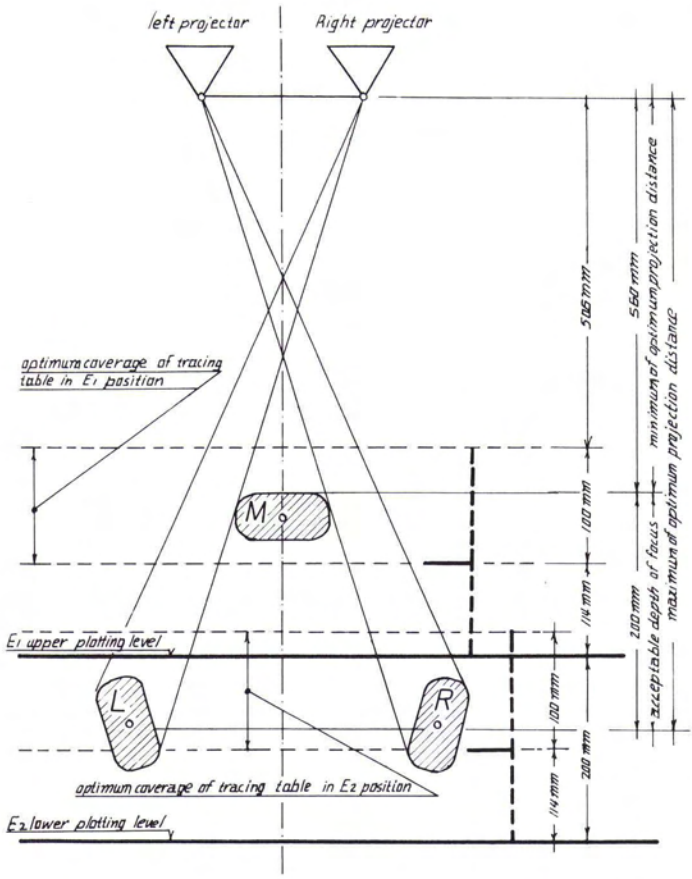

FIG. 3. Projection of photographs by the instrument.

make a cross section and create a situation similar to Figure 1.

Figure 1 shows that the L-object is the symmetrical of M object with respect to the axis of symmetry QQ′, where QQ′ is the intersection between the plane of the left mirror and the horizontal plane in the object space. It becomes the intersection with the vertical plane in the model space. Similarly, the R-object is the symmetrical of M-object with respect to the axis of symmetry QQ″, where QQ″ is the intersection between the right mirror plane and the horizontal plane in the object space. The interrelation between the M- and L- objects is that AA′ is a straight line perpendicular to the QQ′ axis of symmetry and $|AI| = |A'I|$ where I is the intersection between the AA′ and QQ′ lines. This interrelation exists between all points of M- and L-objects. Similar interrelation exists between M- and R- mirror objects with respect to the axis of symmetry QQ″.

The geometric arrangement of Figure 1 allows each camera to "see" all three M-, L-, and R-objects. This is shown in Figure 5.

In conclusion, the assumption can be made that the cameras are the projectors in a plotting instrument. (Compare Figures 1 and 3.) The graphical transformation will be performed in the following steps:

(1) Perform the stereo-plotting (Figure 6)
(2) Make a cross section similar to Figure 1. This cross section will differ from Figure 1 by showing only the mutual visible points from both cameras. More precisely (Figure 2) the cross section of M-model will be the "stereo view" line, the cross section of L-model will be the "left mirror view" line, and finally the cross section of R-model will be the "right mirror view" line.
(3) Reconstruct all points of M-model by transforming graphically the corresponding points from L or R models. This transformation can be done as follows: Assuming that point A′ appears on L model, its corresponding point A does not appear on M model. To determine point A, draw a line A′A perpendicular to QQ′ so that $|A'I| = |AI|$ where I is the intersection between the lines A′A and QQ′ where

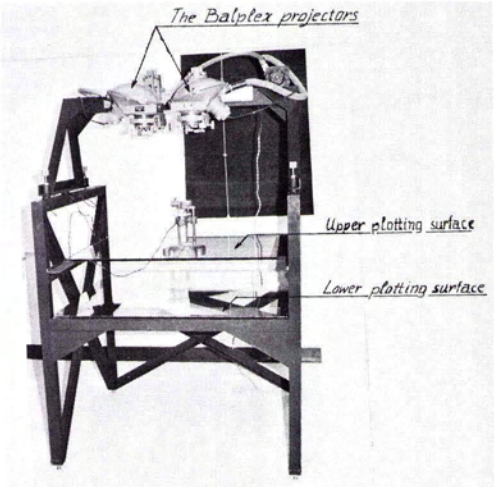

FIG. 4. The plotting instrument.

QQ′ is the intersection between the plane of the left mirror and the vertical plane (that vertical plane creates the cross section). The line QQ′ can be located either from the calibration data of the Bio-photogrammetric Laboratory or by using the common points between L-model and M-model. Those points are located in the double overlapped areas (Figure 2). In this way the point A is determined on the M-model cross section. Similarly all points of M-model can be determined. Analogically the entire cross section of M model can directly plot this by a special instrument, but this is beyond the scope of this article.

The new plotter operates the mirror photographs in the manner shown in Figure 3 by using Balplex 760 projectors. The Balplex projectors are capable of providing the maximum accuracy of 1/10,000 at the photographic distance. This distance in the Bio-photogrammetric Laboratory corresponds to approximately 2 meters, assuming the illustrated geometry. In this case, therefore, the expected accuracy is 0.2 mm, which certainly fulfills the design criteria set for the new Balplex type of instrument. It can be seen in Figure 1 as well as in Figure 3 that the general arrangement of the instrument is as follows:

The scale factor of 2.9 was found to be a good one for plotting mirror photography. Using this scale factor, the models M, L, and R can be established into an acceptable focus in the instrument. Assuming that the distances in the model space are d_1, d_2, d, and pd, the corresponding distances in the object space are D_1, D_2, D, and PD, as shown in Figure 1. The values of the d_1, d_2, d, and pd in the model space using the scale of 1:2.9 are—

$$d_1 = d_2 = \frac{150}{1.16 \times 2.9} = 45 \text{ mm}$$

$$d = \frac{700}{1.16 \times 2.9} = 208 \text{ mm approximately}$$

$$pd = \frac{1888}{1.16 \times 2.9} = 561 \text{ mm}$$

where the 1.16 is the previously mentioned affine factor and the pd is equal to 561 mm, which is the minimum projection distance for the projector. The pd plus D equals 769 mm, which is about the same as the maximum projection distance for the projector. The tracing table optimum vetical coverage is called dt and is approximately 100 mm. The conclusion from these analyses is that the tracing table cannot cover the total

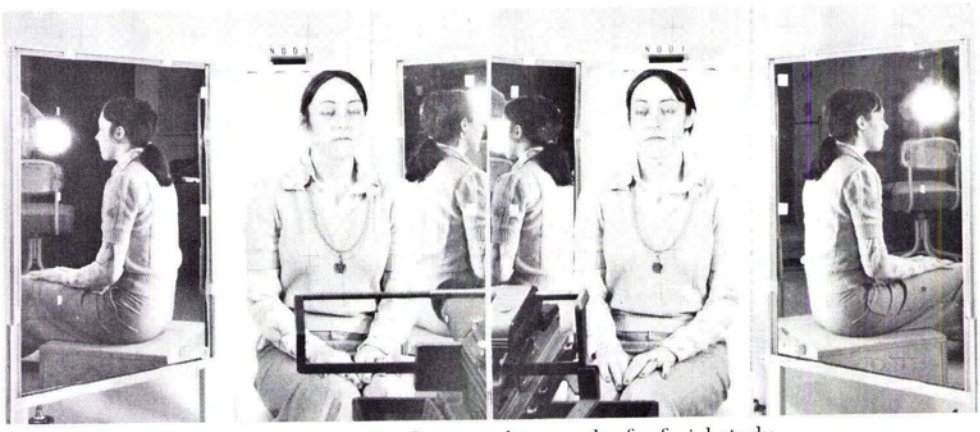

FIG. 5. Stereo pair of mirror photography for facial study.

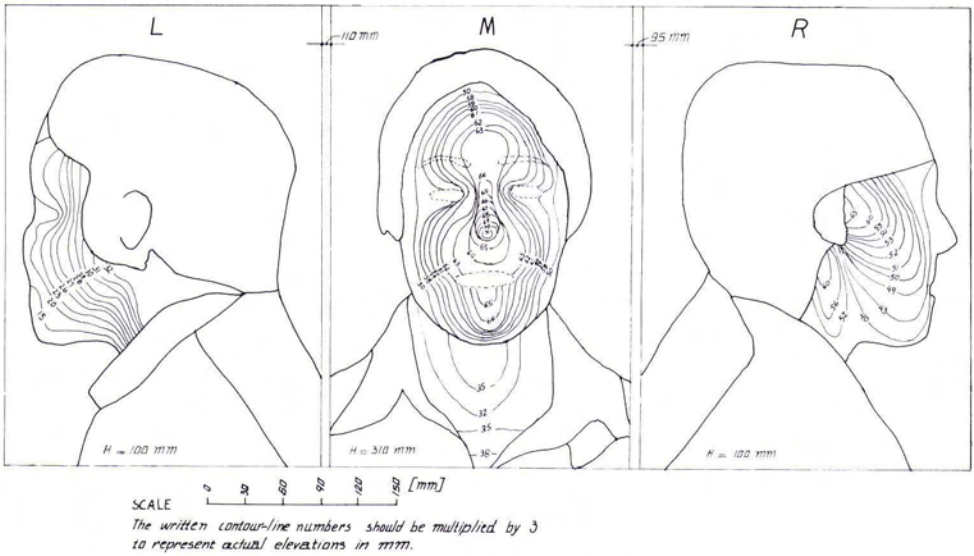

FIG. 6. Maps of mirror photography of Figure 5.

depth of the model since the dt is smaller than d. It can cover only one model, either the M- or the L- and R-models, but not all three models at the same time. In order, therefore, to satisfy the requirement of plotting all three models, it was necessary to separate the plotting into two plotting levels. These two plotting levels are very well indicated in Figure 3 by E_1 as the upper plotting level and E_2 as the lower plotting level. These separate plotting levels are parallel to each other.

The upper or E_1 plotting level permits the plotting of the M-model. The second plotting level is located 200 mm lower which provides for the plotting of the L- and the R-models. This vertical arrangement can readily be seen from Figure 3. In this arrangement, the model remains within the acceptable depth of focus of the instrument. As a consequence, an optimum accuracy can be obtained in the reconstruction of the photograph.

The tracing table is set first on the E_1 plotting surface which is constructed from glass one-half inch thick. In this position the M-model is plotted and then after finishing the M-model, it is moved down to the E_2 plotting surface on which the models L and R are plotted. The E_2 plotting surface consists of one-half-inch-thick glass like the E_1 plotting surface.

Some Plotting Examples

The plotting instrument is shown in Figure 4 with the actual frame and with the plotting levels. Up to the present time, the instrument has been used basically for facial studies and for stump studies. For example, the procedure of plotting for mirror photography in general is the same as the procedure generally used for conventional photography. The M-model is plotted at the upper surface and since the cameras are in a fixed position and only use the fixed orientation elements, the base component in the X-direction and the two rotations are needed to orient the model. The scale is set at 1:3 position. After plotting the middle model M, two points on the metal bar, where the projectors are located, are marked on the plotting paper using a plumb bob. These points are used as registers to connect the plotting of the M-model with the plotting of the L- and R-models. The L- and R-models are plotted on the E_2 lower plotting surface as follows:

The upper plotting surface or glass is moved out from its position in order to give access to the lower plotting surface and the glass plate is moved into the back of the instrument. The two points on the metal bar, as previously mentioned, are marked on the plotting paper which is attached to the lower plotting surface. Then the κ_L and the κ_R orientation elements are moved. These two rotations eliminate the very small parallax existing in the L- and R-models when compared to the M-model. This parallax is due to the auto collimation effect which will be discussed in a later section. The next step is the scale correction which is done with the B_X,

base component. The scale correction also needs to be done due to the fact of the auto collimation effect. Then the plotting of the L- and R-models on the second surface is performed exactly the same way as it is done in a conventional plotter. Needless to say, all of these models also can be digitized. As a consequence, therefore, using *XY* coordinates measured on a coordinatograph, the semi-analytical method can also be used in connection with this mirror photography. A few examples along this line are shown in Figures 5 through 8. Figure 5 shows the photography of an individual for facial studies and Figure 6 shows the plotting of the same individual using that photograph. A photograph is shown in Figure 7 of a human stump and the corresponding plotting of it is shown in Figure 8. The facial study was done by conventional plotting while the stump study was done by digitizing the models.

Calibration of the Plotting Instrument

The reader realizes that this plotting instrument is a modified Balplex Plotter providing affine restitution and it is capable of plotting at two different levels in order to accomodate the mirror photographs. The calibration process of this instrument is, therefore, roughly the same as a conventional Balplex Plotter which is described in detail in the USGS booklet in the following steps:

(1) Calibrating and adjusting the centering device,
(2) Positioning the reflector,
(3) Positioning the projector bulb,
(4) Adjusting the tracing table measuring mark,
(5) Checking the verticality of the tracing table columns,
(6) Checking and calibrating the principal distance of each projector, and
(7) Checking and calibrating the principal point of each projector.

Steps 1, 2, 3, 4, and 5 have been performed in exactly the same way as described by the USGS manual. As a consequence, they will be omitted here. However, steps 6 and 7 are discussed.

Step 6 is the calibration of the principal

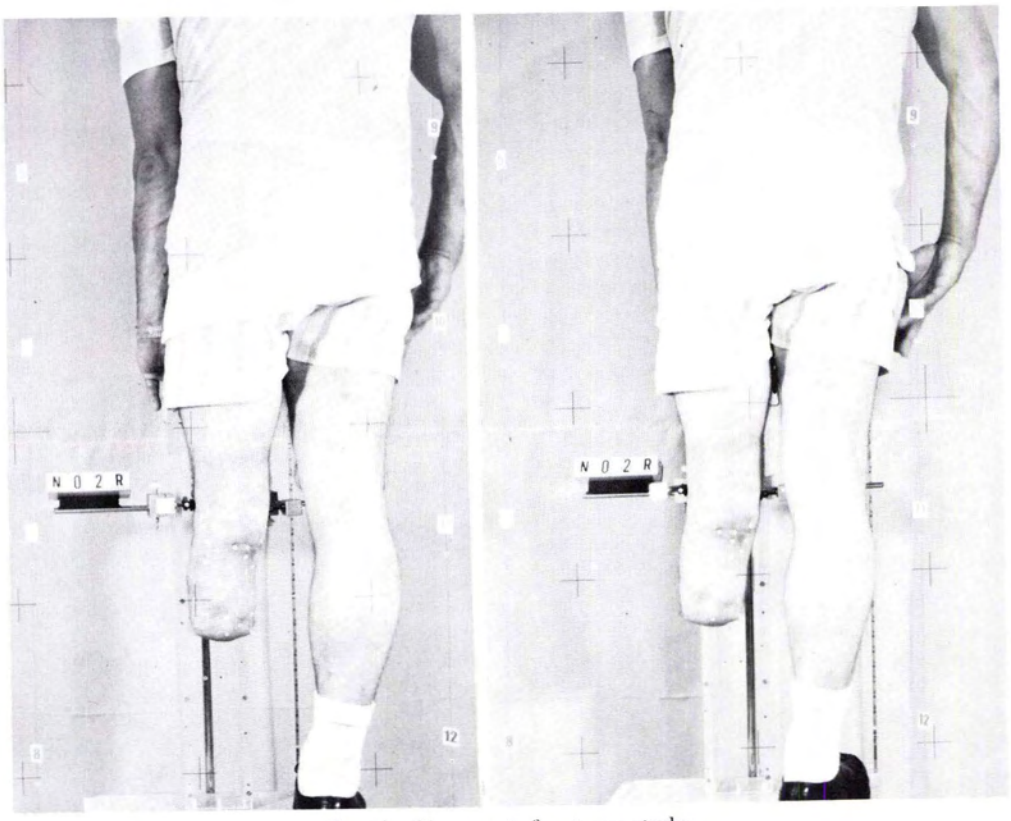

Fig. 7. Stereo pair for stump study.

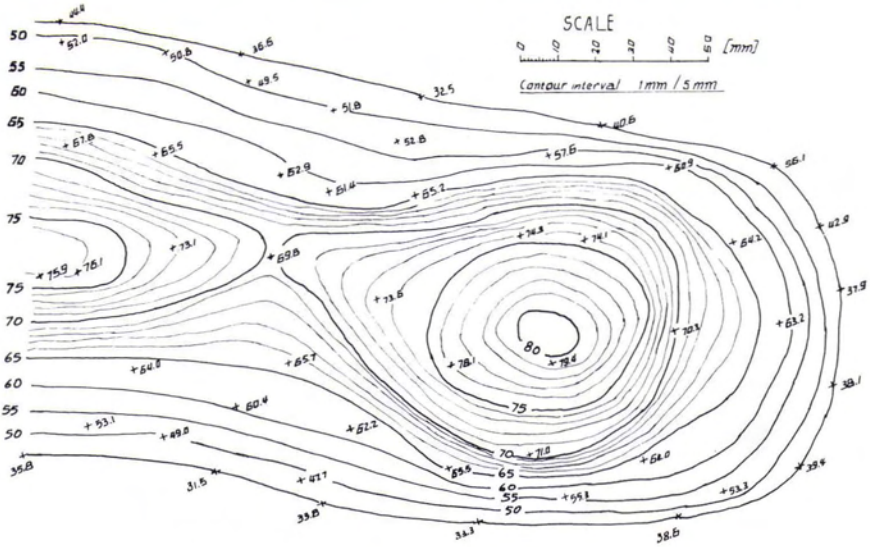

FIG. 8. Map from photographs of Figure 7.

distance for the projector. A grid plate obtained from the Bausch and Lomb Company is put into the projector and this grid plate is graduated in 5 mm intervals. The projector is oriented so that the grid plate is parallel to the plotting surface. Points were selected on the grid plate, two in x and two in y direction which are located on the same lines that pass through the principal point or center intersection of the grid so that they are located in a symmetrical pattern. The tracing table is set at its lower position with the elevation counter pointing at h_1; the measuring mark is centered on one of the grid intersections, which is marked on a plotting paper (point 1L in Figure 9). Then the next point is measured in the same way (point 2L). After this measurement the tracing table is set at its upper elevation and a new set of points is measured (1U, 2U respectively). Using this new set of points and similar triangles, the principal distance of each projector can be determined according to the geometry of Figure 9:

$$\frac{f}{h_2-h_1} = \frac{g}{e_1+e_2} \text{ or } f = \frac{h \cdot g}{e}$$

where

$$e = e_1 + e_2 \text{ and } h = h_2 - h_1$$

This equation gives the principal distance of the projectors. In this particular instrument, it was found that for the left projector the principal distance was 55.086 mm and for the right projector it was 55.085 mm. This principal distance modifies the affine factor to be equal to 1.1598 instead of the previously mentioned 1.16.

Finally, step 7 is the calibration of the projective center. The principal point of each projector was checked and calibrated in the way described in the USGS manual.

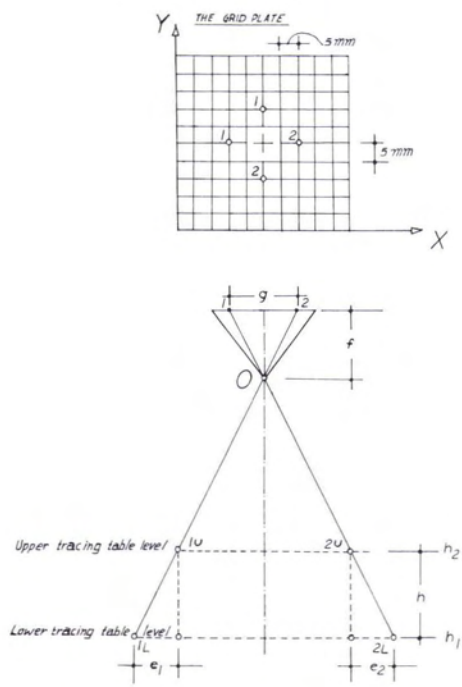

FIG. 9. Determining the principal distances.

Achieved Accuracy

A desirable test to obtain data on the achievable accuracy would be to photograph a controlled test area and compare the coordinates of the surveyed points against those measured in the spatial model of the plotter. There are a number of difficulties with this method. The first one is to establish the control field with sufficient accuracy in the object space. Second is the expense involved in establishing such a control field. There are a number of articles in professional literature which point out the difficulty of achieving the desirable accuracy in the ground measurements of the control field (Karara, 1966; Veress and Tiwari, 1976). Even if this accuracy is achieved, there is considerable expense involved in the establishment of such a project with a significant number of points to be used for the statistical analysis. Because of this fact and because the plotter will mostly be used for graphical evaluation, it was decided to evaluate its relative rather than its absolute accuracy. The relative accuracy was obtained by comparing the results of the new plotter with those obtained by the AP/C. For this purpose five models with premarked points were digitized on the new plotter as well as observed on the AP/C. Besides these premarked points, observations were made on unmarked points so that the total of 407 points were evaluated on both.

The object space coordinates of the points observed by the AP/C were determined by space resection/intersection. The same points were observed on the new plotter and their model coordinates were digitized and transformed to the object space system by three-dimensional transformation. The control points for space resection were determined by classical "ground" method. The three-dimensional transformations of the coordinates of the Balplex model resulted in a system determined by the coordinates of the AP/C. The standard errors obtained from the differences between the two systems are given in Table 1. It can be seen from the test of the five models and from the aveage standard errors that the results are acceptable and adequate for nearly all of the biomedical photogrammetric projects.

Conclusions

The analysis of the results indicates that the accuracy is adequate for mapping of stump, for facial measurements, for movement studies, and for many other quantitative analyses of the human body.

The procedure for stereo-plotting is faster and more economical when compared to analytical photogrammetry in connection with computer graphics.

The major sources of errors of the new plotter are—

- The affine restitution.
- The auto collimation point determination.
- The degree of non-flatness of the mirrors.

This last effect can be discounted if relative motion of points is studied, that is, the movement of certain points is required. This is obtained by comparing two positions of the same point where the distortion is the same in both positions. Thus, its effect is nullified.

The effect of the auto collimation points are described in detail in the USGS manual where its model deformation effect is also illustrated. This effect could be overcome by "drawing" two additional crosses on the centering device as illustrated by Figure 10. In this figure the #146 and #148 refer to the serial number of the cameras by which the photographs are taken. At present this correction does not exist because of the cost involved.

Acknowledgments

The authors would like to express their sincere appreciation to the National Science Foundation for providing the grant.

The would also like to express their ap-

Table 1. Standard Errors Obtained From AP/C and New Plotter Measurements

Model	Model Scale	Plotting Level	σ_x (mm)	σ_y (mm)	σ_z (mm)
1	1:1.878	Lower	±0.25	±0.39	±1.07
2	1:1.864	Lower	±0.43	±0.59	±0.98
3	1:1.877	Lower	±0.38	±0.55	±1.26
4	1:1,878	Lower	±0.29	±0.44	±0.74
	1:3.00	Upper	±0.69	±0.49	±1.50
5	1:3.10	Lower (L)	±0.58	±0.53	±1.13
	1:3.08	Lower (r)	±0.76	±0.69	±2.05
Average			±0.41	±0.51	±1.12

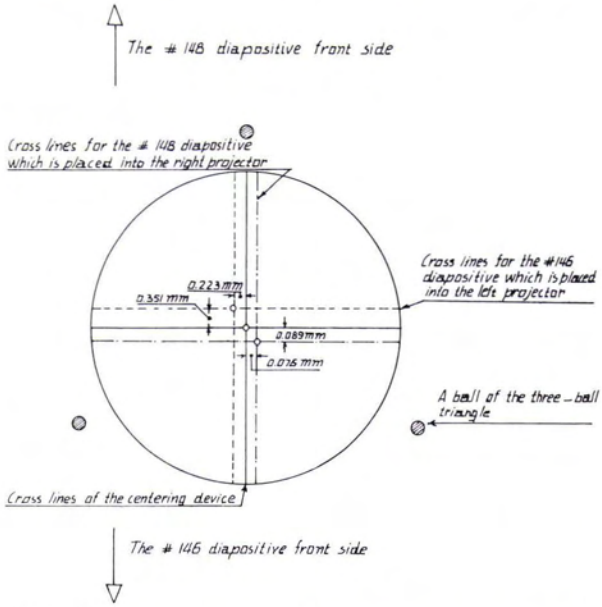

FIG. 10. Design for auto-collimation points correction.

preciation to Dr. Ernest M. Burgess, M. D., head of Prosthetics Research and his staff at the Swedish Medical Center for providing patients and material assistance to execute practical experimentation on the described plotter.

This paper is only a part of the larger research project and the following persons have had responsibilities on the project:

Dr. F. G. Lippert, M.D., Ph.D., supervised the medical procedures and execution of the research. The major emphasis was on the surgical technique of implementation of permanent land marks into the skeleton system. Further, he has selected patients, performed the required surgery, and evaluated the results.

Dr. S. A. Veress, D.Sc., supervised the engineering part of the research. The major emphasis was on the establishment of the Biomedical Laboratory for general studies, emphasizing motion studies. Further, he supervised and formulated the process of x-ray photogrammetry.

Dr. T. Takamoto, Ph.D., was responsible for developing methodology and computer programs. He executed the experimentation for analytical and stereo x-ray photogrammetry.

Mr. R. S. Tiwari was responsible for developing computer programs for the analytical process of external photogrammetry. Further, he has performed the calibration of the camera frame work.

Mr. S. A. Feher was responsible for the design and installation of the camera frames.

Mr. C. H. Schuch was responsible for further refinements to the x-ray photogrammetry considering programs and auxiliary equipment.

Mr. J. N. Hatzopoulos was responsible for the calibration and recalibration of the external photogrammetry lab. He designed a special plotter for the graphical evaluation. (The plotter was not manufactured out of grant funds.)

Mr. G. A. Spolek, M. S. was from Orthopaedic Services, Veterans Administration Hospital, Seattle. (He was not on the grant budget, but his time was donated by the V.A.H.) He was responsible for the design and manufacture of auxiliary equipment for surgical and x-ray purposes.

Finally, the authors would like to thank the Aerial Mapping Company of Oregon and its President, Bela Kalotay, for donating the Balplex projectors which made possible the construction of the described instrument.

REFERENCES

1. Hatzopoulos, J. N., *Design and Calibration of a Plotting Instrument for Close-Range Photogrammetry*, Thesis, University of Washington, 1976.
2. Herron, R. E., "Close Range stereophotogrammetry—A Bridge Between Medicine and Photogrammetric Engineering," *Proceedings of A.S.P.*, 1959 Convention.
3. Karara, H. M., "Stereometric Systems of High Precision," *Proceedings of International Symposium I.S.P.*, 1966.

4. Kratky, V., "Photogrammetric Digital Modeling of Limbs in Orthopaedics," *Photogrammetric Engineering and Remote Sensing*, Vol. 41, No. 6, 1975.
5. Lippert, F. G., "The Three Dimensional Measurements of the Tibia Fracture Motion by Photogrammetry," *Clinical Orthopaedics*, 1974.
6. Lippert, F. G., M. Hussain, and S. A. Veress, "Measurement of Spatial Motion Using Close-Range Analytical Photogrammetric Principles," *Proceedings ASP-ASCM*, 1970.
7. U.S.G.S. Topographic Instructions, *ER-55 Plotter Procedures*, Book 3, Chapter 4F4, Washington, D. C., 1960.
8. Veress, S. A., "The Use and Adoption of Conventional Stereo-Plotting Instruments for Bridging and Plotting of Super Wide Angle Photography, *The Canadian Surveyor*, Vol. 23:359-376, 1969.
9. Veress, S. A., and R. S. Tiwari, "Fixed-Frame Multiple-Camera System for Close-Range Photogrammetry," *Photogrammetric Engineering and Remote Sensing*, Vol. 42, No. 9, September 1976.
10. Veress, S. A., R. S. Tiwari, and J. N. Hatzopoulos, "Mapping of Stump Using Close-Range Photogrammetry," *Proceedings of the ASP Fall Convention*, Seattle, Washington, 1976.

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