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Acquisition of 35-mm Oblique Photographs for Stereoscopic Analysis and Measurement

A Step-by-step approach for obtaining stereoscopic 35-mm oblique aerial photographs and making precise measurements on them is presented.

Introduction

Over the last 10 to 15 years the use of small-format aerial photographs for a variety of environmental, engineering, and planning applications has become quite popular. In particular, hand-held 35-mm cameras are being used more often by scientists, researchers, and technicians who, for reasons related to cost, complexity, temporal constraints, project requirements, and/or personal preferences, have chosen not to contract an aerial survey firm to obtain standard 9- by 9-inch vertical photographs.

The intent of this article is to provide a concise guide on how to obtain and make precise measurements from "hand-held" 35-mm stereoscopic aerial photographs. It is directed at potential users who would like to take their own 35-mm stereoscopic photographs, but are unfamiliar with the proper procedures for doing so. Specific applications of 35-mm photographs in such areas as forestry, agronomy, wildlife management, vegetation analysis, geology, and environmental monitoring are not discussed in this paper because these applications have been addressed in sufficient detail elsewhere (e.g., Dalman, 1978; Evans *et al.*, 1983; Fraga, 1978; Grumstrump and Meyer, 1977; Hall and Walsh, 1977; Johnson and Mulvaney, 1978; Meyer and Gerbig, 1974; Milfred and Kiefer, 1976; Miller, 1974; Mintzer and Spragg, 1978; Nash *et al.*, 1977; Sheirel and Meyer, 1976; Strandberg, 1964). Rather, the emphasis of this paper is on photo mission and measurement considerations basic to most applications of hand-held oblique 35-mm aerial photographs.

Abstract: *Procedures are provided for obtaining stereoscopic 35-mm aerial photographs with a hand-held camera, and for making precise measurements on them. Emphasis is on photo mission and measurement considerations basic to most applications of oblique 35-mm aerial photography. A variety of flight mission factors including equipment, photo center line and flight line determination, aircraft height, film exposure, and time requirements are discussed. Also presented are procedures on how to construct perspective grids for making photographic measurements.*

Types of Oblique Photographs

Aerial photographs can be taken from two basic perspective or viewing angles: vertical and oblique (see Figure 1). In vertical photography, the camera is pointed straight down toward the center of the Earth (i.e., the camera's optical axis is held perpendicular to the ground surface). The angle at which the optical axis is depressed below the "imaginary horizontal" is known as the "depression angle." As shown in Figure 1, the camera is held at a depression angle less than 90 degrees for oblique photography.

Oblique photographs can be categorized as being either high obliques or low obliques. Low oblique photographs (Figure 2) do not show the horizon, and

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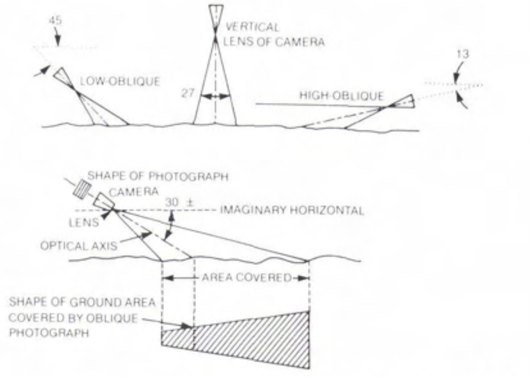


FIG. 1. Orientation of aerial camera for vertical and oblique photography.

are generally taken from a depression angle of approximately 45 degrees. High oblique photographs, which include the horizon, are taken from a depression angle less than 45 degrees (usually around 13 degrees). High oblique photographs, such as the one shown in Figure 3, are often called "horizon trace" photographs. At a certain depression angle, the ground horizon will appear to coincide with the upper limit of the camera field of view. Any photograph taken at a depression angle equal to or greater than this angle will be a low oblique photograph. Conversely, any photograph taken at a depression angle less than this angle will be a high oblique photograph. The exact depression angle which separates high obliques from low obliques will vary with the camera field of view, which is a function of focal length and film size.

Scale and Ground Resolution

The scale on an oblique aerial photograph changes from the bottom (foreground) to the top (background) of the photograph. Because low obliques are taken at a depression angle closer to vertical photographs than high obliques, low obliques show less scale change than high obliques. Table 1 is a list of scale values for estimating the approximate scale of an oblique aerial photograph.

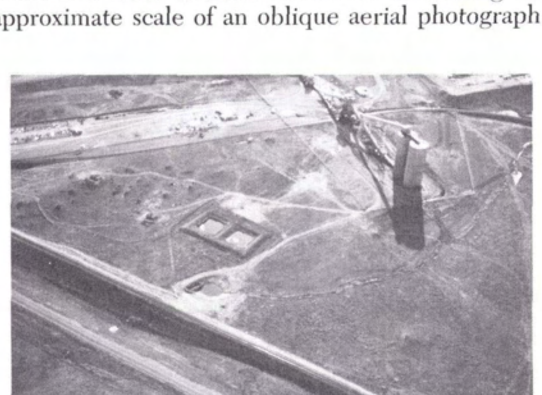


FIG. 2. Low oblique aerial photograph.

FIG. 3. High oblique aerial photograph.

This table shows the approximate scales at various positions (top 1/3, center 1/3, and bottom 1/3) on a 35-mm photograph taken with a standard 50-mm lens at various angles from 1000 feet above terrain. To find the scale when the photograph is taken at other altitudes, multiply the values in the table by the appropriate altitude factor (e.g., for an altitude of 2000 feet, multiply the table value by 2). Procedures for determining scale and other photographic measurements more accurately are discussed later in the article.

Mission Planning

In planning an overflight to acquire 35-mm stereoscopic coverage with a hand-held camera, consideration must be given to a variety of factors including equipment, the length and location of photo center lines and flight lines, aircraft height, film exposure, and mission time requirements. To acquire stereoscopic coverage of a given site, it is recommended that at least a 60 percent overlap between successive frames along a line of flight be

TABLE 1. SCALE ESTIMATES FOR OBLIQUE PHOTOGRAPHS TAKEN AT AN ALTITUDE OF APPROXIMATELY 1000 FEET (ADAPTED FROM EASTMAN-KODAK CO., 1974)

	CAMERA DEPRESSION ANGLE	FRAME POSITION	35mm 50mm lens	2" x 2" 80mm lens
HIGH		Top	1" = INFINITY	1" = INFINITY
OBLIQUES		Center	1" = 2200ft	1" = 1000ft
		Bottom	1" = 1100ft	1" = 500ft
LOW	30°	Top	1" = 1700ft	1" = 1800ft
OBLIQUES		Center	1" = 1000ft	1" = 600ft
		Bottom	1" = 700ft	1" = 400ft
	45°	Top	1" = 900ft	1" = 800ft
		Center	1" = 700ft	1" = 500ft
		Bottom	1" = 600ft	1" = 300ft
	60°	Top	1" = 700ft	1" = 500ft
		Center	1" = 600ft	1" = 400ft
		Bottom	1" = 500ft	1" = 300ft
	75°	Top	1" = 600ft	1" = 400ft
		Center	1" = 500ft	1" = 300ft
		Bottom	1" = 500ft	1" = 300ft

used. Low obliques, because they are more like vertical photographs than high obliques, are recommended for stereo pairs. In either case, a forward overlap greater than 60 percent will improve stereoscopic viewing.

As mentioned, the exact depression angle required for high oblique photographs depends upon the focal length of the camera and the size of the film. For a given film size, the horizon will appear on the photograph if the angle of depression is less than one-half the lens field of view (see Figure 1). A 35-mm camera having a 50-mm lens, for example, has a 27-degree field of view. The horizon will be at or near the top of a high oblique photograph taken with this type of camera system when it is oriented at a 13-degree angle below the horizontal plane of the aircraft. For a low oblique photograph, the depression angle would be 14 degrees or greater. As mentioned earlier, most low oblique photographs are taken at angles of approximately 45 degrees.

With most 35-mm cameras equipped with standard 50-mm lenses, it is recommended that photographs be taken from an altitude of 2000 feet or less to provide for adequate resolution of ground features. (In this case, for example, a flying height of 2000 feet above terrain will ensure that most objects greater than one foot in length are readily identified.) Longer focal length lenses, such as 80- to 200-mm telephoto lenses, will permit photographs to be taken at higher altitudes. However, because they are more sensitive to aircraft motion, the use of these lenses at higher altitudes may result in decreased resolution.

EQUIPMENT: CAMERAS, FILMS, AND FILTERS

A 35-mm camera equipped with a 50-mm lens is recommended for taking oblique aerial photographs. It is also recommended that the camera be a single-lens reflex type, which allows through-the-lens viewing. Automatic cameras with manual override features are particularly useful because automatic exposures may not always work well in an aircraft. This is because the internal metering in the camera may "read" light from the air space in the immediate vicinity of the camera in addition to light at the ground surface. However, when the distance to the ground feature of interest is less than 2000 feet, and the field of view does not include the horizon, an automatic exposure can provide high quality photographs.

Most film for hand-held 35-mm cameras is available pre-packaged in 24- or 36-exposure rolls. It is recommended that twice as much film as required be taken on a mission because film is considerably less expensive than flying time, and experimentation with different exposure settings may be desirable. It is also good practice to have an adequate supply of film with the same batch number because the emulsion on films from different batches can react differently to various lighting conditions.

Some type of filter is generally used when taking aerial photographs. Film factors and film/filter adjustments are usually provided by the manufacturer with each roll of film, and filter factors are also marked on the filters themselves. Film/filter combinations commonly used for both small- and large-format aerial photography are listed in Table 2.

DETERMINING PHOTO CENTER LINE AND FLIGHT LINE PARAMETERS

To ensure proper stereoscopic coverage of a site, the location of the center of each succeeding photograph should be planned before taking the photographs. These succeeding locations will form a line called the "photo center line," a line along the ground surface at which the camera will be pointed as the aircraft proceeds along a parallel flight line. The number of photo center lines required depends upon the size of the area. The following five steps, along with the values given in Table 3, can be used to determine the area encompassed by each photograph, the location of the photo center line, the number of photographs required for a photo center line, and the location of each photograph. For these steps, it is assumed that the camera is held in such a way that the orientation of the resulting photographs is similar to that of the photographs shown in Figures 2 and 3.

- Delineate the boundaries of the particular study area on a suitable topographic map (preferably a USGS 1:24,000-scale topographic map).
- Determine whether the photographs will be high oblique or low oblique and the appropriate altitude for the aircraft. Column E in Table 3 compares the "usable" area available by perspective and altitude.
- Draw a line along the bottom of the study area perpendicular to the path of incoming sunlight; this is the first photo center line (see Figure 4).
- Place a point at one edge of the photo center line, corresponding to the center of the first photograph, and calculate the distance between the centers of the succeeding photographs. Column D in Table 3 shows distances calculated for various altitudes and perspectives.
- Place a series of dots along the photo center line which corresponds to the calculated distance between photographs. (For example, the distance between the centers of high oblique photographs taken with a 50-mm lens, a 60-percent overlap, and at an altitude of 1500 feet, will be approximately 1881 feet).

In order to take oblique photographs along the photo center line at the designated locations, a flight line parallel to the photo center line must be plotted using the following two steps.

- Calculate the distance between the photo center line and the flight line. This distance is a function

TABLE 2. RECOMMENDED FILTERS FOR AERIAL PHOTOGRAPHY (ADAPTED FROM EASTMAN-KODAK CO., 1981)

FILM TYPE	KODAK WRATTEN FILTER	COMMENTS
Panchromatic Black-and-white	No. 12 (known as a "minus blue")	Usually used in combination with an anti-vignetting filter during vertical mapping missions. Reduces atmospheric haze effects.
	No. 25 (absorbs all blue and nearly all green portions of the visible spectrum)	Generally used with infrared film; however, can be used with panchromatic black-and-white film for vegetation surveys. If haze is very bad at lower altitudes, use this filter.
	No. 15 (yellow filter)	Used for oblique aerial photography at less than 5000 feet altitude and under moderately hazy to clear atmospheric conditions.
Color *	2B or HF-3 aerial filter	Used for low- and high-oblique aerial photography at altitudes above 500 feet and when haze is apparent.
	HF-4 or HF-5 (always used in combination with HF-3)	These filter combinations are useful for high altitudes and for very hazy conditions. Use of the HF series is not recommended for use below 500 feet.
Color Infrared	No. 12 (minus blue)	Required for proper exposure.
Black-and-white Infrared	No. 25, No. 29, or No. 70	Used for hazy conditions.
	No. 89B	No. 89B has added haze penetration capabilities; however, exposure is limited to infrared energy.

*These are recommendations for color positive film. With color negative film, color balancing filters are not generally required.

TABLE 3. PHOTOGRAPHIC PARAMETERS FOR OBLIQUE AERIAL PHOTOGRAPHY (ADAPTED FROM EASTMAN-KODAK CO., 1981)

(A) Altitude (feet)	(B) Distance from photo center line to flight line (feet)[1]	(C) Distance between adjacent photo center lines (including 30% overlap) (feet)[1]	(D) Distance between the centers of successive photographs along photo center line (including a 60% overlap)[1]	(E) Area useable in the photograph[2]
HIGH-OBLIQUE – 13 Degree Angle				
500	2166	2133	627	41 acres
1000	4331	4266	1253	152 acres
1500	6497	6399	1881	344 acres
2000	8663	8532	2508	611 acres
2500	10829	10656	3135	959 acres
3000	12994	12798	3762	1376 acres
LOW-OBLIQUE – 45 Degree Angle				
500	500	357	204	6.3 acres
1000	1000	713	408	25.3 acres
1500	1500	1070	612	56.9 acres
2000	2000	1427	816	101.1 acres
2500	2500	1783	1020	158.0 acres
3000	3000	2140	1224	227.6 acres
VERTICAL – 90 Degree Angle				
500	0	168	145	2.0 acres
1000	0	336	290	8.2 acres
1500	0	504	435	18.4 acres
2000	0	672	580	32.7 acres
2500	0	840	725	51.0 acres
3000	0	1008	870	73.5 acres

NOTE: (1) These values apply only to 35mm photographs taken with a 50mm lens.

(2) The area figures for low-oblique and vertical photographs reflect the amount of area visible in the entire photograph, whereas the area figures for high-oblique photographs account for only the portion visible in the lower 70 percent of the photograph. In this case, it is assumed that features in the remaining upper 30 percent of the photograph are too far away to be evaluated adequately.

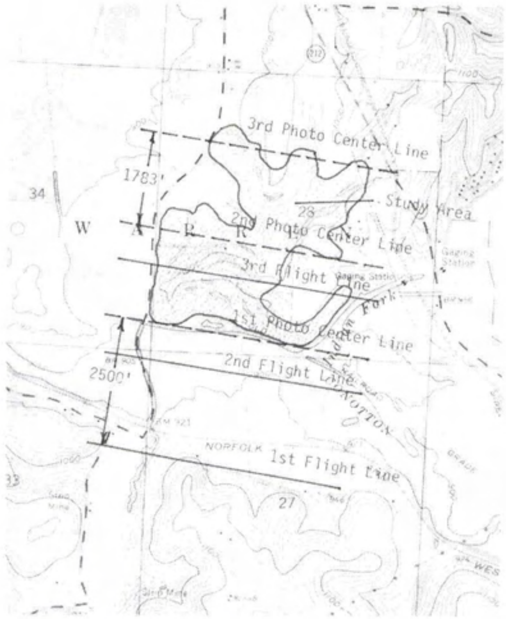


FIG. 4. Plotting of photo center lines and flight lines.

of the aircraft altitude, focal length, and depression angle. Column B in Table 3 lists distances between the photo center line and flight line for low oblique and high oblique photographs taken with a standard 50-mm lens at various altitudes.

- Draw a line on the topographic map parallel to, and at the calculated distance from, the photo center line. This is the flight line. In order to be certain that the sun is behind the aircraft when the pictures are being taken, this second line, the flight line, should be placed away from the photo center line in the direction of the sun (see Figure 4).

Additional photo center lines and corresponding flight lines may be needed to provide complete photographic coverage of a site. Column C in Table 3 provides data required for planning these additional lines. To determine if an additional flight line is needed, the following two steps should be performed:

- Draw a line parallel to the original photo center line at the prescribed distance from the original photo center line. If this parallel line is contained within the study area, it should be included on the map.
- Draw a second flight line parallel to the new photo center line using column B in Table 3 to determine the correct distance.

For example, Figure 4 gives the photo center line and flight line distances for low oblique photographs taken at an altitude of 2500 feet. As shown in columns B and C in Table 3, the distance between a flight line and its respective photo center line is 2500 feet, and the distance between adjacent photo center lines (with 30 percent overlap) is 1783 feet.

Table 4 contains factors that can also be used to determine the various flight plan parameters for 35-mm overflights as discussed above. Compare this table with Table 3 in which actual values are given for the use of 35-mm cameras with 50-mm lenses. In Table 4, factors are given which can be used to compute actual values for flight plan parameters for a wide range of focal lengths.

In the table and accompanying illustrations, D represents the ground distance between the flight line and lower limit of the ground area included in the photograph. C represents the ground distance between the bottom edge of the photograph and the photo center line, B is the ground distance from the flight line to the photo center line, and P refers to the ground distance between the bottom and top edge of the photograph and is used to determine the distance between adjacent photo center lines. For example, if P = 1530 feet for a given set of photographs along a particular photo center line, a sidelap of 30 percent would require that the distance between this and the next photo center line be 1071 feet (1530 feet × 0.70 = 1071 feet). To determine a given flight parameter, simply multiply the flying height (H) by the appropriate factors. For example, if high oblique photographs are to be taken with a 28-mm lens at a height of 1800 feet, the distance between the flight line and the photo center line (i.e., B) is computed as 1800 feet × 2.33 = 4149 feet.

OPTIMUM HEIGHT

A flight altitude of approximately 1500 feet above ground is commonly used for a variety of 35-mm applications. This altitude may not be appropriate, however, for all purposes. In general, large features should be photographed from higher altitudes than smaller features. At an altitude of 1500 feet, the distance from the camera to a feature at the center of the photograph will be approximately 2100 feet at a 45-degree angle, and 6700 feet at a 13-degree angle.

In areas where topographic relief varies greatly, other altitude factors must be considered. As shown in Table 3, changes in altitude require changes in other flight mission parameters. Under actual flying conditions, the aircraft pilot may have to adjust the actual altitude as ground relief changes in order to maintain a constant altitude above terrain.

EXPOSURE REQUIREMENTS

Exposure is a function of the time that light is permitted to reach the film (exposure time), the quantity of light that reaches the film (lens aperture opening), and the sensitivity of the film to light (film speed). Generally, an exposure time or shutter speed of 1/500 to 1/1000 of a second is recommended for aerial photography. The film speed and the exposure time should be set prior to any aperture adjustment.

TABLE 4. TABULATED FLIGHT VALUES FOR OBLIQUE PHOTOGRAPHY (ADAPTED FROM FLEMING AND DIXON, 1981)

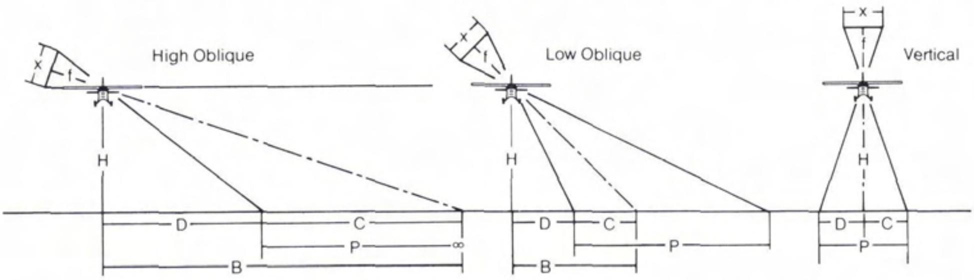
35mm CAMERA 24mm X 36mm FORMAT HORIZONTAL ORIENTATION (X = 24mm)

Focal Length	High Oblique ($\partial = \alpha$)								Low Oblique ($\partial = 45°$)							Vertical ($\partial = 90°$)	
	Ground Distance					Scale No.			Ground Distance				Scale No.			G.D. = S.N	
f (mm)	$\alpha°$	D/H	C/H	B/H	P/H	Sb*/H	Sc*/H	St*/H	D/H	C/H	B/H	P/H	Sb/H	Sc/H	St/H	P/H	Sc/H
18	33.7	.42	1.08	1.50	∞	15.3	30.5	∞	.20	.80	1.0	4.8	14.4	23.9	71.8	1.33	16.9
21	29.7	.59	1.16	1.75	"	14.6	29.3	"	.27	.73	"	3.39	13.1	20.5	47.9	1.14	14.5
24	26.6	.75	1.25	2.0	"	14.2	28.4	"	.33	.67	"	2.67	12.0	18.0	35.9	1.0	12.7
28	23.2	.95	1.38	2.33	"	13.8	27.6	"	.40	.60	"	2.10	10.8	15.4	26.9	.86	10.9
35	18.9	1.29	1.63	2.92	"	13.4	26.9	"	.49	.51	"	1.55	9.17	12.3	18.7	.69	8.71
40	16.7	1.52	1.82	3.34	"	13.3	26.5	"	.54	.46	"	1.32	8.29	10.8	15.4	.60	7.62
50	13.5	1.96	2.20	4.16	"	13.1	26.1	"	.61	.39	"	1.02	6.95	8.62	11.3	.48	6.10
55	12.3	2.18	2.40	4.58	"	13.0	26.0	"	.64	.36	"	.92	6.43	7.84	10.0	.44	5.54
85	8.0	3.47	3.61	7.08	"	12.8	25.6	"	.75	.25	"	.58	4.44	5.07	5.90	.28	3.59
100	6.8	4.11	4.23	8.34	"	12.8	25.6	"	.79	.21	"	.49	3.85	4.31	4.90	.24	3.05
110	6.2	4.53	4.63	9.16	"	12.8	25.5	"	.80	.20	"	.44	3.53	3.92	4.40	.22	2.77
135	5.1	5.58	5.67	11.3	"	12.8	25.5	"	.84	.16	"	.36	2.93	3.19	3.50	.18	2.26

*Sb, Sc, St are photo scale numbers at the bottom edge, center and top edge (respectively) of the photo.

Note: ∂ = *depression angle of hand-held camera*

$\alpha° = \arctan \frac{x}{2f}$ (semi-field angle). Use this value as the depression angle for specific focal lengths for high obliques.



If the photograph is to include the horizon, or when there will be more than 2000 feet between the camera and the photo center line, the aperture setting ("f" stop) should be determined either at a representative ground location prior to boarding the aircraft, or by the exposure settings provided with the film. In either case, a rule of thumb is to adjust these setting by closing the aperture an additional one to two full "f" stops. This adjustment is done regardless of what the light meter indicates while in the air. A final adjustment must be made to compensate for the use of filters (as noted with the filter instructions). If the photograph does not include the horizon, or if it is taken from less than 2000 feet from the subject, the automatic exposure setting or internal metering device may prove adequate.

When in the air, exposure speed and focus should not be changed. Focus should always be set on infinity, and shutter speed should remain at 1/500 to 1/1000. For the untrained aerial photographer, it is advisable, however, to "bracket" the exposures; that is, complete the mission with the settings at the predetermined or automatic positions, then turn around and refly the site(s) at a higher "f" stop and again at a lower "f" stop. This should provide for proper exposure on at least one of the flights. With experience, these extra flights should no longer be necessary.

Exposure interval refers to how often photographs are taken over a specific period of time, and depends primarily on aircraft speed and the ground distance between center points on succeeding photographs. For example, the center points on adjacent high oblique photographs taken along a photo center line with a 35-mm camera having a 50-mm lens will be about 1881 feet apart if the photographs are exposed at an altitude of 1500 feet and with a 60 percent overlap (see Table 3). At an aircraft speed of 90 mph, the exposure interval would be one photograph per every 14.3 seconds (1881 ft. divided by 132 ft. per sec. = 14.3). Recommended exposure intervals for high and low oblique photographs taken at various altitudes and aircraft speeds are listed in Table 5. The use of a camera with a motorized film advance is suggested if photographs must be taken at less than four-second intervals.

TIME REQUIREMENTS

Total flying time should be considered when planning a photographic mission. It takes more time to fly to a site, view the area, and return to the airport than it does to take the photographs. Because the optimum time for taking photographs is usually between two hours before and after local solar noon,

TABLE 5. RECOMMENDED EXPOSURE INTERVALS (IN SECONDS) FOR HIGH AND LOW OBLIQUE PHOTOGRAPHS TAKEN AT VARIOUS ALTITUDES AND AIRCRAFT SPEEDS

	Altitude (Feet)	Aircraft Speed (MPH) 80	90	100	110	120	130	140	150
High Oblique	500	5.4	4.8	4.3	3.9	3.6	3.3	3.1	2.9
	1000	10.7	9.5	8.5	7.8	7.1	6.6	6.1	5.7
	1500	16.1	14.3	12.8	11.7	10.7	9.8	9.2	8.6
	2000	21.4	19.0	17.6	15.6	14.3	13.1	12.2	11.4
	2500	26.8	23.8	21.3	19.5	17.8	16.4	15.3	14.3
	3000	32.2	28.5	25.6	23.4	21.4	19.7	18.4	17.1
Low Oblique	500	1.7	1.5	1.4	1.3	1.2	1.1	1.0	0.9
	1000	3.5	3.1	2.8	2.5	2.3	2.1	2.0	1.9
	1500	5.2	4.6	4.2	3.8	3.5	3.2	3.0	2.8
	2000	7.0	6.2	5.6	5.1	4.6	4.3	4.0	3.7
	2500	8.7	7.7	6.9	6.3	5.8	5.3	5.0	4.7
	3000	10.5	9.3	8.3	7.6	7.0	6.4	6.0	5.6

the aircraft should arrive at the site early enough to allow photo acquisition to take place within that time period. Time needed to reach the proper altitude, and the time required for an optional practice flight, should also be considered.

TIPS FOR ENSURING A SUCCESSFUL MISSION

In addition to the flight planning considerations discussed in the previous sections, some other procedures that can help ensure a successful photographic mission include the following:

- Before the flight, explain the purpose of your mission to the pilot. Show the pilot the type of features to be photographed and devise hand signals, or some other method of communication, to let the pilot know when you are taking pictures. If the pilot understands the requirements, the aircraft can be controlled to minimize motion and provide a good view of the area or features.
- In some cases, it may be desirable to take photographs while the aircraft is traveling towards the feature to minimize distortion caused by movement. This can be accomplished by having the pilot reduce the throttle, bank, and "side-slip" the aircraft towards the feature.
- Prevent the camera and the upper part of your body from touching the aircraft. This will minimize the effect of vibration.
- If possible, shoot with the passenger window open. This eliminates glare and reflection caused by the glass. If you have to shoot through the window, use an 81A filter, or an equivalent haze filter, to compensate for the slight blue-green tint inherent in the acrylic glass used in most light airplane windows. Also, wear a long-sleeved dark shirt or jacket to reduce the chance of creating unwanted window reflections, and always use a lens shade.
- With a high-wing monoplane, the best shooting angles are in front and behind the struts. The front angle is best for tracking a subject, if you take care not to get the propeller in the frame. With a mid- or low-wing monoplane, pictures may have to be taken in a steep turn to avoid photographing the wing.

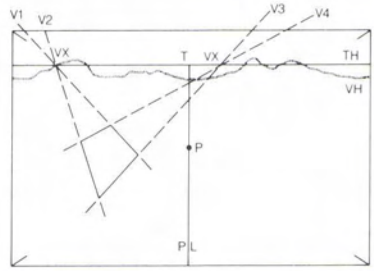


FIG. 5. Perspective relationships on an oblique aerial photograph.

PHOTOGRAPHIC MEASUREMENTS

Due to inherent scale changes associated with the angle at which they are taken, 35-mm oblique photographs require some form of perspective correction or rectification in order to make precise scale, area, and distance measurements. One way to rectify oblique photographs is to construct a perspective grid for the photograph by locating various points and lines. These include the "principal point," the "principal line," the "visible" and "true" horizon traces, the "nadir" line, the "isoscale" line, the "isocenter," and a "scale" line. The following discussion on constructing such grids is based on procedures developed by Strandberg (1964).

It is recommended that 35-mm aerial photographs which are to be rectified be enlarged to a 6- by 9-inch size. In doing so, an "enlargement factor" must be calculated. For a photograph that has been enlarged to a 6- by 9-inch size, the enlargement factor is calculated as

$$\frac{25.4\text{mm/inch (9 inches)}}{36\text{mm}} = 6.35.$$

(Note that the format of a 35-mm slide measures 24 by 36 millimetres.) To construct a perspective grid for an enlarged 35-mm print, the following steps should be performed in sequence.

(1) Place a large piece of tracing paper over the photograph to be rectified. This is necessary because some of the lines that have to be drawn may be quite long.

(2) Locate the "principal point" of the photograph. The principal point is the approximate center of the photograph, and is located at the point where straight lines drawn from opposite corners of the picture cross. This is labeled P in Figures 5, 6, and 7.

(3) Locate the "true horizon" (TH) line by using the following procedure:

(a) If a level (visible) horizon can be seen (such as is provided by smooth terrain or a water surface), draw a line along it. This is line VH

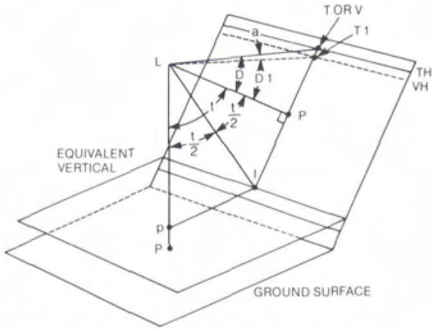


FIG. 6. The geometry of rectification.

in Figure 5 and Figure 6. Then proceed with steps 4 through 8.

(b) If a level horizon cannot be seen, as in the case of a low oblique photograph, or if the horizon is irregular, it may be possible to use an alternate method. Because straight, parallel lines converge at the true horizon (TH) when extended, suitable lines may be extended from the sides and ends of rectangular buildings or cultivated fields (see Figure 5). The points on the true horizon where these lines converge are called "vanishing points." If lines are extended from two or more sets of parallel lines, the position of the true horizon can be found by drawing a straight line through all the vanishing points (VX). (Note that, for low oblique photographs, the true horizon (TH) will be drawn above the photograph.)

(c) If this alternate method can be used to locate the true horizon, steps 5 and 6 can be eliminated. In place of these steps, measure the distance along the principal line (found in step 4) from the true horizon (TH) to the principal point (P). Divide this distance by the focal length of the camera times the enlargement factor. The answer is the tangent value of the depression angle (angle D in Figure 6). The proceed with steps 4, 7, and 8.

(4) Draw in the "principal line" of the photograph. The principal line (PL) is drawn at right angles to both the true and visible horizon and passes through the principal point.

(5) Calculate the depression angle of the photograph by using the following steps. (This angle, angle D in Figure 6, is the angle down from the true horizon at which the camera was pointed when the picture was taken.)

(a) If a level horizon is present, the angle at which the camera was depressed below the visible (or apparent) horizon must be determined. This angle is determined by dividing the distance from the visible horizon to the principal point by the focal length multiplied by its enlargement factor. The answer is the tangent of the depression angle.

(b) Determine the size of the angle between the visible and true horizon (angle "a" in Figure 6). This angle acts as a "correction factor" which must be added to the angle found in step 5a. This angle is calculated (in minutes of arc) by deriving the square root of the flying height in feet. For example, if the angle in step 5a was determined to be 12 degrees, and the photograph was taken at 2500 feet, the size of the depression angle is 12 degrees plus 50 minutes of arc (12° 50″).

(6) Locate the "true horizon" line (TH). By referring to trigonometric tables, find the tangent of the total depression angle (found in step 5b). Multiply the tangent of this angle by the focal length of the camera times its enlargement factor. Then place a point this distance up from the principal point, and draw a line through this point, parallel with the visible horizon.

(7). Using the following steps, calculate the "tilt angle," and plot the position of the "isoscale" line and the "nadir."

(a) Subtract the size of the angle found in step 5b from 90 degrees. The answer is the tilt angle (shown as "t" in Figure 6).

(b) Locate the "isoscale" point (shown as I in Figure 7). This point is located by measuring down from point P along the line TP the distance derived by multiplying the focal length times its enlargement factor by the tangent of one-half the tilt angle. For example, if the depression angle (D) is 25 degrees, the tilt angle is 90 degrees minus 25

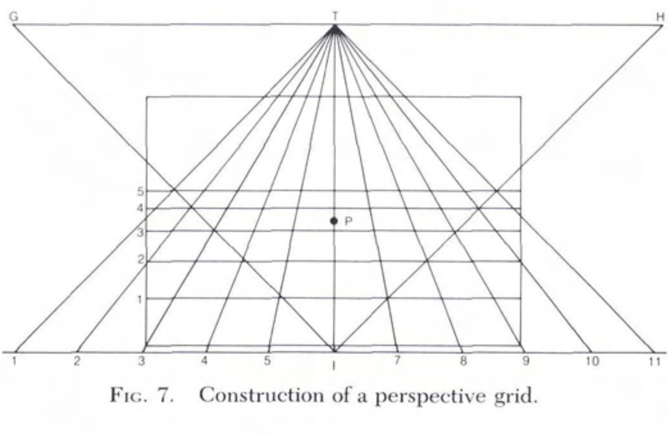


FIG. 7. Construction of a perspective grid.

degrees, or 65 degrees. One-half of the tilt angle is 32 degrees 30 minutes, and the tangent of that is 0.63707. Thus, the distance from P to I is computed as

$$50\text{mm}\ (6.35)\ (0.63707) = 202.27\text{mm}.$$

(8) As shown in Figure 7, make two tick marks (G and H) on either side of T (on the true horizon line) at the same distance as TI.
(9) Draw a line through I perpendicular to the principal line (TI). This line is the "isoscale" line. Distances measured along this line are at the same scale (as if the photograph were a vertical photograph taken with the same camera from the same altitude). After this is done, the basic scale formula f/H can be used to calculate distances.
(10) Using the basic scale formula, calculate and place a series of tick marks along the isoscale line equal to 209 feet on the ground. (Note that a square acre is approximately 209 feet on a side.) For example, if the photograph were taken with a 35-mm camera with a 50-mm lens from an altitude of 1500 feet, the contact scale along the isoscale line would be calculated as

$$\text{Scale} = f/H = \frac{0.16\ \text{ft}}{1500\ \text{ft}} = \frac{1}{9375}\ \text{or}\ 1{:}9375$$

However, because a 6- by 9-inch enlargement of a 35-mm photograph has been enlarged 6.35 times, the enlarged scale is actually 1:1476. Reduced to a scale of 1:1476, 209 feet is equal to 1.70 inches. Consequently, the tick marks should be placed 1.70 inches along the isoscale line in both directions from point I.

(11) Draw a series of lines, one from each tick mark, which converge at T. The resulting fan-shaped array of lines should extend far enough to the sides so that the entire picture area of the photograph is included within the array. These lines will become the Y-axis margins of the final perspective grid squares.
(12) As shown in Figure 7, draw 45-degree diagonal lines GI and HI across the fan-shaped array of lines. These diagonal lines establish the X-axis of each of the perspective grid squares.
(13) With a straightedge, draw lines across the array of lines through each intersection of GI and HI with the Y-axis lines drawn in step 4. These parallel lines form the X-axis lines of each individual perspective grid squares.

As seen in Figure 7, the spacing between successive lines (and therefore the scale of the photograph) decreases considerably as one progresses from the principal point toward the top of the photograph. Because of the rapid decrease in scale, little is gained by extending the X-axis lines more than about one inch above the principal point on a 6- by 9-inch enlargement of a high oblique photograph.

After the above steps have been completed, area measurements can be made on the photograph with the aid of the one-acre perspective grid. Linear measurements can also be made by using the isoscale line as a reference scale. Figures 8 and 9 illustrate perspective grids constructed for high and low oblique aerial photographs.

Quick, though less accurate, estimates of size and photographic scale can also be made by using various factors listed in Table 4. Sb, Sc, and St in the table represent the photographic scale at the bottom edge, center, and top edge of the photograph, respectively, and can be used to determine ground distance in these areas. As with the other factors in this table, scale in these areas is determined by multiplying the flying height (H) by the appropriate factors.

In using this table, it should be understood that the factors apply to 35-mm slides and negatives. For determining various scale and distance relationships for prints, the appropriate enlargement factor (as discussed above) must also be applied.

Summary

In this paper procedures are provided for obtaining oblique 35-mm aerial photographs with a hand-held camera, and for making precise measurements on them. It is directed at scientists, researchers, and technicians who would like to take their own 35-mm stereoscopic photographs, but are unfamiliar with the proper procedures for doing so. Specific applications of 35-mm photographs in such areas as forestry, agronomy, wildlife management,

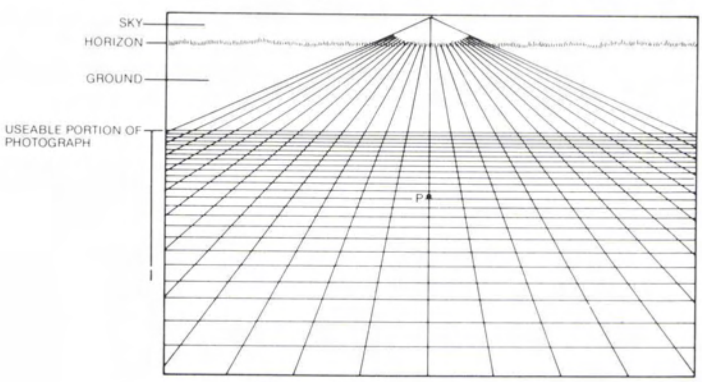


Fig. 8. High oblique perspective grid.

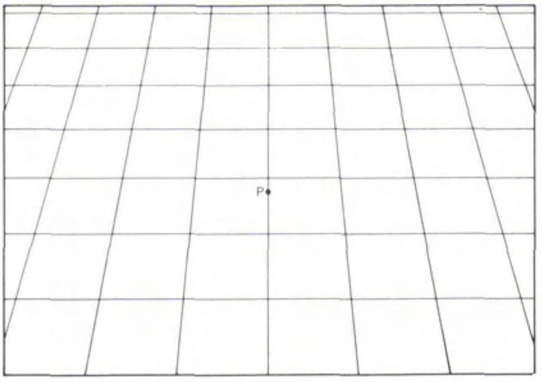


FIG. 9. Low oblique perspective grid.

geology, and environmental monitoring are not addressed herein since they have been more than adequately discussed elsewhere.

The emphasis of the paper is on photo mission and measurement considerations common to most applications of oblique 35-mm aerial photography. A variety of flight mission factors including equipment, photo center line and flight line determination, aircraft height, film exposure, and time requirements are discussed. Also presented are tips on how to ensure a successful photo mission and procedures on how to construct a perspective grid for making photographic measurements.

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